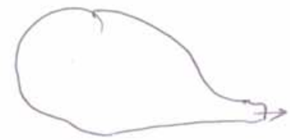
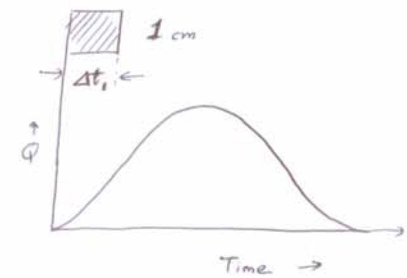
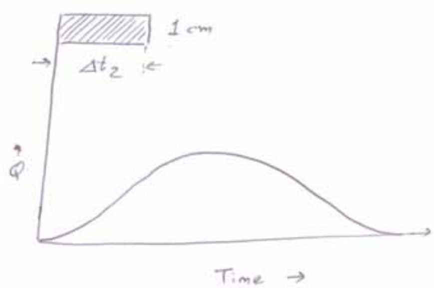
Unit Hydrograph Applications

Suppose we have for a watershed a Δt_1 - hr unit hydrograph





Then is it possible for us to obtain a unit hydrograph for the same watershed having different rainfall duration



We want to find a Δt_2 - hr unit hydrograph for the watershed.

How will we proceed ?

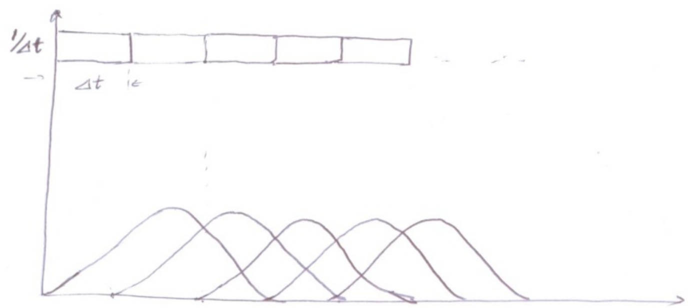
→ We will use a method called S - hydrograph.

S - hydrograph is the step response from a linear watershed that is subjected to an excess rainfall of unit intensity (say 1 cm/hr) for an indefinite period.

(Recall the step response function of a linear system).

The step response and pulse response was related as such:

$$h(t) = \frac{1}{\Delta t}\left[g(t) - g(t-\Delta t)\right]$$



→ Beginning at time $t=0$, if we apply a unit pulse of intensity $1/\Delta t$ for a duration Δt, then you have

$$h(t) = \frac{1}{\Delta t}\left[g(t) - g(t-\Delta t) \right]$$

→ Again we have a response from the second pulse beginning at time Δt to $2\Delta t$

$$h(t-\Delta t) = \frac{1}{\Delta t}\left[g(t-\Delta t) - g(t-2\Delta t) \right]$$

→ Similarly for a pulse from $2\Delta t$ to $3\Delta t$,

$$h(t-2\Delta t) = \frac{1}{\Delta t}\left[g(t-2\Delta t) - g(t-3\Delta t) \right]$$

and so on.

If we super-pose all the pulse responses, we get the response for a continuous input of intensity $1/\Delta t$

i.e. $$h(t) + h(t-\Delta t) + h(t-2\Delta t) + \cdots$$

$$= \frac{1}{\Delta t}\left[g(t) - g(t-\Delta t) \right] + \frac{1}{\Delta t}\left[g(t-\Delta t) - g(t-2\Delta t) \right] + \cdots$$

i.e. $$h(t) + h(t-\Delta t) + h(t-2\Delta t) + \cdots = \frac{1}{\Delta t}\, g(t)$$

Or if we use a continuous step input of intensity 1 cm/hr, then

$$g(t) = \Delta t \left[h(t) + h(t - \Delta t) + \cdots \right]$$

→ Theoretically this $g(t)$ is the same S-hydrograph.

It should be smooth.

However if observational data are used, then it may be not that smooth.

→ That is we first construct the S-hydrograph.

Then the S-hydrograph is shifted to required duration say Δt_1 hour.

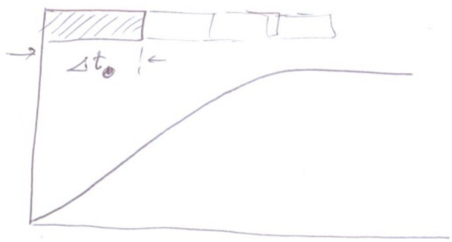
i.e. $$g_1(t) = g(t - \Delta t_1)$$

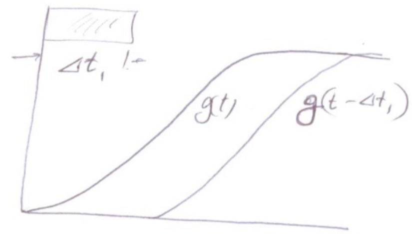
We can obtain the unit hydrograph of Δt_1-hour as such:

$$h_1(t) = \frac{1}{\Delta t_1} \left(g(t) - g(t - \Delta t_1) \right)$$

~~Example~~: The S-Hydrograph Method

* Construct S-hydrograph from unit hydrograph of duration Δt. (i.e. from Δt - Unit hydrograph)
* Move the S-hydrograph by Δt_1 time
* So you get $g_1(t) = g(t - \Delta t_1)$
* Subtract both hydrographs at each instant time t.





$$h_1(t) = \frac{1}{\Delta t_1}\left[g(t) - g(t-\Delta t_1) \right]$$

Example.

Unit hydrograph in $\frac{m^3}{s} \cdot \frac{1}{cm}$

From a 0.5 hr unit hydrograph, determine 1.5 hr unit hydrograph for a watershed. The data are:

Time (h)	0.5 hr UH ($m^3/s \cdot cm$)	S-hydrograph $g(t)$ (m^3/cm)	S-hydrograph $g(t-1.5h)$ (m^3/cm)	1.5 hr UH ($\frac{m^3}{s} \cdot \frac{1}{cm}$)
0.5	4.513	8123.4	–	1.504
1.0	12.053	29818.8	–	5.522
1.5	26.172	76928.4	–	14.246
2.0	27.993	127315.8	8123.4	22.073
2.5	16.309	156672.0	29818.8	23.491
3.0	5.060	165780.0	76928.4	16.454
3.5	4.256	173440.8	127315.8	8.542
4.0	3.061	178950.6	156672.0	4.126
4.5	1.932	182428.2	165780.0	3.083
5.0	0	182428.2	173440.8	1.664
5.5		182428.2	178950.6	0.644
6.0		182428.2	182428.2	0
6.5				
7.0				

→ We need to shift S-hydrograph by 1.5 hrs.

→ The 1.5 hr unit hydrograph is developed like that.

Instantaneous Unit Hydrograph

Recall the impulse response function of a linear system.
Similarly if we have a unit amount of input, that is impulsively applied to the watershed (say 1 cm of excess rainfall) – the corresponding response function is instantaneous unit hydrograph (IUH).

→ Please note that this is a theoretical concept.

→ However, IUH's are useful – because they characterise the response of the watershed without taking into account rainfall duration.

Recall convolution integral

$$Q(t) = \int_0^t u(t-\tau)\, I(\tau)\, d\tau$$

If $Q(t)$ and $I(t)$ are having the same dimensions then $u(t-\tau)$ is having dimension $[T^{-1}]$.

$$0 \le u(l) \le \text{some positive peak value} \quad ; \text{ for } l > 0$$

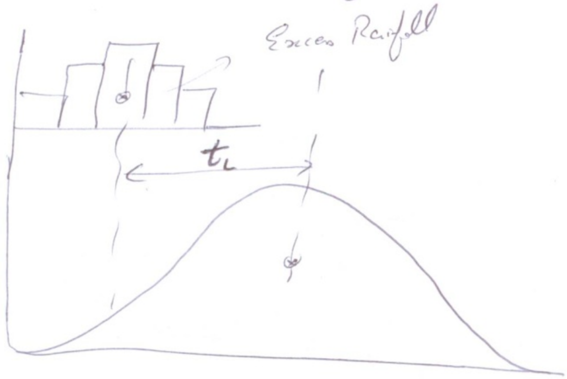
$$u(l) = 0 \quad ; \text{ for } l \le 0$$

$$u(l) \to 0 \quad \text{as } l \to \infty$$

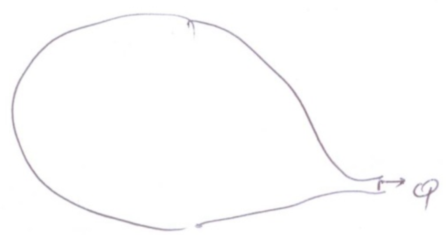
i.e. $$\int_0^\infty u(l)\, dl = 1$$

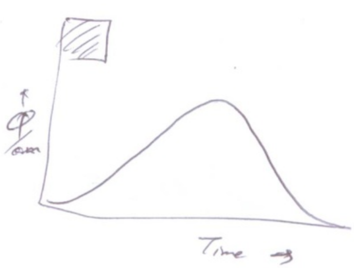
Also. $$\int_0^\infty u(l)\, l\, dl = t_L$$ → the time lag of the IUH.

This time lag actually means that difference in time of the centroids of excess rainfall hyetograph and direct runoff hydrograph.



→ From this study on unit hydrographs, it is clear to you





UH is a deterministic, lumped, unsteady hydrological model.