LECTURE 38

18-APRIL-2017

DIMENSIONAL ANALYSIS

- In this fluid mechanics course, you have seen two ways of analytically analyzing the fluid flow:
  - The integral or the control volume approach
  - The infinitesimal or the differential approach
- You came up with expressions to analyze different types of flow (e.g., steady or unsteady, compressible or incompressible, viscid or inviscid, etc.)
- You have also seen that the above expressions may be general in nature, however, when you try to solve them; you were able to apply them only in simplified cases.
  (Or else you may have to use high-end computational fluid dynamics).
- If you recall the history in our introduction class, we suggested that there were basically two types of developments in fluid mechanics history:
  - One group of scientists worked on analytical ways.
  - Second group of scientists explored experiments.
- That is, experiments are also an equally important aspect in the study of fluid mechanics.
- Experimental studies are also required to validate the concepts developed using control volume approach as well as differential approach, since experiments are the only realistic proof.
- When you do experiments, there will be large number of data sets. It may become cumbersome and you will find it difficult to represent those data appropriately.
- Dimensional analysis is a way:
  - To plan
  - To present
  - To interpret

  the experimental data in an appropriate and simplified form.

- In dimensional analysis, you will use methods to reduce number and complexity of experimental variables that affect a given physical phenomenon.

- In science subjects, you have studied that the four basic dimensions are:
  Mass [M], Length [L], Time [T]. Temperature [θ].
  i.e. in a dimensional system, any variable can be expressed in [$MLT\theta$] system
  i.e. [$M^aL^bT^c\theta^d$].
- As there are four basic or primary variables M, L, T and θ, you can do dimensional analysis by grouping the given variables into dimensionless form. This will help in saving time and money. (How?)

e.g. If you want to analyze force F on a complex body immersed in a stream of fluid that depends on the body length L, stream velocity V, fluid density ρ, fluid viscosity μ
i.e.   F = f(L, V, ρ, μ)

As you require now to develop relation of F with respect to the variables L, V, ρ and μ:

- You are now required to conduct several combinations of experiments to ultimately come up with a reasonably good relation of F with L, V, ρ and μ.

⇨ If you do dimensional analysis, you can still come up with a very good relation of force with respect to length, velocity, density and viscosity.

Q. What are the dimensionless quantities possible according to theory of dimensional analysis?

For a n-dimensional variable problem, you can have (n-k) dimensionless quantities where k=the number of primary variables (1, or 2, or 3, or 4).

In this case of F = f(L, V, ρ, μ) there are 5-variables F, L, V, ρ, μ.

You can have two dimensionless parameters.

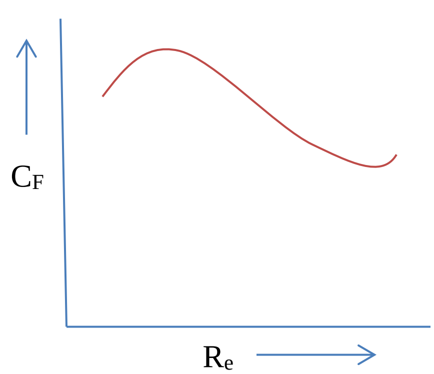
➔ Now you check:

$$\frac{F}{\rho V^2 L^2} = g\left(\frac{\rho V L}{\mu}\right)$$

The quantity $\frac{F}{\rho V^2 L^2} = C_F$ is a dimensionless force coefficient.

$\frac{\rho V L}{\mu}$ is the dimensionless Reynolds number.

You can now plot:



⇨ The dimensional analysis helps in thinking and planning the experiments to validate theories.

⇨ The above example suggests that the :

- Dimensional force coefficient is a function only of the Reynolds number. That's why we were able to easily plot them.

⇨ Dimensional analysis can benefit in providing appropriate scaling laws.
(e.g. You cannot build many prototypes of the experimental object and study the effect of each variables. That may be costly).

You can develop small models and test on it and scale the results.
To scale the results, you need to apply the scaling laws on the models.

e.g. A similarity of model and prototype can be achieved as such:
$R_{e_m} = R_{e_p}$, $C_{F_m} = C_{F_p}$
i.e. $\frac{F_p}{F_m} = \frac{\rho_p}{\rho_m} (\frac{V_p}{V_m})^2 (\frac{L_p}{L_m})^2$ (The scaling law for force)

This is achieved by considering the dimensionless numbers for model and prototype to be same.

<u>Principle of Dimensional Homogeneity</u>
If you have an equation that describes a physical process, then this equation will be dimensionally homogeneous, if each of its additive terms have same dimensions.
Recall the expression for displacement of a falling body:

$S = S_0 + v_o t + \frac{1}{2} g t^2$ .....$\boxed{1}$

[L] [L] [$LT^{-1}T$] [$LT^{-2}T^2$]

[L] [L]

Equation $\boxed{1}$ is dimensionally homogeneous.
Similarly, for <u>inviscid</u>, <u>incompressible</u>, <u>irrotational</u> fluid flow, you can have the Bernoulli's equation in <u>steady</u> state:

$$\frac{p}{\rho} = \frac{v^2}{2} + gz = constant$$

[L^2T^{-2}] [L^2T^{-2}] [L^2T^{-2}]

Hence this is also dimensionally homogeneous.