20/01/2016

LECTURE 6

<u>Hydrostatics Pressure</u>:

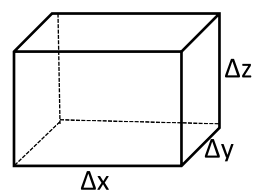
In the last class, we discussed about the fluid property called <u>pressure</u>.

- Pressure is a scalar quantity.
- In the hydrostatic conditions, we have seen that by static balance of forces

$$\Delta \vec{F}_p + \rho \vec{g} \Delta x \Delta y \Delta z = 0$$

or, $-\vec{\nabla} p \Delta x \Delta y \Delta z + \rho \vec{g} = 0$

or, $\vec{\nabla} p = \rho \vec{g}$



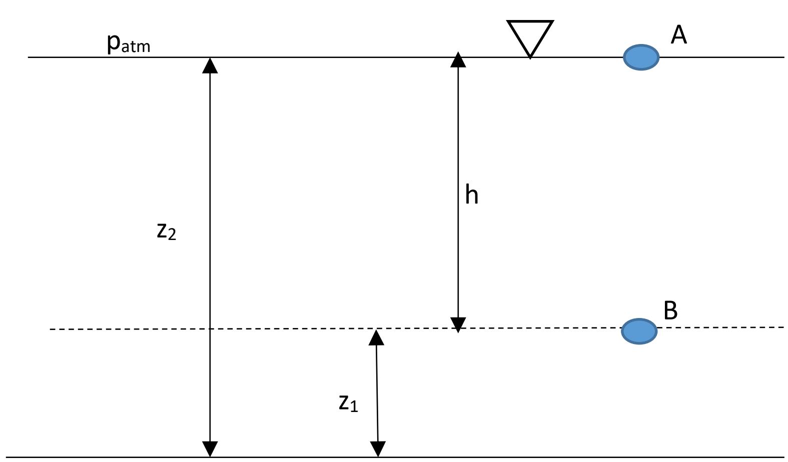
We have seen in hydrostatic condition,

$\frac{\partial p}{\partial x} = 0$ and $\frac{\partial p}{\partial y} = 0$, which means that there will be no change in pressure in horizontal direction in the hydrostatic condition.

The hydrostatic principle can also be used to evaluate pressure below the surface of water,

If $\vec{g} = 0\ \hat{\mathrm{i}} + 0\ \hat{\mathrm{j}} - g\ \hat{\mathrm{k}}$, and $\vec{\nabla} p = \frac{\partial p}{\partial x}\ \hat{\mathrm{i}} + \frac{\partial p}{\partial y}\ \hat{\mathrm{j}} + \frac{\partial p}{\partial z}\ \hat{\mathrm{k}}$

Then $\vec{\nabla} p = \rho \vec{g}$ will give $\frac{\partial p}{\partial z} = -\rho g$



On surface, pressure is atmospheric, i.e., $p_{atm} = 0$ in gage scale.

From a datum line at the bottom, let the elevation to water surface be z_2, & the elevation to an arbitrary point B existing below the water surface as z_1. ‘h’ is the depth of point B from the water surface.

As, $\frac{\partial p}{\partial z} = \frac{dp}{dz}$ in hydrostatic condition

$$\frac{dp}{dz} = -\rho g$$

So, $dp = -\rho g dz$

$$\int_B^A dp = -\rho g \int_B^A dz$$

$$p_A - p_B = -\rho g (z_2 - z_1)$$

$$p_A = p_{atm} = 0$$

So, $p_B = -\rho g (z_2 - z_1) = \rho g h$

i.e. $p_B = \rho g h$

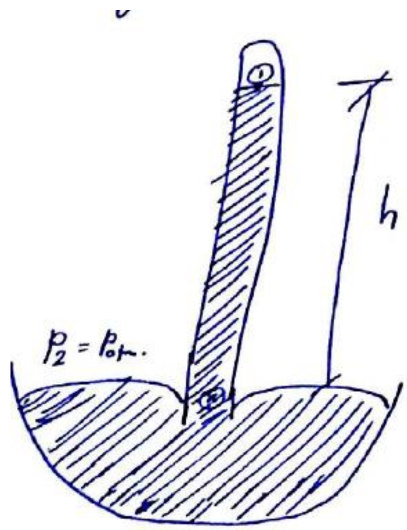
This is the principle used to evaluate pressure below the surface of water.

- ✓ Many <u>Barometers</u> works on the principle of hydrostatic or it is uses the hydrostatic formula.
- ✓ A Barometer is used to measure atmospheric pressure.
- ✓ We can make a Barometer in laboratory using mercury as the liquid.

For example at 20°C, $_{Hg}$ = 13300 kg/m^3 & Atmospheric pressure = 101.35 kPa

If we invert a tube filled with mercury into a vessel consisting of mercury, presuming that the tube is sufficiently long

- On inverting the tube into the reservoir of mercury, in static condition, the mercury dip little bit at the closed end at the top.
- At the top, the mercury will be in equilibrium with its vapor.



However, the vapor pressure of mercury is very small & negligible. Therefore, the upper space may be almost treated as vaccum. Also, $P_1 = 0$

$P_2 = P_{atm}$

So, $p_2 = \rho_{Hg}gh$

Where h = 0.761 m at 20°C.

That is the atmospheric pressure is = 101.35 kPa at 20°C.

- Hydrostatic principles are also used in manometry.
- The principle of manometry suggests that the difference in pressure at two different elevations is directly proportional to the difference in elevations.

i.e, p_1 = pressure at elevation z_1

& p_2 = pressure at elevation z_2

Then defining $\Delta p = p_2 - p_1$ & $\Delta z = z_2 - z_1$

The manometry suggests

$\Delta p = - \rho.g. \Delta z$

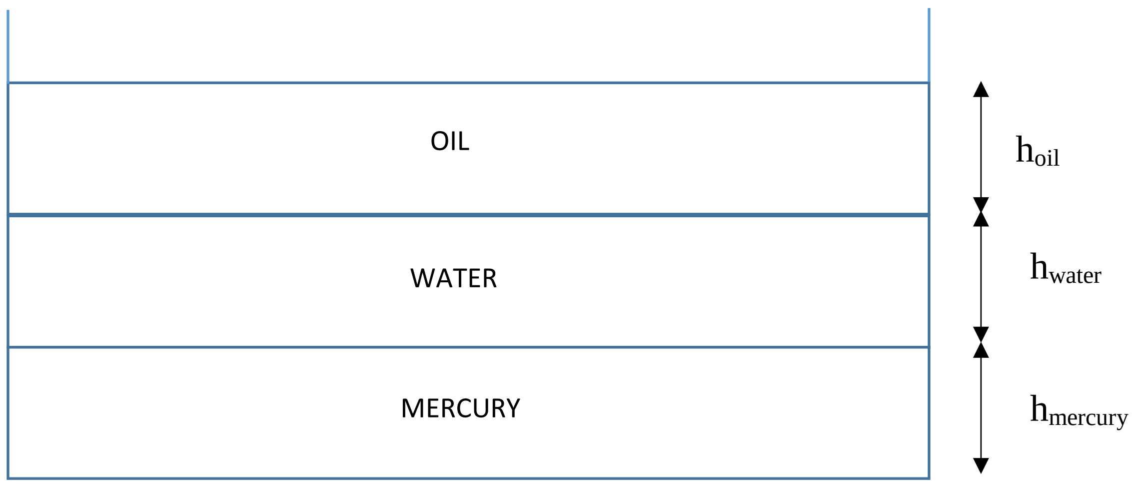
Or, $|\Delta z| = \dfrac{\Delta p}{\rho g}$

Also, $p_2 = p_1 - \rho g(z_2 - z_1)$

Or, $p = p_0 + \rho gh$

Principles of manometry can be applied to any layers of immiscible fluids as well.

Suppose in a jar, if it is filled with mercury first for the height h_{Hg} , water for a height h_w and oil for a height h_{oil}.



To find the pressure at bottom of the jar (say p_0)

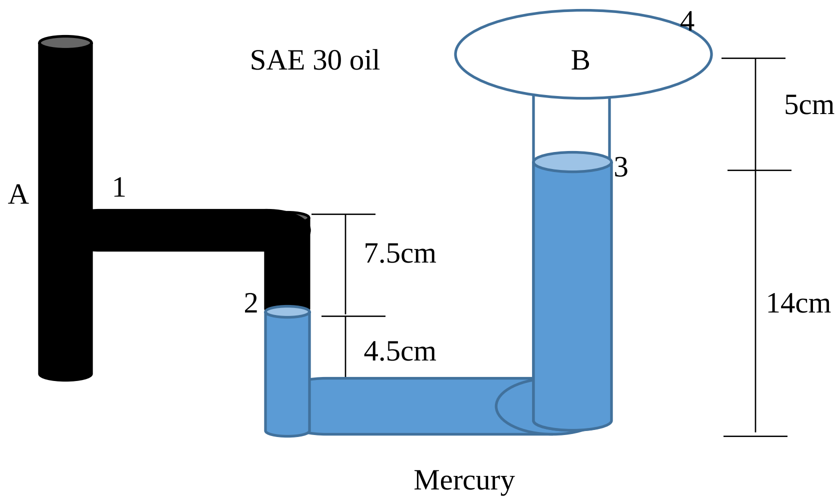
$$p_0 = p_{atm} + \rho_{oil}.g.h_{oil} + \rho_{water}.g.h_{water} + \rho_{mercury}.g.h_{mercury}$$

Manometer principle can be applied to any static column. It need not to be that one end should be opened to the atmosphere.

<u>Example</u>: (As adopted from FM White's text Book)

Water is flowing through a pipe. To measure the pressure of water in the pipe, a pressure gage(B)(that works on principle of manometry) is attached to the pipe at portion (A). If the pressure at point B is 100kPa, estimate the pressure at A.

Given, $(\rho g)_{mercury}$ = 133100 N/m^3; $(\rho g)_{water}$ = 9790 N/m^3; $(\rho g)_{SAE\text{-}Oil}$ = 8720 N/m^3



<u>Answer</u>:

We assume that water, mercury & SAE oil are immiscible and have distinct interfaces.

We will apply manometry rules to evaluate pressure at A.

To use manometry principle, we will give identification number at interface & end points as shown.

The pressure at 4 is already given = 100 kPa = p_B

Manometry works at the principle $\Delta p = -\rho g \Delta z$

i.e, $p_A - p_B = p_1 - p_4$

$p_1 - p_4 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4)$

$= (\rho g)_{water}(z_1 - z_2) - (\rho g)_{Hg}(z_2 - z_3) - (\rho g)_{SAE}(z_3 - z_4)$

You can see that, $z_1 - z_2 = +7.5$cm

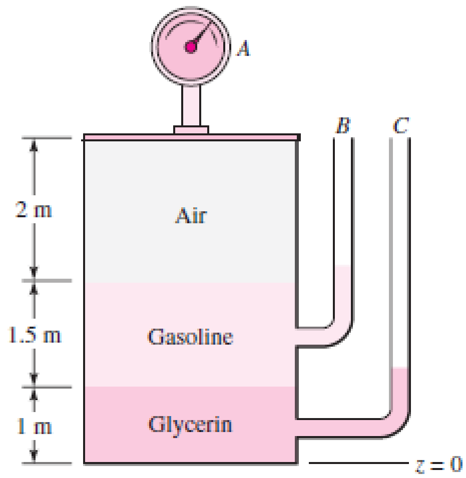
$z_2 - z_3 = -9.5$cm

$z_3 - z_4 = -5.0$ cm

So, $p_1 - p_4 = -(9790*0.075) - (133100)*(-0.095) - (8720)*(-0.05) = 12346.25$ Pa

Given $p_4 = 100$ kPa, So, $P_1 = P_4 + 12.346 = 112.346$ kPa

<u>Quiz Question</u>:



(Source: Fluid Mechanics by F.M. White)

Pressure gage A reads 1.5 kPa (gage). The fluids are at 20^0C. Determine the elevations z, in meters, of the liquid levels in the open piezometer tubes B and C. Find the elevation Z.

$\rho_{Gasoline} = 680\ kg/m^3$

$\rho_{Glycerin} = 1260\ kg/m^3$

$\rho_{Kerosene} = 804\ kg/m^3$