LECTURE - 17
15 - FEB - 2010

SURFACE - WATER SYSTEM

In the last class we have briefly seen that on the surface of earth, the flow of water can be

→ Overland flow
→ Channel flow

We have also seen the concept of → travel time
→ Time of concentration, etc.

When we discussed on overland flow and channel flow → the flow processes were considered to be in steady state.

While recalling our earlier lectures - we have applied RTT for various hydrologic processes → atmospheric, sub-surface, surface
→ So we will stick to the same principles hereafter.
→ Now as the hydrological processes can be modeled ~~with~~ using systems concept with randomness, space, and time as variability for the hydrologic parameters.

→ In most of the cases we used (especially for surface water flow) - steady state conditions.

→ That is when you applied Manning's equation for uniform flow in a channel:

$$Q = \frac{1}{n} A R^{2/3} S_o^{1/2}$$

This is a ~~stead~~ deterministic, lumped, steady flow model.

→ Now in the coming chapters again we will deal with situations that are unsteady and/or distributed in nature.

A General Hydrologic System Model

When you applied continuity equation to an hydrologic system using RTT

$$\frac{DM}{Dt} = 0 = \frac{d}{dt}\iiint_{cv} \rho \, dU + \iint_{cs} \rho \, \vec{v}\cdot\hat{n} \, dA$$

$$\iiint_{cv} \rho \, dU = \text{Storage} = \rho S$$

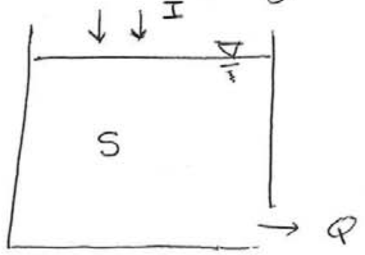
$$\iint_{cs} \rho \, \vec{v}\cdot\hat{n} \, dA = \text{Outflow} - \text{Inflow}$$

As water is incompressible.

we arrived at the relation

$$\frac{dS}{dt} = I - Q$$

Consider a hypothetical reservoir



There is inflow and outflow from reservoir as well as storage of water in the reservoir.

Here, the inflow to the reservoir can vary with respect to time. Similarly outflow from the reservoir can also vary w.r.t time.

→ Definitely the storage of water can rise or fall in the reservoir w.r.t time and is $S(t)$.

→ Also change in storage is due to time variations in inflow and outflow.

∴ In a general form we can suggest

$$S = f\left(I, \frac{dI}{dt}, \frac{d^2I}{dt^2}, \ldots, Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}, \ldots\right) \rightarrow (1)$$

↘ This is storage function

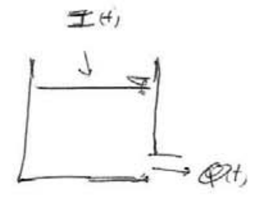
⇒ Now consider the situation where

$S = kQ \rightarrow (2)$ where k is a constant.

Equation (2) definitely belongs to a particular category of Equation (1).

$S = kQ$ describes a linear reservoir.

Linear System in Continuous Time



We know

$$\frac{dS}{dt} = I(t) - Q(t)$$

Also earlier we suggested

$$S = f\left(I, \frac{dI}{dt}, \frac{d^2I}{dt^2}, \ldots\ldots, Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}, \ldots\ldots\right)$$

In mathematical courses you have already seen that if $A = f(B, C, D)$ then $A = m_1 B + m_2 C + m_3 D$ is a linear.

$$\text{lly} \quad S = a_1 Q + a_2 \frac{dQ}{dt} + a_3 \frac{d^2Q}{dt^2} + \cdots + a_n \frac{d^{n-1}Q}{dt^{n-1}}$$
$$+ b_1 I + b_2 \frac{dI}{dt} + b_3 \frac{d^2I}{dt^2} + \cdots\cdots + b_m \frac{d^{m-1}I}{dt^{m-1}}$$

in which $a_1, a_2, a_3, \ldots, a_n, b_1, b_2, b_3, \ldots, b_m$ are constants and derivatives $> \frac{d^{n-1}Q}{dt^{n-1}}$ as well as $> \frac{d^{m-1}I}{dt^{m-1}}$ are neglected.

→ If you can relate as shown above, then the storage function describes a linear system.

→ That is we expressed storage as linear equation with constant coefficients.

→ As the coeffts $a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_m$ are constants — the system now becomes time-invariant.

→ That is the way the system processes input ~~and~~ to output does not change with time.

→ Now differentiate this linear equation of S

$$\therefore \frac{dS}{dt} = a_1 \frac{d\varphi}{dt} + a_2 \frac{d^2\varphi}{dt^2} + \cdots + b_1 \frac{dI}{dt} + b_2 \frac{d^2 I}{dt^2} + \cdots$$

" " " " "

i.e. $I - \varphi =$

or re-arranging

$$a_n \frac{d^n\varphi}{dt^n} + a_{n-1} \frac{d^{n-1}\varphi}{dt^{n-1}} + \cdots + a_1 \frac{d\varphi}{dt} + \varphi$$

$$= -\left[b_m \frac{d^m I}{dt^m} + b_{m-1} \frac{d^{m-1} I}{dt^{m-1}} + \cdots + b_1 \frac{dI}{dt} + I \right]$$

i.e. $[\quad]\,\varphi = [\quad]\,I$

↘ Some function of differential operators on φ — $N(D)$

⇒ Some function of diff. operators on I. — $M(D)$

$$\therefore \varphi = \left[\frac{M(D)}{N(D)}\right] I(t)$$

where $\frac{M(D)}{N(D)}$ → transfer function of the system.

(Response of the output to a given input sequence).

Now recall linear reservoir

$$S = k Q.$$

In this case $a_1 = k$ and

$$a_2, a_3, \ldots, a_n, b_1, b_2, \ldots, b_m = 0$$

$$\therefore \quad \frac{dS}{dt} = I - Q$$

$$\text{i.e.} \quad k \frac{dQ}{dt} = I - Q$$

$$\text{or} \quad k \frac{dQ}{dt} + Q = I$$

Solve this equation to get Q.

Response Functions of Linear Systems

We have seen for a linear system

$$\varphi(t) = \left[\frac{M(D)}{N(D)}\right] I(t)$$

→ In these cases we are suggesting that I, φ, and S are not varying w.r.t space.

→ They are all lumped spatially.

→ They vary only w.r.t time (unsteady).

∴ We are now going to study deterministic, lumped, unsteady models.

In mathematics you have studied that:

→ For solving linear differential equations

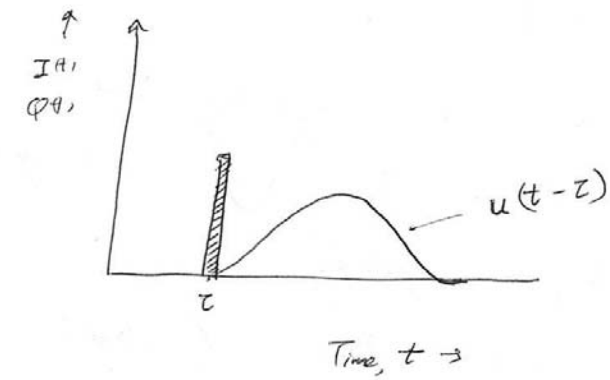
* If some $f(\varphi)$ is a solution for the linear diff. equation, then if you multiply $f(\varphi)$ with a scalar c, i.e. $c\,f(\varphi)$ is also a solution (Principle of Proportionality).

* If two solutions are there for diff equation i.e. $f_1(\varphi)$ and $f_2(\varphi)$ then $f_1(\varphi) + f_2(\varphi)$ is also a solution (Principle of Superposition).

Impulse Response Function

→ For the linear system there are certain characteristics

→ It has an unique response to an unit impulse



For a unit impulse at time τ, the response is $u(t-\tau)$.