15/02/2017

LECTURE – 15

Green Gauss Theorem

Yesterday, we continued discussions on index notations that can be used to represent tensors.

In index notations we saw the

- Gradient of a scalar
- Divergence of a vector
- Cross product of vector etc.

We also saw that a second rank tensor can be written as a sum of a symmetric & asymmetric tensor.

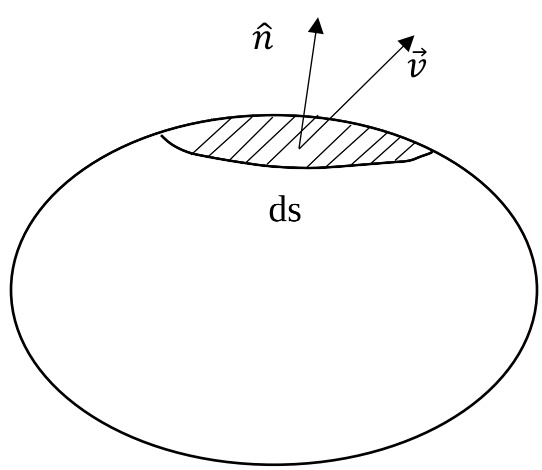
i.e, $B_{ij} = \frac{1}{2}[B_{ij} + B_{ji}] + \frac{1}{2}[B_{ij} - B_{ji}]$

First part is symmetric & second part is asymmetric

Today we will briefly discuss about Green Gauss Theorem etc.

Green Gauss Theorem relates the volume & surface integrals.

- The most common form of Gauss's theorem is the Gauss divergence theorem which you have studied in previous courses.



If you have a volume U formed by surface S, the Gauss divergence theorem for a vector $\vec{v}$ suggest that

$$\iint_S \vec{v}.\hat{n}dS = \iiint_U \nabla.\vec{v}dU$$

where, S = bounding surface on closed surface

U = volumetric domain

$\hat{n}$ = the unit outward normal vector of the elementary surface area ds

- This Gauss Divergence theorem can also be represented using index notations.

$$\iint_S v_i n_i dS = \iiint_U \frac{\partial v_i}{\partial x_i} dU$$

For any scalar multiple or factor for vector $\vec{v}$, say $\alpha\vec{v}$, the Green Gauss can be represented as

$$\iint_S (\alpha\vec{v}.\hat{n})dS = \iiint_U \nabla.(\alpha\vec{v})dU$$

i.e. $$\iint_S (\alpha\vec{v}.\hat{n})dS = \iiint_U \alpha(\nabla.\vec{v})dU + \iiint_U (\vec{v}.\nabla)\alpha dU$$

or $$\iint_S \alpha v_i n_i dS = \iiint_U \alpha \frac{\partial v_i}{\partial x_i} dU + \iiint_U v_i \frac{\partial \alpha}{\partial x_i} dU$$

Gauss's theorem is applicable not only to a vector $\vec{v}$.

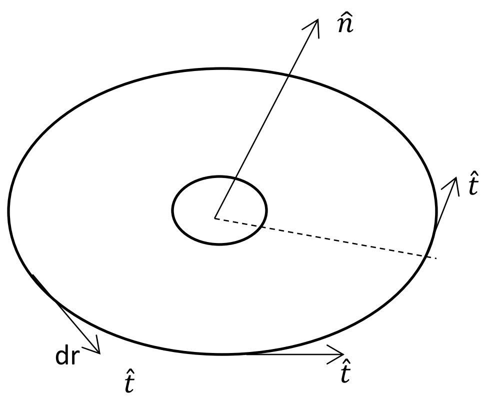
It can be applied to any tensor $\overline{\overline{B}}$ (say)

$$\iint_S \alpha B_{ij} n_i dS = \iiint_U \alpha \frac{\partial B_{ij}}{\partial x_i} dU + \iiint_U B_{ij} \frac{\partial \alpha}{\partial x_i} dU$$

Stoke's Theorem

Stoke's theorem relates the integral over an open surface S, to line integral around the surface's bounding curve (say C)

You need to appropriately choose the unit outward normal vector to the surface



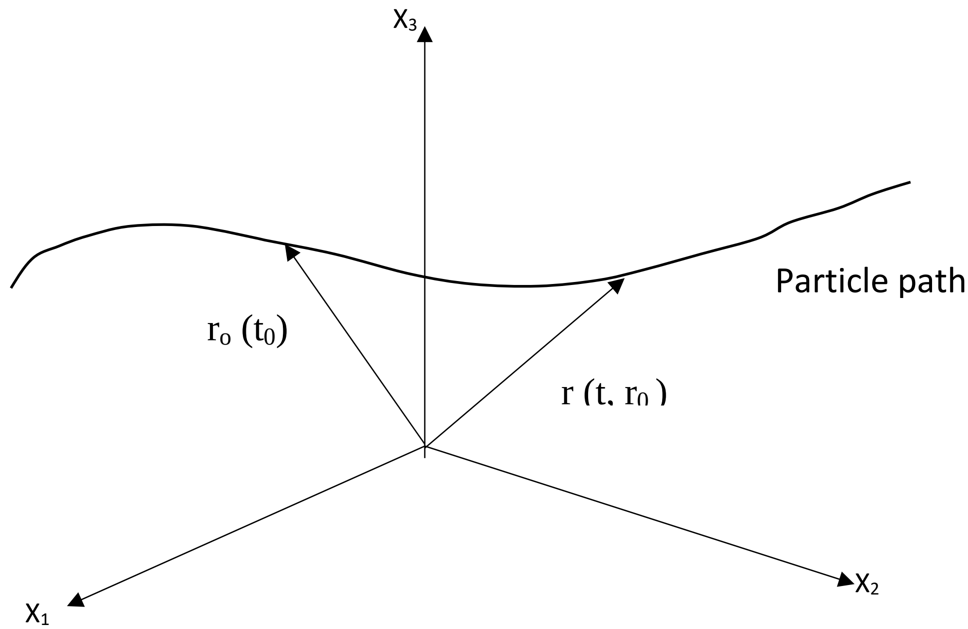
$$\vec{v}.d\vec{r} = v_i dr_i$$
$$dr_i = t_i.dr$$

$$\iint_S (\nabla \times \vec{v}).\hat{n} dS = \oint_C \vec{v}.d\vec{r}$$

$$i.e. \iint_S \varepsilon_{ijk} \frac{\partial v_k}{\partial x_j} n_i dS = \oint_C v_i t_i dr$$

Kinematics of Fluid Flow

- You know kinematics is the study of motion without reference to the forces or stresses that produce that motion.
- You have also seen the concept of fluid particles & that the collection of particles form the continuum of the fluid.
- You have also seen Lagrangian & Eularian description of the flow field.
- Lagrangian description is based on the motion of fluid particles.



- The path of the particle is given in the X_1, X_2,X_3 coordinate system (as drawn)

Initially the position vector of the particle was $\vec{r}_0(t_0)$

Therefore $\vec{r}_0$ is a known as referred value.

Based on the particle at $\vec{r}_0$ at time t_0 , that particle will be having positions at different times.

$$\vec{r} = \vec{r}(\vec{r}_0, t)$$

- The velocity of the field particle will be $\vec{v} = \dfrac{d\vec{r}(\vec{r}_0, t)}{dt}$
- The acceleration of the particle will be, $\vec{a} = \dfrac{d^2\vec{r}(\vec{r}_0, t)}{dt^2}$
- This descriptions of velocity & accelerarion (i.e, the Lagrangian approach) is same as that you has studied in mechanics

Any property $F = F(\vec{r}(\vec{r}_0, t), t)$

- As was suggested earlier , it may be cumbersome to describe fluid motion through Lagrangian approach
- Moreover, the medium is getting completely deformed during the motion.
- In Eularian approach, <u>we need to interpret velocity & acceleration vectors</u>.
- In Eularian description, we will involve the Eularian vectors x & time t.
- Any fuild property (say F) , depends directly on $\vec{x}$ & t.

$$F = F(\vec{\text{x}}, t)$$

- As we are not following individual fluid particles, we have to relate Lagrangian & Eularian descriptions.
- At any snapshot time 't' , if we merge the Lagrangian & Eularian coordinate, then

  $F(\vec{r}(\text{t}, \vec{r}_0, t_0), t) = \text{F}(\vec{\text{x}}, \text{t})$

  Where $x = \vec{r}(t, \vec{r}_0(t_0))$