LECTURE 24
13-MARCH-2013

LEVEL POOL ROUTING

In the last day's lecture we were discussing on routing.

* Hydrologic routing
* Hydraulic routing

→ In hydrologic or lumped routing you have

* Horizontal or level pool routing
* Canal routing

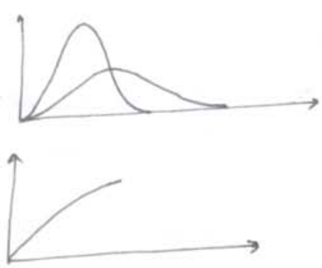
Based on continuity equation

$$\frac{dS}{dt} = I - Q$$

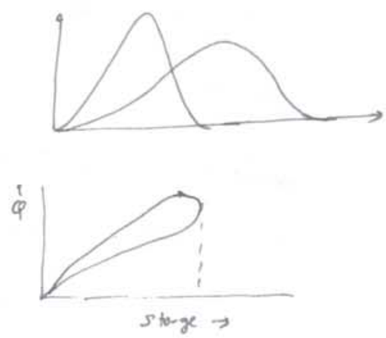
we have seen that we need to find storage function to solve the system differential equation.

$$S = f\left(Q, \frac{dQ}{dt}, \ldots, I, \frac{dI}{dt}, \ldots\right)$$

⇒ You have invariable and variable storage versus discharge relationships.



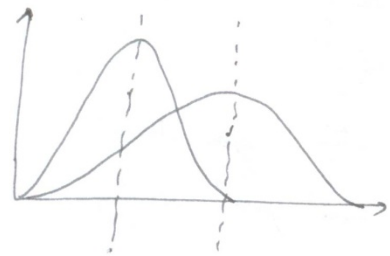
Invariable



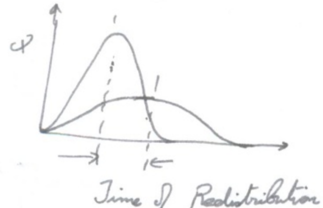
Variable

→ Invariable storage you see in horizontal pools.

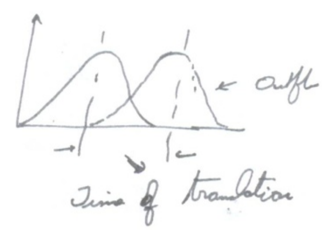
Routing means you are identifying the outflow discharge from inflow. Naturally you are going to change the peak and time to peak.



If there is horizontal pool



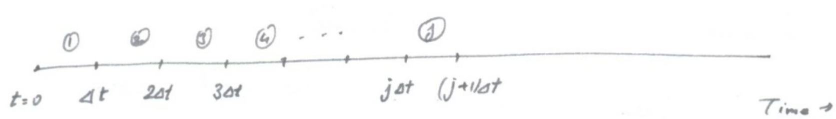
In a canal where the inflow & outflow magnitudes are same



LEVEL - POOL ROUTING

As discussed earlier the method used to route inflow hydrograph in reservoirs or horizontal water surface pools.

⇒ The method suggest that discretise the time domain into small time steps of Δt. This Δt can be the same one used in inflow hydrograph ordinates.



The continuity equation

$$\frac{dS}{dt} = I - Q$$

is now approximated as

$$\int dS = \int I\,dt - \int Q\,dt$$

j	Time	Inflow
1	0	0
2	Δt	I_1
3	$2\Delta t$	I_2
4	$3\Delta t$	I_3
	$4\Delta t$	I_4

At any time interval j:

$$\int_{S_j}^{S_{j+1}} dS = \int_{j\Delta t}^{(j+1)\Delta t} I\,dt - \int_{j\Delta t}^{(j+1)\Delta t} Q\,dt$$

$$S_{j+1} - S_j = \frac{I_j + I_{j+1}}{2}\Delta t - \frac{Q_j + Q_{j+1}}{2}\Delta t$$

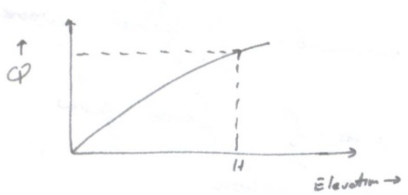
Rearrange the terms and bring all the unknowns on one side:

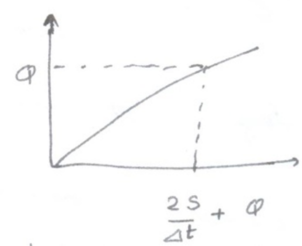
$$\frac{2S_{j+1}}{\Delta t} + Q_{j+1} = \left(I_j + I_{j+1}\right) + \left(\frac{2S_j}{\Delta t} - Q_j\right)$$

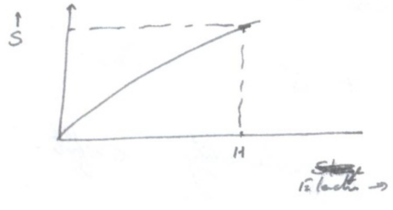
Also we can infer

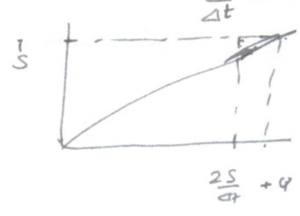
$$\left(\frac{2S}{\Delta t} - Q\right) = \left(\frac{2S}{\Delta t} + Q\right) - 2Q$$

For level pools, you have







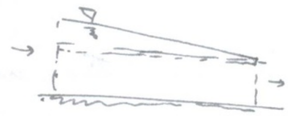


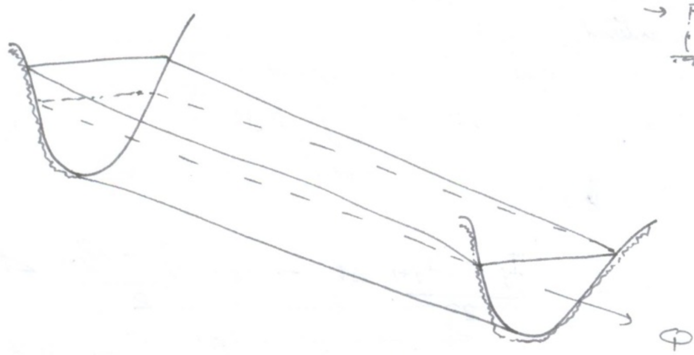
Please go through example problem:

MUSKINGUM METHOD for RIVER ROUTING

→ This is a hydrologic routing method used in rivers or canals.

→ Here you have variable discharge - storage relationships.





Storage in the system consists of

= Prism storage + Wedge storage

Prism storage → constant volume based on constant cross section along the length of channel.

Wedge storage → Due to advancement of flood wave.

Let us assume cross-sectional area of flood flow proportional to discharge at the section

Volume of prism storage = KQ

$K \rightarrow$ proportionality coefft.

Volume of wedge storage = $KX(I - Q)$

$X \rightarrow$ weighing factor $\quad 0 \le X \le 0.5$

$\therefore$ In Muskingum method:

$$S = KQ + Kx(I - Q)$$

i.e. $$S = K\left(xI + (1-x)Q\right)$$

This is a linear model for routing flow in streams.

The method is:

* Discretize time domain 

* $$S_j = K\left(xI_j + (1-x)Q_j\right)$$

$$S_{j+1} = K\left(xI_{j+1} + (1-x)Q_{j+1}\right)$$

$$\therefore S_{j+1} - S_j = K\left(xI_{j+1} + (1-x)Q_{j+1}\right) - K\left(xI_j + (1-x)Q_j\right) \rightarrow (1)$$

Also: $$\int dS = \int I\,dt - \int Q\,dt$$

i.e. $$S_{j+1} - S_j = \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_j + Q_{j+1}}{2}\Delta t \rightarrow (2)$$

Comparing (1) and (2):

$$K\left(xI_{j+1} + (1-x)Q_{j+1}\right) - K\left(xI_j + (1-x)Q_j\right) = \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_j + Q_{j+1}}{2}\Delta t$$

i.e. $$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j$$

where $$C_1 = \frac{\Delta t - 2Kx}{2K(1-x) + \Delta t}$$

$$C_2 = \frac{\Delta t + 2Kx}{2K(1-x) + \Delta t}$$

$$C_3 = \frac{2K(1-x) - \Delta t}{2K(1-x) + \Delta t}$$

Please note that

$$C_1 + C_2 + C_3 = 1.0$$

You may go through example 8.4.1 from the text book.