Lecture 11 **30-January-2013**

Green – Ampts Infiltration

Green - Ampt Method

If you see in Horton's equation $f = f_c + (f_0 - f_c)e^{-kt}$

and Philip's equation $\cancel{F(t)}$ $f(t) = \frac{1}{2} S t^{-1/2} + K$,

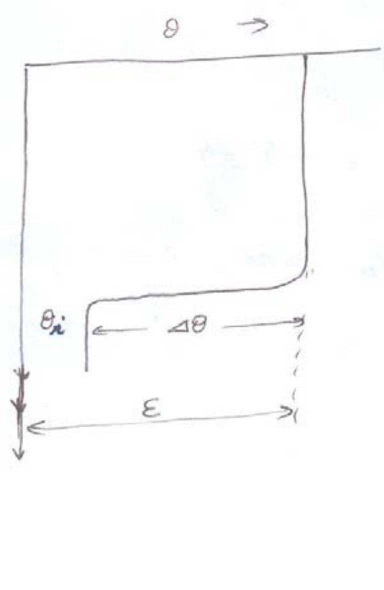
they are based on approximating the solutions to the Richards equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial \theta}{\partial z} + K\right)$$

→ This Richard's equation is a more accurate representation of unsaturated flow in soils. It incorporates many properties and to solve this equation is not that easy.

→ Instead of approximating the solutions, if we can approximate the theory of flow, but that have an accurate solution → How to be done?

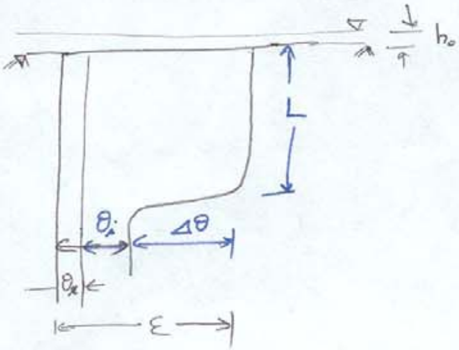
Green and Ampt suggested moisture profile as follows:



→ There is a sharp wetting front

→ Above wetting front fully saturated

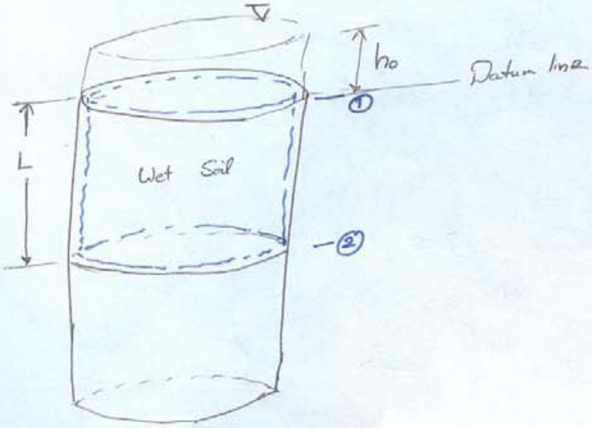
→ Below front, whatever initial moisture content θ_i



Water is ponded to a small depth h_0 on the soil surface.

* Water front penetrated to depth L from ground surface.

Let us consider a vertical column of soil (unit area).



~~total~~
We are considering the control volume in the column.
→ This is between wetting front and ground surface.

Increase in water stored in CV due to infiltration $= L(\varepsilon - \theta_i) = L\,\Delta\theta$

$= F(t)$

Recall Darcy's law: $q = -K \frac{\partial h}{\partial z}$

The direction of q taken earlier was +ive upwards

$$q = -f$$

$$\therefore f \approx K \frac{\Delta H}{\Delta z}$$

i.e. $f = K \dfrac{H_1 - H_2}{z_1 - z_2}$

Q: What is H_1 ? $\rightarrow$ $H_1 = h_o$

What is H_2 ? $\rightarrow$ H_2 = Suction head + Datum head

$= \psi - L$

Suppose if we consider for the case of infiltration only the magnitude of suction head as ψ (because it causes suction of water in the downward direction.

$$H_2 = -\psi - L$$

$$\therefore f = \frac{K\left[h_o - (-\psi - L)\right]}{L}$$

If $h_o \approx 0.0$, then $f \approx K \dfrac{\psi + L}{L}$

Q: What is the infiltrated depth L ?

The cumulative infiltration, $F = L\,\Delta\theta$

$$\therefore L = \frac{F}{\Delta\theta}$$

$$\therefore f = K \frac{\psi + F/\Delta\theta}{F/\Delta\theta} = K \frac{\psi\Delta\theta + F}{F} = \frac{dF}{dt}$$

(Infiltration Theory)

$\therefore$ To solve, use method of separation.

$$\frac{F}{F + \psi\Delta\theta}\, dF = K\,dt$$

(Please note that we are using the magnitude of suction head in ψ. Actually ψ will be negative ~~+~~ -ive in the expression however, it is +ive.)

$$\int_0^{F(t)} \left[\left(\frac{F + \psi\Delta\theta}{F + \psi\Delta\theta}\right) - \left(\frac{\psi\Delta\theta}{F + \psi\Delta\theta}\right)\right] dF = \int_0^t K\,dt$$

i.e. $F(t) - \psi\Delta\theta \left[\ln\left(F(t) + \psi\Delta\theta\right) - \ln\left(\psi\Delta\theta\right)\right] = Kt$

$$F(t) - \psi\Delta\theta \ln\left(1 + \frac{F(t)}{\psi\Delta\theta}\right) = Kt$$

This is cumulative infiltration expression using Green-Ampt's method.

$$f(t) = \frac{dF}{dt} = K\left[\frac{\psi\Delta\theta}{F} + 1\right]$$

⇒ The cumulative infiltration expression is non-linear and implicit. You need to use either

→ Method of Bisection
→ Method of successive approximation
→ Newton's- Raphson method, etc. to solve for F, etc.

Properties of GA Parameters

The parameters are: → Hydraulic conductivity K
Porosity, ε
Wetting front soil suction head ψ
(Only magnitude)

Various scientists have studied the relationships between moisture content θ and suction head ψ as well as ~~[illegible]~~ θ and K.

In soils you can define the term

Saturation, $\sigma = \dfrac{\text{Volume of water}}{\text{Volume of voids}}$

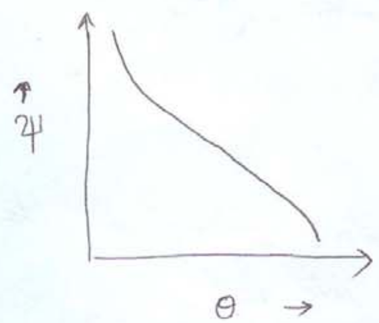
$$0 < \sigma < 1.0$$

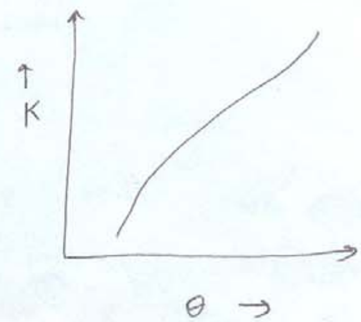
$$\therefore \theta = \sigma \varepsilon$$

Effective saturation, $\sigma_e = \dfrac{\theta - \theta_r}{\varepsilon - \theta_r}$ $\left(\text{or} = \dfrac{\sigma - \sigma_r}{1 - \sigma_r} \right)$

$\theta_r \rightarrow$ Residual moisture content

There are
→ Brooks and Corey relations, etc.
→ Van Genuchten relations, etc

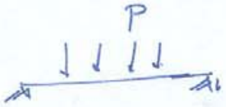




Q: If the soil ~~layer~~ considered for infiltration have multiple layers, how the infiltration equation looks?
→ We are going to ask you in TUTORIAL.

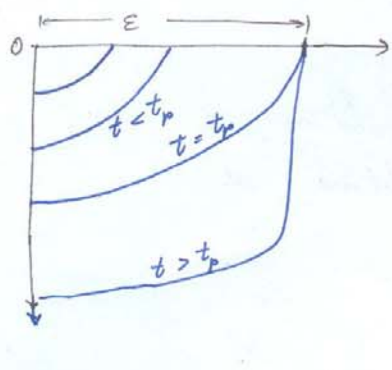
Infiltration due to Ponding

→ Earlier we assumed ponding depth $h_o \approx 0.0$
→ What happens if it is not negligible?



* During rainfall, if the intensity of rain > infiltration capacity of soil → ponding occurs.

* Ponding will start only after certain time when the top portion gets completely saturated.
* This time is called ponding time t_p.



Consider a dry soil

We can find an expression for ponding time.
(Mein and Larson, 1973)

We are using G A model.

There are three principles

(i) ~~Bea~~ Before ponding of water occurs, all the rainfall water is infiltrated into soil.

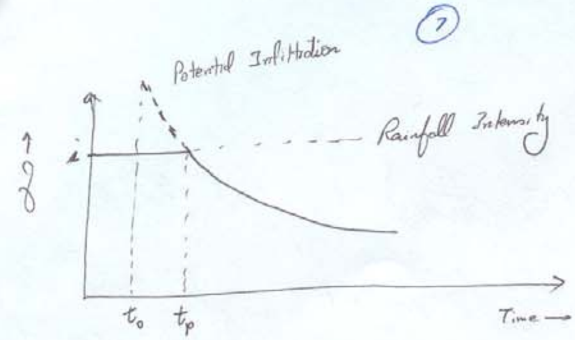
(ii) The potential infiltration rate is function of cumulative infiltration.

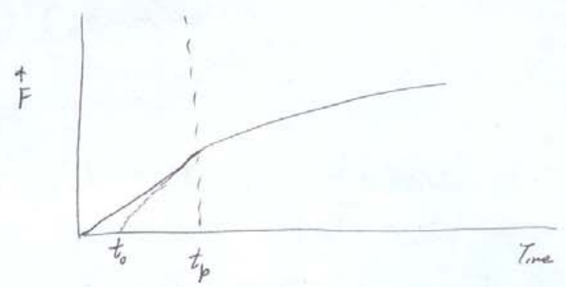
(iii) Ponding occurs when potential infiltration < rainfall intensity.

$$f = K\left(\frac{\psi\Delta\theta}{F} + 1\right)$$

$\Rightarrow$ Before ponding, $f = i = K\left(\frac{\psi\Delta\theta}{F} + 1\right)$

Now $F = it$, $\therefore$ $i = K\left[\frac{\psi\Delta\theta}{it} + 1\right]$





→ From the diagram here

At the ponding time beginning.

$$F = i t_p$$

$$\therefore i = K\left(\frac{\psi\Delta\theta}{i t_p} + 1\right)$$

or $$\boxed{t_p = \frac{K \psi\Delta\theta}{i(i-K)}}$$

→ This is ponding time for constant rainfall.

Q: How will you obtain infiltration rate after ponding?

→ Let us extend the cumulative infiltration curve from t_p backwards. It intersects at time $t_0 < t_p$.

→ If the infiltration rate curve f vs t is also extended backwards upto time t_0, then the theoretical explanation can be given.

From $t_0 < t < t_p$ → Assumed Infiltration occurs at potential infiltration rate f

→ The amount infiltrated during $t_0 < t < t_p$ should be equal to it

$\therefore$ At $\Delta t = t_p - t_0$, $F = F_p$ (ie. within the time t_0 to t_p)

Using the curved infiltration rate fig:

$$F_p - \psi\Delta\theta \ln\left(1 + \frac{F_p}{\psi\Delta\theta}\right) = K(t_p - t_0)$$

Note: $F_p = i t_p$

In $t > t_p$,

$$F - 2\psi\Delta\theta \ln\left(1 + \frac{F}{2\psi\Delta\theta}\right) = K(t - t_0)$$

$$\therefore \quad F - F_p - 2\psi\Delta\theta\left[\ln\left(\frac{2\psi\Delta\theta + F}{2\psi\Delta\theta}\right) - \ln\left(\frac{2\psi\Delta\theta + F_p}{2\psi\Delta\theta}\right)\right] = K(t - t_p)$$

Note: $\underline{\underline{F_p}} = i t_p$.

or $$F - F_p - 2\psi\Delta\theta \ln\left[\frac{2\psi\Delta\theta + F}{2\psi\Delta\theta + F_p}\right] = K(t - t_p)$$

$$\sim F = i t_p + 2\psi\Delta\theta \ln\left[\frac{2\psi\Delta\theta + F}{2\psi\Delta\theta + i t_p}\right] + K(t - t_p)$$