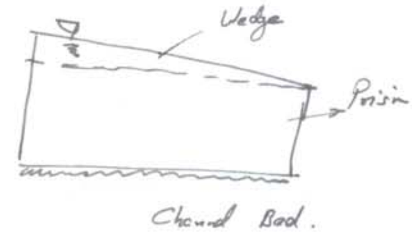
LECTURE 26
15-MARCH-2013

MUSKINGUM METHOD for RIVER ROUTING

→ Yesterday, we started the discussion on river flow routing by lumped methods

→ You have variable discharge - storage relationships for such cases.

→ We assumed that storage of water in the system consisted of



- * Wedge storage
- * Prism storage

→ The ~~dis~~ storage function was

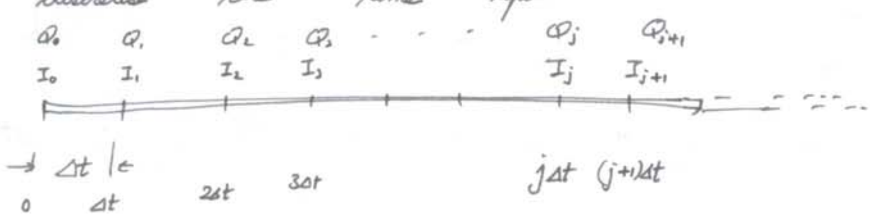
$$S = K\left(XI + (1-X)Q\right)$$

a linear model

where K → proportionality constant $[T^{-1}]$

X → a fraction $0 \le X \le 0.5$

As discussed in level pool routing, again we need to discretise the time span into smaller time steps



∴ Storage in the system between time ~~step~~ $j\Delta t$ and $(j+1)\Delta t$ is given by:

$$S_j = K(x I_j + (1-x) Q_j)$$

$$S_{j+1} = K(x I_{j+1} + (1-x) Q_{j+1})$$

and

$$S_{j+1} - S_j = K(x I_{j+1} + (1-x) Q_{j+1}) - K(x I_j + (1-x) Q_j) \rightarrow (1)$$

Recall continuity equation

$$\frac{dS}{dt} = I - Q$$

From this we arrived at the step:

$$\int dS = \int I\, dt - \int Q\, dt$$

and subsequently

$$S_{j+1} - S_j = \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_j + Q_{j+1}}{2}\Delta t \rightarrow (2)$$

∴ Comparing (1) and (2):, we get

$$K(x I_{j+1} + (1-x) Q_{j+1}) - K(x I_j + (1-x) Q_j)$$

$$= \frac{I_j + I_{j+1}}{2}\Delta t - \frac{Q_j + Q_{j+1}}{2}\Delta t$$

Rearranging the terms we can now write

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j$$

where $C_1 = \dfrac{\Delta t - 2KX}{2K(1-X) + \Delta t}$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X) + \Delta t}$$

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X) + \Delta t}$$

$$\therefore \; C_1 + C_2 + C_3 = 1$$

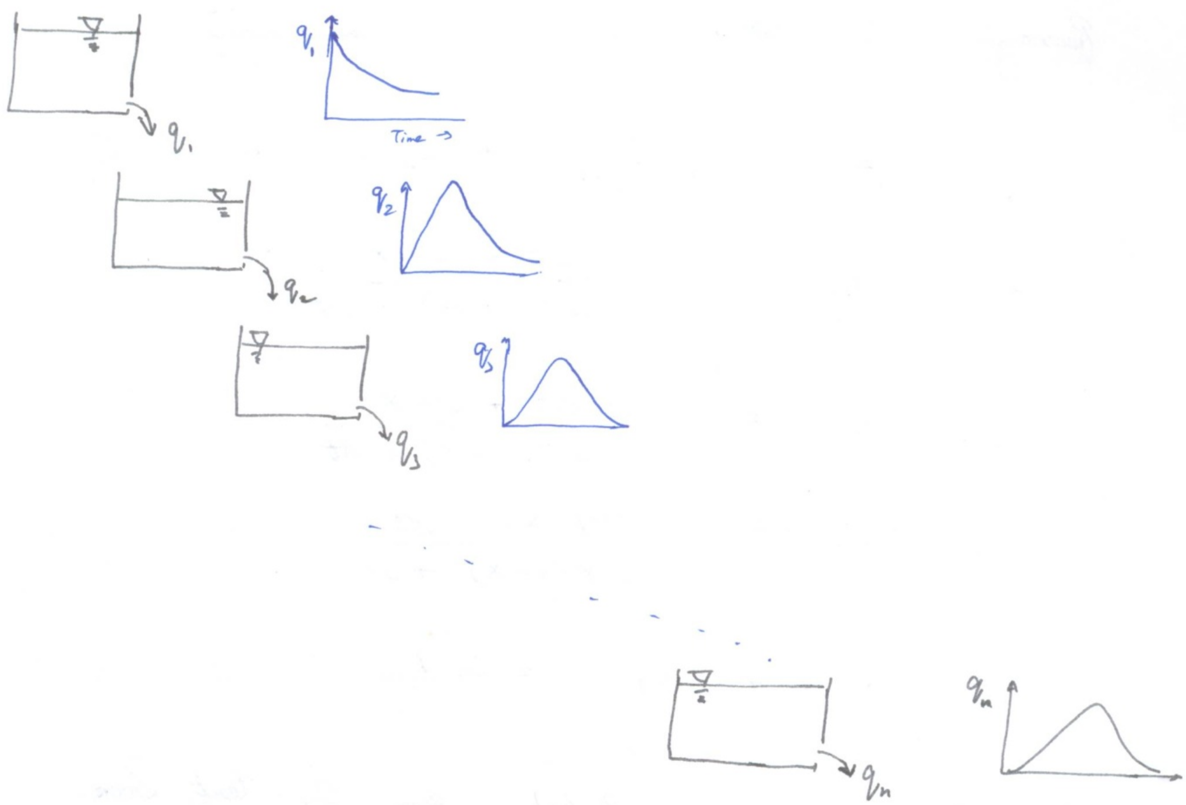
Work out Example 8.4.1 from the text book.

Flow Routing using Linear Reservoir Model

→ There is a method of routing inflow hydrograph using a series of 'n' linear reservoirs.

→ Recall for a linear reservoir $S = kQ$.

→ The entire watershed is approximated by a series of 'n' linear reservoirs



By connecting linear reservoirs in series, you need to find the outflow from the n^{th} reservoir.

→ Then it is routing the inflow that is ~~applied~~ suggested at first linear reservoir.

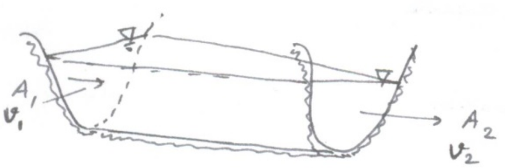
→ We are not going to work out the theory:

DISTRIBUTED FLOW ROUTING

In last chapter we studied hydrologic routing where it is suggested that routing parameters vary only with respect to time. The spatial quantity is lumped.

→ However, in nature we require routing to vary w.r.t space
  ∵ flow rate, velocity, depth, etc. vary in space.

→ For that purpose, we use distributed flow routing. Mostly we will be using distributed flow routing in rivers or channel flows, etc.



For a cross section of a river or channel, we now assume that
- → Average cross sectional velocity in the direction of flow is considered
- → No transverse velocity and vertical velocity.

∴ Predominantly the flow is one-dimensional along the reach of the river.

We use Saint Venant's equations for routing in a one-dimensional flow through channels.

Saint - Venant Equations

To derive Saint ~~Venant~~ Venant's equation, we require following assumptions:

(i) Flow is one-dimensional

(ii) Flow varies gradually along the channel
→ hydrostatic pressure condition can be implemented.

(iii) The longitudinal axis is approximated as a straight line.

(iv) Bottom slope of channel is small or negligible
The channel bed is fixed.

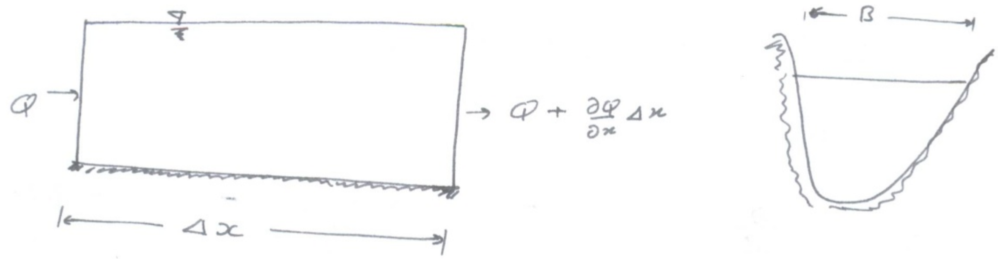
(v) Fluid is incompressible.

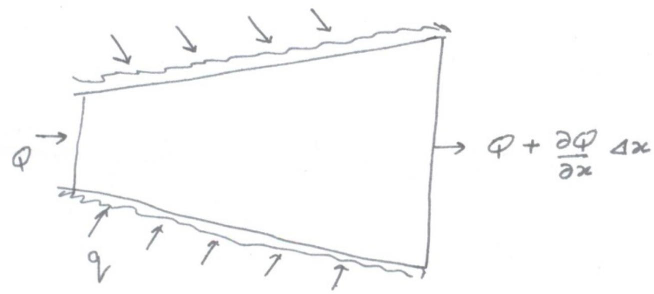
The Continuity Equation

Recall from Reynold's transport theorem, we had arrived at the continuity equation in the following form:

$$\frac{DB}{Dt} = 0 = \frac{d}{dt}\iiint_{cv} \rho \, d\forall + \iint_{c.s} \rho \, \vec{V} \cdot \hat{n} \, dA$$

We will apply this continuity equation to a channel elemental control volume of length Δx.





→ There is an inflow Q to the control volume on the left side

→ There is also lateral inflow entering the c.v.

It is q → the discharge per unit length of the channel. $q \to \frac{L^3}{T\,L}$

∴ The lateral inflow rate is $= q\,\Delta x$

$$\therefore \iint_{c.s} \rho\,\vec{V}\cdot\hat{n}\,dA = \rho\left(Q + \frac{\partial Q}{\partial x}\Delta x\right) - \rho\left(Q + q\Delta x\right)$$

∴ ~~The continuity~~ Also $\frac{d}{dt}\iiint_{cv} \rho\,dV \approx \frac{\partial}{\partial t}\left(\rho A \Delta x\right)$

Volume of channel element $= A\,\Delta x$

$A \to$ average cross sectional area of the element

Also the control volume is fixed in space.

∴ You have now

$$\cancel{\frac{\partial}{\partial t}\iiint} \quad \frac{\partial}{\partial t}(\rho A \Delta x) + \rho\left(Q + \frac{\partial Q}{\partial x}\Delta x\right) - \rho(Q + q\Delta x)$$

i.e. $$\boxed{\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0}$$

This is the conservation form of continuity equation

Can be applied for both

→ Prismatic and

→ Non-prismatic channels.

There is a non-conservation form of continuity equation :

The average flow velocity ~~across~~ in a cross section = V

For a unit width of channel cross section

(Neglect lateral inflow)

A = Depth of flow × 1 = y

$Q = VA = Vy$

Then we have : $$\frac{\partial}{\partial x}(Vy) + \frac{\partial y}{\partial t} = 0$$

or $$V\frac{\partial y}{\partial x} + y\frac{\partial V}{\partial x} + \frac{\partial y}{\partial t} = 0$$

The momentum equation for one-dimensional river flow:

Momentum Equation

$$\sum \vec{F} = \frac{d}{dt} \iiint_{cv} \vec{V} \rho \, dU + \iint_{cs} \vec{V} \rho \, \vec{V} \cdot \hat{n} \, dA$$

For an unsteady non-uniform flow in a channel, the forces acting on the elemental control volume are:

* Gravity force F_g
* Frictional force F_f
* Contraction / Expansion force, F_e
* Wind shear force, F_w
* Net pressure force, F_p

i.e. $$\sum \vec{F} = F_g + F_f + F_e + F_w + F_p$$

The Gravity Force: Volume of fluid $= A \Delta x$

Weight $= \rho g A \Delta x$

For a bed slope S_0 which is very small, you have a component of gravitational force normal to the bed surface:

$$F_g = \rho g A \Delta x \, S_0$$

Frictional force, F_f : The frictional force acts on the sides and bottom of CV.

If $\tau_o \rightarrow$ Bed shear stress

$P \rightarrow$ Wetted perimeter

Then $\tau_o = \rho g \frac{A}{P} S_f$

$S_f \rightarrow$ friction slope from Manning's equation

$$\therefore F_f = -\tau_o P \Delta x$$

$$= -\rho g \frac{A}{P} \cdot P \Delta x \, S_f$$

$$= -\rho g A S_f \Delta x$$