LECTURE 21

06-MARCH-2013

The Unit Hydrograph

Recall in the last class we discussed on discrete form of convolution equation based on pulse response function of the linear hydrologic system.

That was $$Q_n = \sum_{m=1}^{n \leq M} P_m U_{n-m+1}$$

where U_i ; where the discrete values of pulse response function.

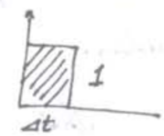
The Unit Hydrograph → is a unit pulse response function of a linear hydrologic system.
The method was proposed by Sherman (1932).
"Unit hydrograph is the direct runoff hydrograph resulting from 1 unit depth (say 1 cm) of excess rainfall precipitated uniformly over the drainage area (or watershed area) at a constant rate for an effective duration".

Q: What is the purpose of unit hydrograph ?
S: As said earlier UH is an unit pulse response function of a linear hydrologic system.
Therefore, using this unit response, one can

generate direct runoff for various rainfall depths from the watershed.

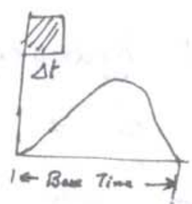
Basic Assumptions in UH Model

→ Excess rainfall suggested should be of constant intensity within the effective duration



→ Excess rainfall is uniformly distributed throughout the whole drainage area

→ The base time of direct runoff (DRH) resulting from an excess rainfall of pulse time width Δt is constant.



→ The watershed in consideration is assumed to remain unchanged.

This UH model is applicable only to small watersheds (0.5 to $25\ km^2$).

UH is time invariant

Same response from model

Unit hydrograph as said earlier is unit pulse response function.

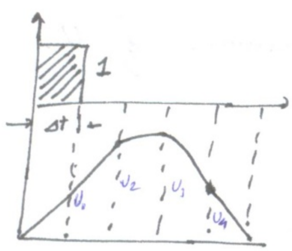
Similarly, we have step response function called S - hydrograph

→ impulse response function called instantaneous unit hydrograph.

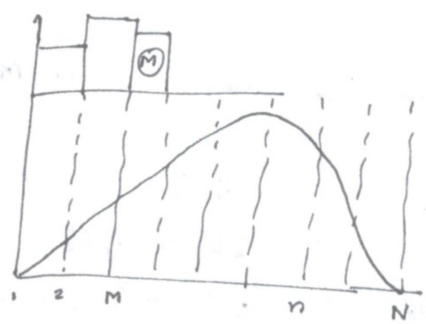
To Derive Unit Hydrograph

Recall the discrete convolution equation

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$



Time.



Time

Suppose if we have the measurements and for a storm event having M pulses it was observed there were measurements for discharge Q_n,

$n = 1, 2, 3 \ldots, N$.

Beyond N there is no discharge due to that storm

⇒ Then using deconvolution we can derive the ordinates of unit hydrograph $U_1, U_2, \ldots$

Q: For the given observation how many ordinates will be there for unit hydrograph (i.e $U_1, U_2, \ldots$)?

S: The number of observations for discharge

Q_n ; $n = 1, 2, 3, \ldots N$

The number of rainfall pulses $= M$

$\therefore$ ~~The~~ ~~last~~ For $N > M$ (this has to be always)

the response from M^{th} pulse begins at $M-1$ and up to N.

i.e. $U_1, U_2, U_3, \ldots, U_{N-M+1}$

$\Rightarrow$ So we have $N-M+1$ unknowns of U.

$n=1$; $Q_1 = P_1 U_1$

$n=2$; $Q_2 = P_2 U_1 + P_1 U_2$

$n=3$; $Q_3 = P_3 U_1 + P_2 U_2 + P_1 U_3$

$\vdots$

$n=M$; $Q_M = P_M U_1 + P_{M-1} U_2 + \cdots + P_1 U_M$

$n=M+1$; $Q_{M+1} = 0 + P_M U_2 + P_{M-1} U_3 + \cdots + P_2 U_M + P_1 U_{M+1}$

$\vdots$

$n=N-1$; $Q_{N-1} = 0 + 0 + 0 + \cdots + P_M U_{N-M} + P_{M-1} U_{N-M+1}$

$n=N$; $Q_N = 0 + 0 + \cdots + P_M U_{N-M+1}$

Example

Find the half-hour unit hydrograph for the given data

	Time (Pulse) (h)	Excess Rainfall (cm)	Direct Runoff (m^3/s)
(1)	0-0.5	2.7	12.14
(2)	0.5-1.0	4.9	54.56
(3)	1.5	4.6	150.29
(4)	2.0	-	259.07
(5)	2.5		301.46
(6)	3.0		222.27
(7)	3.5		111.25
(8)	4.0		52.38
(9)	4.5		39.78
(10)	5.0		23.55
(11)	5.5		8.88

Solution.

Here in the given data $M = 3$

$N = 11$

$\therefore$ Number of coordinates for UH; $N - M + 1 = 11 - 3 + 1 = \underline{9}$

i.e. $U_1, U_2, U_3, \ldots, U_9$

So we have now 9 unknowns.

$n=1;\quad Q_1 = 12.14 = P_1 U_1$

$\therefore U_1 = \dfrac{12.14}{2.7 \times 10^{-2}} = 449.63 \ m^3/s$

$n=2;\quad Q_2 = 54.56 = P_2 U_1 + P_1 U_2$

$\therefore U_2 = 1204.75 \ m^3/s$

$n=3;\quad Q_3 = 150.29 = P_3 U_1 + P_2 U_2 + P_1 U_3$

n ; $\quad U_3 =$

$n=4;\quad Q_4 = 259.07 = 0 + P_3 U_2 + P_2 U_3 + P_1 U_4$

$\therefore\ U_4 =$

$n=5;\quad Q_5 = 301.46 = 0 + 0 + P_3 U_3 + P_2 U_4 + P_1 U_5$

$\therefore\ U_5 =$

Similarly find U_6, U_7, U_8, U_9.

Unit Hydrograph Application

What is the use of unit hydrograph?

Ans: It is clear to you unit hydrograph is unit pulse response of a watershed.

It is a characteristic of a watershed.

$\therefore$ On obtaining unit hydrograph for a particular duration storm for a watershed (i.e. say ½-hr UH, or 1-hr U.H, 2-hr UH, ... like this).

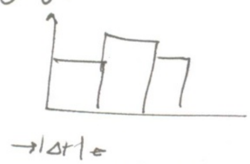
you can determine direct runoff hydrograph for any rainfall of that duration.

Again by incorporating base flow, you get streamflow hydrographs.

Q: If you are given actual rainfall hyetograph and same (Δt hr) unit hydrograph, how will you compute streamflow hydrograph?

Ans. * From the rainfall hyetograph deduct abstractions to obtain excess rainfall hyetograph.

* Correlate the time interval of excess rainfall hyetograph and unit hydrograph



$\rightarrow|\Delta t|\leftarrow$

Excess Rainfall Hyetograph

$\rightarrow|\Delta t|\leftarrow$

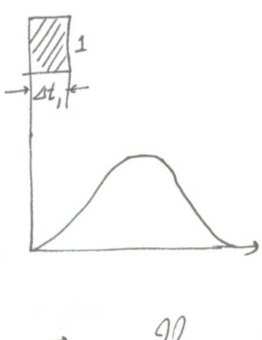
Unit Hydrograph.

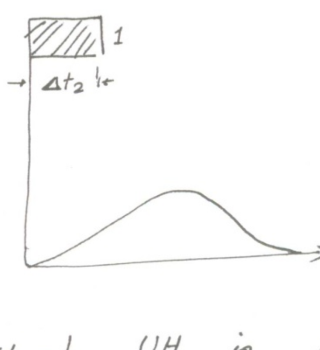
* Use discrete convolution equation.

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$

* This will give ordinates of direct runoff hydrograph.
* Add baseflow to DRH to obtain streamflow hydrograph.

Unit Hydrographs for Different Rainfall Durations





→ If a ~~unit~~ Δt_1-hr UH is given to you for a particular watershed, then you can derive Δt_2-hr Unit hydrograph for that particular watershed.

→ This is done by S- hydrograph method.

S- hydrograph is the step response from a linear watershed that is subjected to an excess rainfall of intensity (say 1 cm/hr) for an indefinite period.