LECTURE 41
21- APRIL -2017

MODELLING and SIMILARITY

- The dimensional analysis what you are studying is used in modelling studies.
- You saw the Buckingham Π theorem.

- How will you do it practically?

  - Selection of variables is important. It requires good experience to decide which variable is important, etc.
  - Based on the requirement, the engineer can judge and suggest
    - That viscous flows can be neglected or not.
    - Temperature effects can be neglected or not.
    - Whether surface tension is important or not.
    - Gravitational force is important or not, etc.

  - Once the variables are selected, you need to perform analysis. Identify Π 's.
  - The experimenters will try to seek similarity between the model and the prototype designed.

- On any phenomena, the engineer may come up with a non-dimensional relation.
  $\Pi_1 = f\left(\Pi_1, \Pi_2, \Pi_3, \ldots\ldots\ldots\ldots\ldots, \Pi_k\right)$
- The flow conditions of a model test will be completely similar to the prototype, if all the dimensionless parameters of the model (say Π_2, Π_3, Π_4,…… Π_k) is same as the prototype.

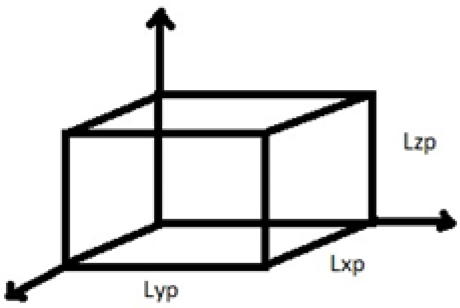
  i.e. You require
  $\Pi_{2m} = \Pi_{2p}$; $\Pi_{3m} = \Pi_{3p}$; ……

- It is very difficult to achieve complete similarity between a model and a prototype.
- However, by compromising various features, you can have:
  - Geometric similarity between model and prototype
  - Kinematic similarity
  - Dynamic similarity
  - Thermal similarity

- **<u>Geometric similarity</u>**

  If all the length dimensions [L] of the model and the prototype are having same linear scale ratio, the model and prototype is said to be geometrically similar.

  For a body, if the dimensions in the x, y, z directions are L_{xp}, L_{yp}, L_{zp}
  Then, if you have a miniature model of dimensions L_{xm}, L_{ym}, L_{zm} .



And if $\frac{L_{x_m}}{L_{x_p}} = \frac{L_{y_m}}{L_{y_p}} = \frac{L_{z_m}}{L_{z_p}} = \text{constant},$

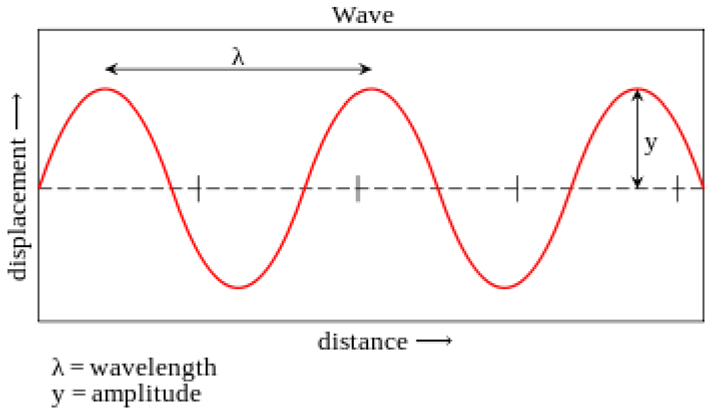
Then this model is **geometrically similar** to the prototype)

- You can visualise homologous points between the model and prototype.

- **<u>Kinematic Similarity</u>**
  - If the model and prototype have same length scale ratio (i.e. geometric similarity) and in addition have same time scale ratio, then they are kinematically similar.
  - For time scale equilibrium, you may require evidence of Reynolds Number and Mach Number, etc.
  - For fluid flows you have seen that one can neglect or incorporate effects of viscosity (or friction).
  - For a frictionless flow like free surface flow, the model and prototype can be made kinematically similar. (How???)
    - You have to provide the same Froude Number.

- Recall earlier, we had described Froude's Number as $Fr = \frac{U}{\sqrt{gL}}$

For example in a Wave Motion Modelling Studies:
The prototype wave:



(Source :https://en.wikibooks.org/wiki/Alevel_Physics_(Advancing_Physics)/What_is_a_wave%3F)

- In the wave modelling studies, the actual or prototype wave may be large and its wave length can be large.
- A model of the wave can be developed in laboratory by providing kinematically similar conditions.
- In kinematic similarity, the Froude number have to be equal.

$$\text{i.e.} \quad Fr_m = \frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}} = Fr_p$$

$$\text{or} \quad \frac{V_m{}^2}{gL_m} = \frac{V_p{}^2}{gL_p}$$

$$\text{or} \quad \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$$

If $\frac{L_m}{L_p} = \alpha$, Then $\frac{V_m}{V_p} = \sqrt{\alpha}$

$$\frac{t_m}{t_p} = \frac{L_m / V_m}{L_p / V_p} = \sqrt{\alpha}$$

- If viscosity, and surface tension, etc. play important role in flow of the fluid, then kinematic similarity may not be sufficient. You may have to go for dynamic similarity.

Dynamic Similarity:

- The model and the prototype are dynamically similar, if they have same length scale ratio, same time scale ratio and same force ratio.
- From Newton's law,for any fluid particle-the sum of pressure force , gravity force, and friction force is equal to the net force or inertial force.

$$\text{i.e.} \quad \underbrace{\vec{F}_p}_{\text{Pressure force}} + \underbrace{\vec{F}_g}_{\text{Gravity force}} + \underbrace{\vec{F}_f}_{\text{Friction force}} = \underbrace{\vec{F}_i}_{\text{Inertia force or Net force}}$$

- The dynamic similarity consider that

$$\frac{\vec{F}p_m}{\vec{F}p_p} = \frac{\vec{F}g_m}{\vec{F}g_p} = \frac{\vec{F}f_m}{\vec{F}f_p}$$

- In this flow through the sluice gate experiment, the force polygon at homologous points will have exactly the same shape if they are dynamically similar. This is possible, only if $Re_p = Re_m$ and $Fr_m = Fr_p$.

Example (Adopted from FM White's Fluid Mechanics)

- The pressure drop due to friction in a long circular pipe is a function of average velocity,density,viscosity,and pipe length and diameter.

  i.e. $\Delta P = f(V, \rho, \mu, L, D)$

- Try to interpret how ΔP varies with V and use Pi-Theorem to rewrite the above function in dimensionless form.

Solution:

You are provided the information $\Delta P = f(V, \rho, \mu, L, D)$

- These are six basic variables $\Delta P, V, \rho, \mu, L, D$ (i.e. n=6)
- The basic dimensions are M,L,T (i.e. k=3)
- We expect j = n-k = 6-3 = 3 ∏ groups.

D (cm)	**L (cm)**	**Q (m^3/h)**	**ΔP (P_a)**	**ρ (kg/m^3)**	**μ (kg/ms)**	**V (m/s)**
1.0	5.0	0.30	4680	680	$2.92 * 10^{-4}$	1.06
1.0	7.0	0.60	22300	680	$2.92 * 10^{-4}$	2.12
1.0	9.0	1.00	70800	680	$2.92 * 10^{-4}$	3.54
2.0	4.0	1.00	2080	998	0.0010	0.88
2.0	6.0	2.00	10500	998	0.0010	1.77
2.0	8.0	3.10	30400	998	0.0010	2.74
3.0	3.0	0.50	540	13550	$1.56 * 10^{-3}$	0.20
3.0	6.0	1.70	9600	13550	$1.56 * 10^{-3}$	0.67
3.0	4.0	1.00	2480	13550	$1.56 * 10^{-3}$	0.39

- As we went to see how ΔP varies with V, we are not going to keep it as repeating variable.

$\therefore$ We will keep ρ,μ,D as repeating variable

$$\Pi_1 = \rho^a \mu^b D^c \Delta p \qquad \rightarrow M^0 L^0 T^0$$

$$\Pi_2 = \rho^d \mu^e D^f V \qquad \rightarrow M^0 L^0 T^0$$

$$\Pi_3 = \rho^g \mu^h D^i L \qquad \rightarrow M^0 L^0 T^0$$

Solving for exponents, you get:

$$\Pi_1 = \frac{\rho D^2 \Delta P}{\mu^2} \ , \ \Pi_2 = \frac{\rho V D}{\mu} , \ \Pi_3 = \frac{L}{D}$$

$\therefore$ We will get a dimensionless expression

$$\frac{\rho D^2 \Delta P}{\mu^2} = f\left(\frac{\rho V D}{\mu}, \frac{L}{D}\right)$$

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

As our objective is to find ΔP versus V,

We have to plot Π_1 versus Π_2 with Π_3 as a parameter.

i.e. $\frac{L}{D} = 500$ for the first row of the data

$$\frac{\rho D^2 \Delta P}{\mu^2} = 3.73 \times 10^9$$

$$\frac{\rho V D}{\mu} = 24700$$

From second row of the data,

$$\frac{L}{D} = 700\ , \qquad \frac{\rho D^2 \Delta P}{\mu^2} = 1.78 \times 10^{10}$$

$$\frac{\rho V D}{\mu} = 49370$$

Like this you evaluate for each row. Nine data points you will get.

- For same $\frac{L}{D}$ ratio , you will see ΔP is increasing linearly with L.