22/04/2017

LECTURE – 42

PIPE FLOWS:

Yesterday, for the example problem

$\Delta p = f(V, \rho, \mu, L, D)$

We came up with the non – dimensional relation

$$\pi_1 = f(\pi_2, \pi_3)$$

$$\text{or,} \quad \frac{\rho D^2}{\mu^2}\Delta p = f\left(\frac{\rho V D}{\mu}, \frac{L}{D}\right)$$

You can plot π_1 versus π_2 with π_3 as a parameter.

Or,

You can plot π_1 versus π_3 with π_2 as a parameter.

As you need to find Δp versus V, let us first go with π_1 versus π_2 with π_3 as a parameter.

From the first row of the data

i.e. $\mu_{water} = 0.001 kg/\text{ms}$, evaluate $\frac{\rho D^2 \Delta p}{\mu^2} = 3.73 \times 10^9$

$$\frac{\rho V D}{\mu} = 24700$$

From the second row, $\frac{L}{D} = 700$, $\quad \frac{\rho D^2 \Delta p}{\mu^2} = 1.78 \times 10^{10}$

$$\frac{\rho V D}{\mu} = 49370$$

Like this, you evaluate for each row of the data. You will get nine data points.

For same $\frac{L}{D}$ ratio, you will see that Δp is increasing linearly with L.

As L is present only in the non-dimensional parameter $\pi_3 = \frac{L}{D}$, (where π_1 & π_2 is devoid of it), you can now state

$$\pi_1 = f(\pi_2, \pi_3) = \frac{L}{D} g(\pi_2)$$

i.e., $$\frac{\rho D^2 \Delta p}{\mu^2} = \frac{L}{D} g(\pi_2) = \frac{L}{D} g\left(\frac{\rho V D}{\mu}\right)$$

or, $$\frac{\rho D^3 \Delta p}{L \mu^2} = g\left(\frac{\rho V D}{\mu}\right)$$

where, g( ) is a function.

(Please note that this g is not acceleration due to gravity)

By plotting the available data you will see that

$$\frac{\rho D^3 \Delta p}{L \mu^2} = 0.155 \left(\frac{\rho V D}{\mu}\right)^{1.75}$$

Most of the Newtonian smooth pipe flows correlate in this same manner.

This is the principle in many pipe flow problems.

VISCOUS FLOWS OR PIPE FLOWS:

One of the most important application of solving linear momentum & continuity equations are in pipe flows.

Recall, you cannot have a fully general solution to the Navier -Stokes equation.

Based on the nature of the problem & type of fluid flow, we can get particular solutions.

As said earlier you may have to neglect or sometimes incorporate the effects of

- Viscosity
- Gravity
- Compressibility
- Pressure, etc.

As a civil engineer, you will be mostly dealing with incompressible fluid (or water) in many practical problems.

We will see the application to pipe flows, where as a civil engineer, you may have to design pipe flow network.

Pipe flows are generally viscous.

Reynold's number plays a prominent part in understanding the pipe flow.

Based on the Reynold's number, a pipe flow is classified as

- ✓ Laminar
- ✓ Turbulent

Example (As adopted from FM White's Fluid Mechanics):

From Reynold's experiments on pipe flows of various diameters, it was observed that there is a critical Reynold's number $Re_{critical} \approx 2300$. Below this value, pipe flow is laminar. For a flow through a 5 cm diameter pipe, at what velocity will this critical Reynold's number appear at 20°C. a) For airflow, b) For water flow.

The input parameters are:

For air, $\mu_{air} = 1.80 \times 10^{-5} kg/\text{ms}$ **,** $\frac{L_e}{D} \approx 1.6 \times \text{Re}^{\frac{1}{4}}$ $\rho_{air} = 1.205 kg/m^3$

For water, $\mu_{water} = 0.001 kg/\text{ms}$ **,** $\rho_{water} = 998 kg/m^3$

Solution:

For pipe flow, the Reynold's number is defined as $Re = \frac{\rho V D}{\mu}$

$$\text{Re}_{critical}\Big|_{air} = 2300 = \frac{\rho V D}{\mu} = \frac{1.205 \times V \times 0.05}{1.80 \times 10^{-5}}$$
$$or, V_{air}\Big|_{critical} = 0.689 m/s$$

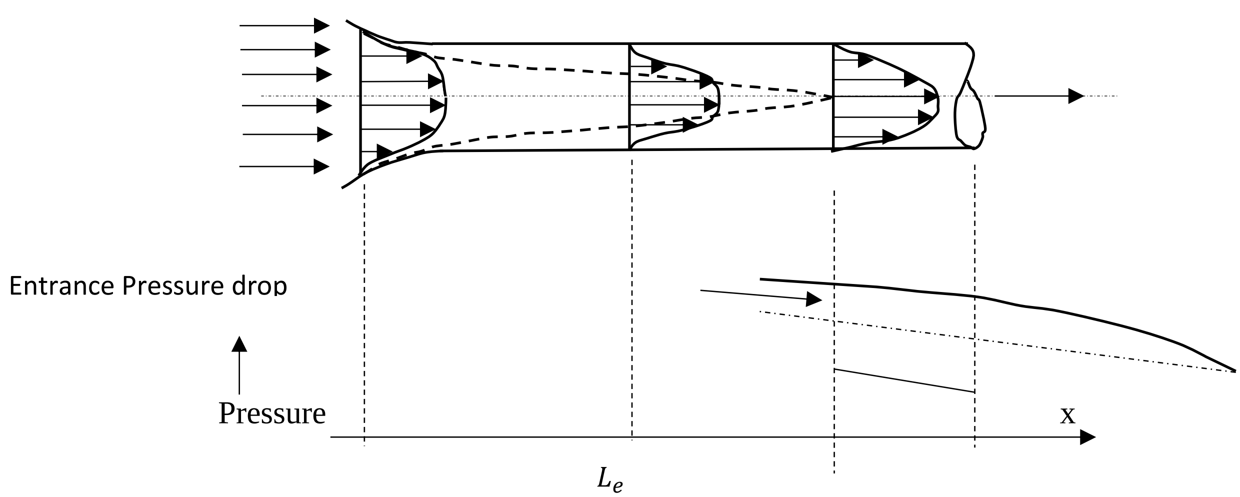
$$\text{Re}_{critical}\Big|_{water} = 2300 = \frac{\rho V D}{\mu} = \frac{998 \times V \times 0.05}{0.001}$$
$$or, V_{water}\Big|_{critical} = 0.0461 m/s$$

Fully developed pipe flow:

Usually in pipe flow design, you take "fully developed pipe flow."

"A fully developed pipe flow is the one in which the effects of viscosity are fully present & the pipe entrance effects are not taken into account."

Consider an incompressible pipe flow



Let us assume that the pipe is connected to an inviscid flow stream.

Therefore, there will be no effect of viscosity at the entrance. The velocity profile will look like the first portion.

The boundary layer starts growing with respect to pipe direction & merges at length L_e , where the full effects of viscosity is witnessed in velocity profile.

From the length L_e onwards the pipe will be having fully developed viscous flow.

From dimensional analysis done by other scientists and experts, it is observed that the entrance length L_e is a function of the Reynold's number.

In laminar flow: $\frac{L_e}{D} \approx 0.06\,\mathrm{Re}$

For turbulent flow: $\frac{L_e}{D} \approx 1.6 \times \mathrm{Re}^{\frac{1}{4}}$

Let us the pipe is connected to an inviscid flow stream.

Therefore, there will be no effect of viscosity at the entrance. The velocity profile will look like the first portion.

The boundary layer starts growing with respect to pipe direction & merges at length L_e , where the full effects of viscosity is witnessed in velocity profile.

From the length L_e onwards the pipe will be having fully developed viscous flow.

From dimensional analysis, it is observed that the entrance length L_e is a function of the Reynold's number.

In laminar flow: $\frac{L_e}{D} \approx 0.06\,\mathrm{Re}$

For turbulent flow: $\frac{L_e}{D} \approx 1.6 \times \mathrm{Re}^{\frac{1}{4}}$

Example (Adopted from FM White's Fluid Mechanics)

A 2 cm diameter pipe is 20 m long and delivers water at 8 × 10^{-4} m^3/s at 20^0C. What fraction of this pipe is taken as entrance region?

Solution: Given Q = 8 × 10^{-4} m^3/s

$$V_{avg} = \frac{Q}{A} = \frac{8 \times 10^{-4}}{\frac{\pi}{4} \times (0.02)^2} = 2.546\, m/s$$

For water at 20^{0}C , $\rho = \frac{998kg}{m^3}$ *and* $\mu = 0.001kg/ms$

⇨ $R_e = \frac{\rho VD}{\mu} = \frac{998 \times 2.546 \times 0.02}{0.001} = 50820$

⇨ Hence the pipe is turbulent.

Now $\frac{L_e}{D} \approx 1.6 {R_e}^{\frac{1}{4}}$

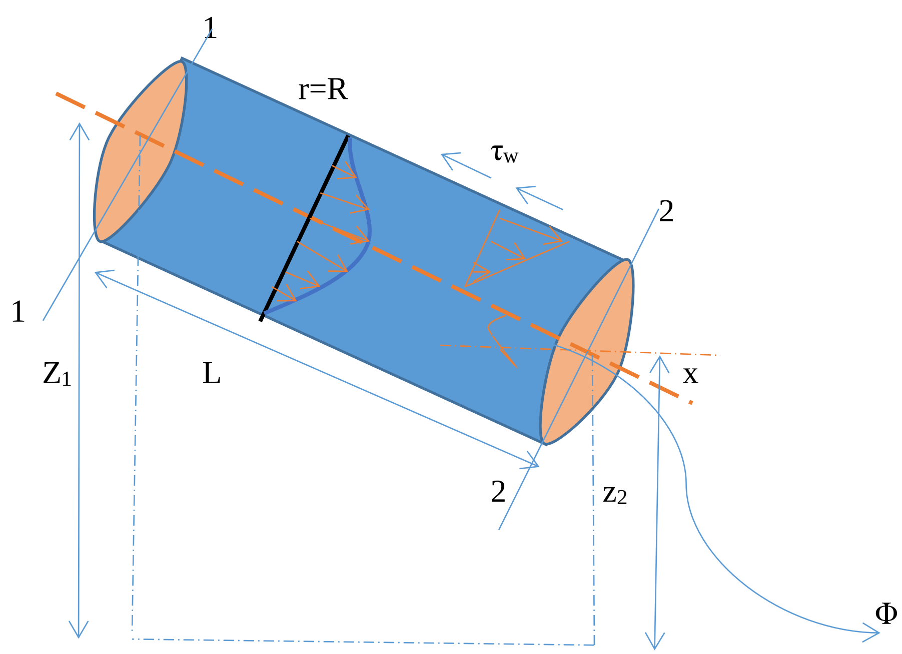
⇨ $L_e = 1.6 \times (50820)^{\frac{1}{4}} \times 0.02 = 0.48$ m

⇨ $L_e = 0.48 \ll 20$ m

⇨ You can approximate fully developed pipe flow for this pipe throughout.

<u>Head loss Friction Factor</u>

- In various pipe flow problems, we need to analyse the head loss (i.e. the energy head loss).
- For pipe flow analyses, let us again use the control volume. The flow is <u>incompressible</u> and <u>steady</u> (assumed).



The length of pipe = L = x_1 - x_2

- ⇨ The one-dimensional continuity equation suggests

$$Q_1 = Q_2 = constant$$

As pipe diameter is same at 1-1 and at 2-2 (the control volume portion),
$A_1 = A_2 = A\ (Area\ of\ cross\ section) and\ v_1 = v_2 = v$

- ⇨ The steady flow energy equation (please note that this is NOT INVISCID) is:

$$\frac{p_1}{\rho g} + \alpha \frac{{v_1}^2}{2g} + z_1 = \frac{p_2}{\rho g} + \alpha \frac{{v_2}^2}{2g} + z_2 + h_f$$

Where h_f is the energy loss.

- ⇨ As the flow is steady and also the velocity profile is same throughout, $\alpha_1 = \alpha_2$

So, $\boxed{h_f = \Delta z + \frac{\Delta p}{\rho g}}$
Change in height of hydraulic grade line.

- ⇨ If we apply momentum equation to the control volume:
Consider pressure, gravity and shear forces:

$$\sum F_x = \Delta p \left(\frac{\pi}{4} D^2\right) + \rho g \left(\frac{\pi}{4} D^2\right) L sin\phi - \tau_w (2\pi R) L$$

- ⇨ $\sum F_x = \dot{m}(v_2 - v_1)$ in steady state condition.

As $v_1 = v_2$ so, $\sum F_x = 0$
i.e. $\tau_w (2\pi R) L = \Delta p \pi R^2 + \rho g \pi R^2 L sin\phi$

- ⇨ Shear stress, $\tau_w = R\rho g \left[\frac{\Delta p}{\rho g} + \Delta z\right]$
- ⇨ $\tau_w = \rho g R h_f$
- ⇨ Shear stress is related to the head loss, so, $h_f = \frac{\tau_w}{\rho g R}$
- ⇨ From literature, it is available about Darcy-Weisbach friction factor ‘f’, and how head loss is defined, based on friction factor, i.e. $h_f = \frac{f L V^2}{D 2g}$ (Refer to your lab manual)
- ⇨ The friction factor in pipe flow analysis is some function of
=$function(R_e, \frac{\varepsilon}{D}, duct\ shape)$
- ⇨ Where , $\varepsilon = wall\ roughness\ height$

Laminar Fully Developed Pipe Flow

For a fully developed Poiseuille flow in a round pipe of diameter D, radius R:

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right) \; where \; u_{max} = \left(-\frac{dp}{dx}\right)\frac{R^2}{4\mu}$$

And $\left(-\frac{dp}{dx}\right) = \frac{\Delta p + \rho g \Delta z}{L}$

As, $v = \frac{Q}{A} = \frac{u_{max}}{2}$

$$v = \left[\frac{\Delta p + \rho g \Delta z}{L}\right]\frac{R^2}{8\mu}$$

$$Q = \int u dA = \pi R^2 v = \frac{\pi R^2}{8\mu}\left[\frac{\Delta p + \rho g \Delta z}{L}\right]$$

Since $\tau_w = \left|\mu \frac{du}{dr}\right|_{r=R} = \frac{4\mu V}{R} = \frac{8\mu V}{D} = \frac{R}{2}\left[\frac{\Delta p + \rho g \Delta z}{L}\right]$

As $h_f = \Delta z + \frac{\Delta p}{\rho g}$

so $\frac{8\mu V}{D} = \frac{R\rho g}{2L}\left[\Delta z + \frac{\Delta p}{\rho g}\right]$

$$\frac{8\mu V}{D} = \frac{R\rho g}{2L} h_f$$

Or, $h_f = \frac{32\mu L V}{\rho g D^2} = \frac{8\mu L V}{\rho g R^2}$

$h_f = \frac{8\mu V L}{\rho g R^2} = f_{laminar}\frac{2L}{R}\frac{V^2}{2g}$

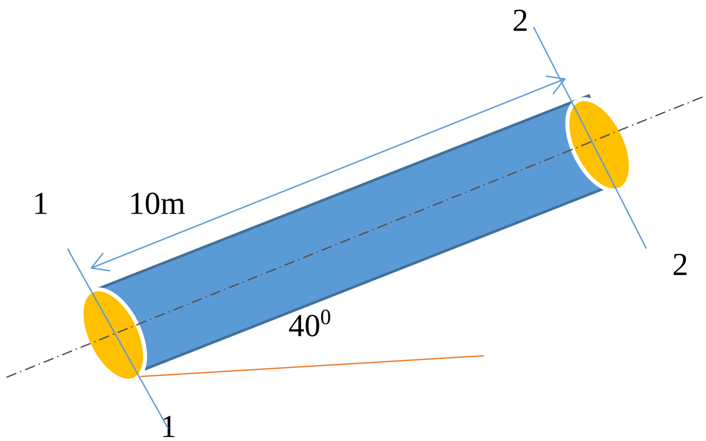
Where, $f_{laminar} \rightarrow friction\ factor = function\left(\frac{1}{R_e}\right)$

Example (Adopted from FM White's Fluid Mechanics)

An oil with ρ=900kg/m^3 and μ=0.18kg/ms flows through an inclined pipe. Two sections, section 1 and section 2 are 10 m apart. Assume steady laminar flow.

- ***a) Check whether flow is up or down.***
- ***b) Compute h_f between 1 and 2.***
- ***c) Compute the discharge Q.***
- ***d) Velocity, V.***
- ***e) The Reynolds number.***
  Given following inputs: p_1=350000Pa, z_1=0.0, p_2=250000Pa, z_2=10sin40^0=6.43m, D=6cm.

Solution:



The flow of oil will be in the direction of falling hydraulic gradient.

As the flow is steady, $v_1 = v_2 \quad so \ \frac{{v_1}^2}{2g} = \frac{{v_2}^2}{2g}$

$$(HGL)_1 = \frac{p_1}{\rho g} + z_1 = \frac{350000}{9.8 \times 900} + 0.0 = 39.64m$$

$$(HGL)_2 = \frac{p_2}{\rho g} + z_2 = \frac{250000}{9.8 \times 900} + 6.43 = 34.74m$$

So, $(HGL)_1 > (HGL)_2$

a) The flow occurs from 1 to 2.
b) Head loss between 1 and 2, $h_f = 39.64 - 34.74 = 4.9m$
c) Discharge, Q for circular pipe,
$$Q = \frac{\pi R^2}{8\mu}\left[\frac{\Delta p + \rho g \Delta z}{L}\right] = \frac{\pi R^4 \rho g}{8\mu L} h_f = \frac{\pi \times 900 \times 9.81 \times (0.03)^4}{8 \times 0.18 \times 10} \times 4.9 = 0.0076\, m^3/s$$
d) Velocity, $V = \frac{Q}{A} = \frac{0.0076}{\pi \times (0.03)^2} = 2.7\frac{m}{s}$
e) Reynolds number, $R_e = \frac{\rho V D}{\mu} = \frac{900 \times 2.7 \times 0.06}{0.18} = 810\ (Laminar\ flow)$