**07/03/2017**

**Lecture 21**

# Conservation of Linear Momentum using RTT

- Before mid-semester exam, we have seen the
  1. Derivation of Reynolds Transport Theorem (RTT),
  2. Application of RTT in the Conservation of Mass principle
  3. RTT was written in the form:

$$\left.\frac{DB}{Dt}\right|_{system} = \frac{d}{dt}\iiint_{CV} \beta\rho dU + \iint_{CS} \beta\rho(\vec{v}.\hat{n})dA$$

If the control volume is moving, then

$$\left.\frac{DB}{Dt}\right|_{system} = \frac{d}{dt}\iiint_{CV} \beta\rho dU + \iint_{CS} \beta\rho(\vec{v}_r.\hat{n})dA$$

Where $\vec{v}_r$ is relative velocity = $= \vec{v} - \vec{v}_s$

In index notation, for a deformable and moving control volume, we can write:

$$\left.\frac{DB_{ij....}}{Dt}\right|_{system} = \frac{d}{dt}\iiint_{CV} \beta_{ij....}\rho dU + \iint_{CS} \beta_{ij....}\rho v_{r_k} n_k dA$$

- RTT can be applied for the principle of conservation of linear momentum.
  - In this case, the extensive property will be the linear momentum ( $m\vec{v}$ ) and the corresponding intensive property will be $\vec{v}$
    i.e., B = $m\vec{v}$ and β = $\vec{v}$

$$\left.\frac{D(m\vec{v})}{Dt}\right|_{system} = \frac{d}{dt}\iiint_{CV} \vec{v}\rho dU + \iint_{CS} \vec{v}\rho(\vec{v}_r.\hat{n})dA$$

In physics, you have studied Newton's second law, which states that the rate of change of linear momentum in a system is the net force acting on it. So,

$$\left.\frac{D(m\vec{v})}{Dt}\right|_{system} = \sum\vec{F} = \frac{d}{dt}\iiint_{CV} \vec{v}\rho dU + \iint_{CS} \vec{v}\rho(\vec{v}_r.\hat{n})dA$$

- We should note that:
    - $\vec{v}$ is velocity relative to an inertial non-accelerating co-ordinate system.
    - $\sum \vec{F}$ is the vector sum of all forces acting on the system material (consists of surface forces and body forces).

- The body forces can be gravity force, electromagnetic force, etc.
- The surface forces are pressure force, viscous force, etc.

## The Pressure Force

- Recall, in the chapter on hydrostatics, we described about the pressure force.
- The force due to pressure is always perpendicular and inwards to the plane, that is considered.
- So, if you have a control volume of any shape, then the net pressure force acting on the surface of the control volume will be:

$$\vec{F}_{pres} = \iint_{CS} p(-\hat{n})dA$$

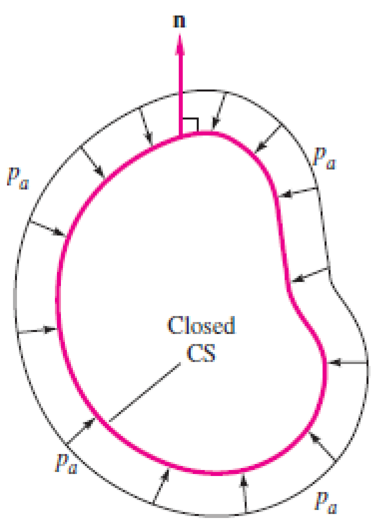


Fig.1: Uniform pressure force on a body
(Source: Fluid Mechanics by F.M. White)

- On a closed surface, if the pressure has uniform value (or magnitude) $p_a$ , then the net pressure force

$$\vec{F}_{pres} = \iint_{CS} p(-\hat{n})dA = -p\iint_{CS} \hat{n}dA = 0$$

(This result is independent of the shape of the control surface).

For example, a control volume formed by enclosed surface, where outside uniform atmospheric pressure $p_{atm}$ acts,

Then

$$\vec{F}_{pres} = \iint_{CS} p_{atm}(-\hat{n})dA = 0$$

**Example: (Adopted from FM White's Fluid Mechanics)**

A nozzle is used to control the water exit and is of the following shape. The water pressure at the entrance of the nozzle is 280 kPa. The atmospheric pressure is 103 kPa. Diameter at section 1 is $D_1$=7 cm, and at section 2 is $D_2$=2.5 cm. Estimate the net pressure force.

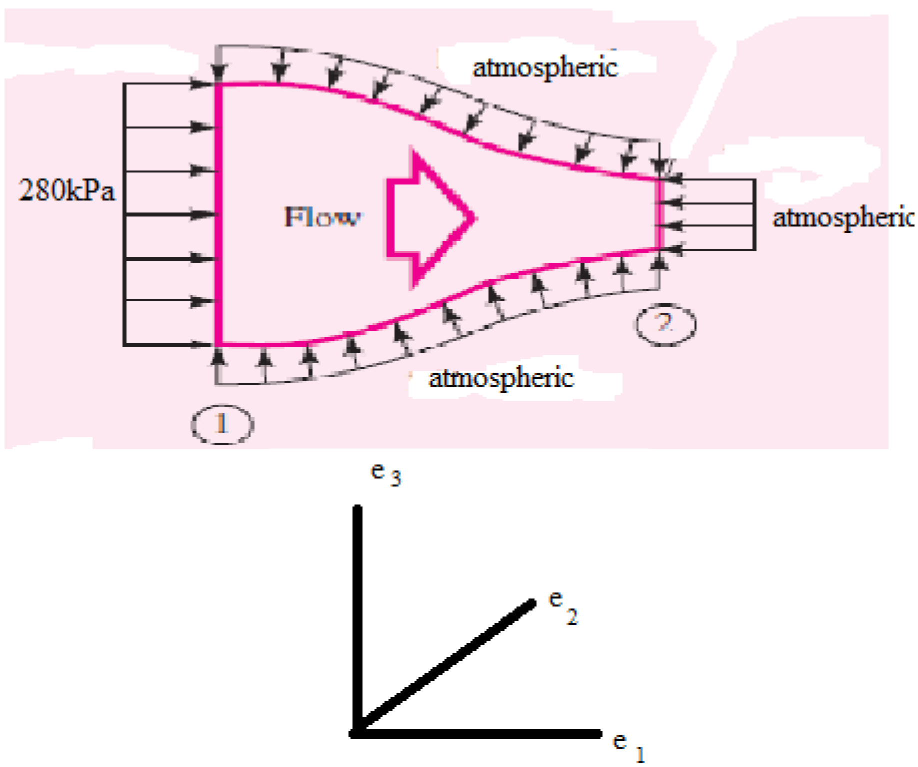


Fig. 2: Problem Statement and reference coordinate axes

(Source: Fluid Mechanics by F.M. White)

**Solution:** We need to first frame a suitable control volume (as good as the free body diagram you have studied in solid mechanics).

The control volume is shown:

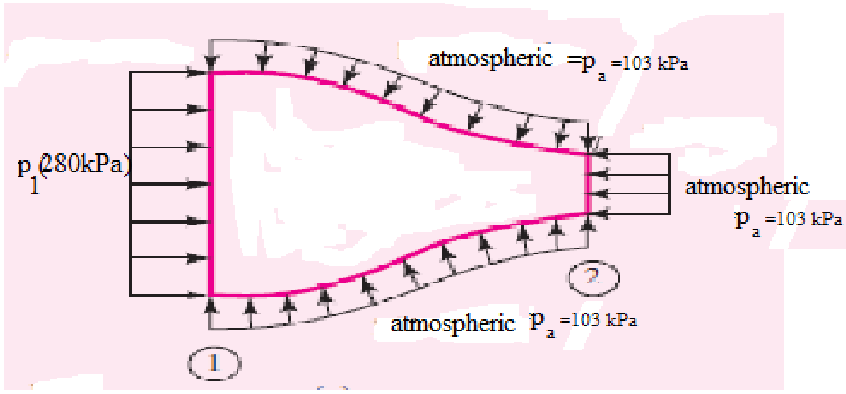

Fig. 2: Representation of the control volume
(Source: Fluid Mechanics by F.M. White)

- You have inlet at section 1 and outlet at section 2.
- The atmospheric pressure of 103 kPa acts throughout all surfaces of the control volume.
- Then we can use gage pressure

  $P_{gage} = p - p_{atm}$

  Acting on specific portions of the control surfaces.

  And $\vec{F}_{pres} = \iint_{CS} p_{gage}(-\hat{n})dA$

  Now, $p_{gage} = 280 - 103 = 177$ kPa

$$\vec{F}_{pressure} = \iint_{CS} 177 \times 10^3 (-\hat{n})dA$$

⇨ Net pressure force $= 177 \times 10^3 \times \frac{\pi}{4} \times (0.07)^2$

$$= 681\hat{e}_1 \quad \text{N}$$

⇨ $\vec{F}_{pressure} = 681\hat{e}_1 + 0\hat{e}_2 + 0\hat{e}_3$

In the application of RTT to linear momentum principle

i.e., $\left.\frac{D(m\vec{v})}{Dt}\right|_{system} = \frac{d}{dt}\iiint_{CV} \vec{v}\rho dU + \iint_{CS} \vec{v}\rho(\vec{v}_r.\hat{n})dA$

If the openings on the surfaces permit one-dimensional inflow and outflow and if the flow is steady, then

$$\sum \vec{F} = \sum \dot{m}_{outlet}\vec{v}_{outlet} - \sum \dot{m}_{inlet}\vec{v}_{inlet}$$

**Example: (Adopted from FM White's Fluid Mechanics)**

A fixed vane turns a water jet of cross sectional area A through an angle $\theta$ without changing its velocity magnitude. The flow is steady, pressure is $p_a$ everywhere and friction on the vane is negligible.

a) Find the components of vane force in $x_1$ and $x_2$ directions.

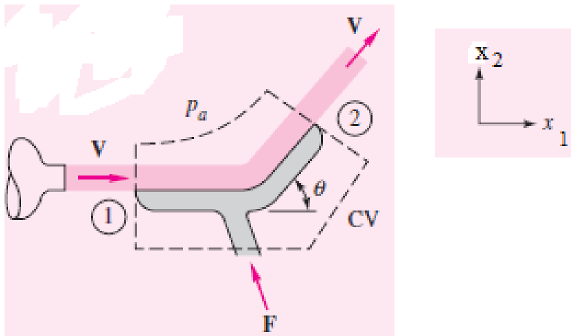


Fig. 3: Problem figure
(Source: Fluid Mechanics by F.M. White)

**Soln**: The fixed vane is shown above.

- Now you need to visualize appropriate control volume (similar to free body diagram).
- The dotted line shows the control volume.

- As the vane leg is cut by the control surface, you need to provide a reaction vane force.

- As the vane is fixed, (it is not moving), the net force acting on the control volume should be the reaction vane force.

i.e., $\sum \vec{F} = \vec{F}_{vane} = \dot{m}(\vec{v}_2 - \vec{v}_1)$
$\dot{m} = \rho A v$

The components are:

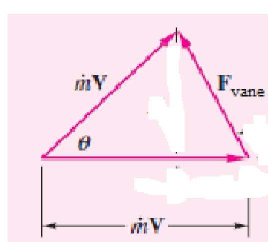


Fig. 4: Force diagram
(Source: Fluid Mechanics by F.M. White)

$$F_{x_1} = \dot{m}(v_{2_{x_1}} - v_{1_{x_1}})$$

$$F_{x_2} = \dot{m}(v_{2_{x_2}} - v_{1_{x_2}})$$

$$F_{x_1} = \rho Avv(\cos\theta - 1)$$

$$F_{x_2} = \rho Avv\sin\theta$$

$$\vec{F}_{vane} = \rho Av^2[(\cos\theta - 1)\hat{e}_1 + \sin\theta\hat{e}_2]$$