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Lecture 10

Buoyancy and Stability

- Last class, we discussed about the **buoyant force**.
- To interpret the stability of a floating body, we introduced you the concept of **metacenter.**
- The position of metacenter (M), whether it is below G or above G will suggest the floating body is stable or not as shown in Fig. 1 below.



- In that tilted portion of the floating body, if M is above G, it will have a restoring moment that will bring back the body into the original position.
- If M is below G, overturning moment prevails and it will overturn the body.
- The distance between M and G is called **metacentric height**.
- How can we explain the stability based on metacentric height???
 - > You need to visualize a floating body and relate stability with respect to waterline area.

Consider a cross section of a vessel of length L.



Fig. 2: Stability concept based on waterline area (Source: Fluid Mechanics by Frank White)

- The original portion of the vessel is shown in Fig. A. It is symmetric to y-axis. For our easiness, let G be the center of gravity of the floating body and it lies on water surface. You can visualize a water plane cod in the perpendicular direction to the paper and the plane length is L. Also you adopt x-y coordinate with origin passing through G.
- > When the vessel is tilted by an angle θ (Fig. B), the submerged volume of the vessel changes.

The center of buoyancy will shift to new position B'.

This B' will be the centroid of the submerged volume aobde (Fig. B).

To evaluate the centroid B', let it be at a distance \bar{x} from the y-axis.

 $\Rightarrow \text{ The volume of the submerged portion} = V_{aobde}$ Hence, moment of the volumes :

$$\overline{x}V_{aobde} = \int_{codea} xdV + \int_{obd} xdV - \int_{coa} xdV$$

For volume codea, $\int x dV = 0$, because the body is symmetric with respect to y-axis.

Again, $dV = L dA = L x \tan\theta dx$

$$\Rightarrow \quad \overline{x}V_{aobde} = 0 + \int_{obd} xLx \tan \theta dx - \int_{coa} xL(-x \tan \theta) dx$$

Recall, the water line area = (cod) *L in the initial portion.

Similarly, here L dx = elemental waterline area

$$\overline{x}V_{aobde} = \tan\theta \left[\int_{obd} x^2 L dx + \int_{coa} x^2 L dx\right]$$

If we integrate for -x to +x, the quantity L dx, it will give you the initially evaluated waterline area.

$$\Rightarrow L dx = dA_{waterline}$$
$$\Rightarrow \overline{x}V_{aobde} = \tan\theta \int x^2 dA_{waterline}$$
$$\Rightarrow = \tan\theta I_0$$

 \Rightarrow Where I₀ is the area moment of inertia of the waterline footprint of the body. This area moment of inertia is about the tilt axis passing through O.

$$\Rightarrow \overline{x}V_{aobde} = \overline{x}V_{submerged} = I_o \tan \theta$$

$$\overrightarrow{\text{MB}} \Rightarrow \text{represents distance between M and B}$$

$$\overrightarrow{\text{MG}} \Rightarrow \text{represents distance between M and G}$$

$$\overline{x}V_{submerged} = I_o \tan \theta$$

$$\text{Or } \frac{\overline{x}}{\tan \theta} = \frac{I_0}{V_{submerged}}$$

For small tilting, we can assume MBB' to be a right triangle

$$\overline{MB} = \frac{\overline{x}}{\tan \theta} = \frac{I_0}{V_{submerged}}$$

 \Rightarrow The metacentric height, MG = MB - GB

$$\overline{MG} = \frac{I_0}{V_{submerged}} - \overline{GB}$$

- > While designing the vessel, you should note that \overline{MG} is positive. Body is stable for \overline{MG} positive.
- > If \overline{GB} is negative, then the \overline{MG} is always positive.

Example: (Adopted from Fluid Mechanics text book by F. M. White)

A barge has a uniform rectangular cross section of width 2L and vertical draft height H.

- a) Determine the metacentric height for a small tilt angle.
- b) The range of ratio L/H for which the barge is statically stable, if G is exactly at the waterline.

Ans. Assuming the barge has length 'b' into the paper.



Fig.3 : Problem figure.

The waterline area of the barge initially will be $b \times 2L$. Area moment of inertia of the waterline = $I_0 = b \frac{(2L)^3}{12}$ Submerged volume V_{submerged} = 2LbH Obvious from the shape of vessel and configuration given,

$$\overline{MG} = \frac{I_0}{V_{submerged}} - \frac{H}{2}$$
$$= \frac{\frac{8bL^3}{12}}{2LbH} - \frac{H}{2} = \frac{L^2}{3H} - \frac{H}{2}$$

$$\overline{MG}$$
 to be positive, $\frac{L^2}{3H} - \frac{H}{2} > 0$
 $L^2 > \frac{3}{2}H^2$