28/03/2017

LECTURE – 30

Differential Approach for Energy:

Last day we derived the Navier - Stoke's momentum equations for incompressible fluid.

We also stated that Navier - Stoke's equations along with the continuity equation are the most fundamental equations to represent incompressible fluid flows in any domain.

Today, we will briefly discuss about the conservation of energy principle using differential approach.

You may recall the Reynold's Transport Theorem for energy for a control volume:

$$\frac{DE}{Dt} = \frac{d}{dt}\iiint_{cv} e\rho dU + \iint_{cs} e\rho(\vec{v}.\hat{n})dA \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where, E is the extensive property, energy in the system

e is the intensive property

Also recall, $e = \hat{u} + \frac{1}{2}v^2 + gz$

where, $\hat{u}$ is internal energy per unit mass

v^2 is kinetics energy per unit mass

gz is the potential energy per unit mass

You may also recall that

$$\frac{DE}{Dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

Where, Q is heat added to the system

W is the work done by the system

Subsequently for any fixed control volume, we came up with the expression:

$$\frac{dQ}{dt}-\frac{dW_s}{dt}-\frac{dW_v}{dt}=\iiint_{cv}\frac{\partial}{\partial t}(e\rho)dU+\iint_{cs}(e+\frac{p}{\rho})\rho(\vec{v}.\hat{n})\,\mathrm{dA} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Therefore, in the differential analysis, again you can take the same elementary volume ΔxΔyΔz as the control volume & you can apply the energy equation (2) in that.

As the volume is elemental, you can consider that no shaft produces through the surfaces of the volume inside it.

Therefore, $\frac{dW_s}{dt} = 0$

Hence, $\frac{dQ}{dt}-\frac{dW_s}{dt}-\frac{dW_v}{dt}=\iiint_{cv}\frac{\partial}{\partial t}(e\rho)dU+\iint_{cs}(e+\frac{p}{\rho})\rho(\vec{v}.\hat{n})\,\mathrm{dA}$

Adopting similar procedure, as done for conservation of mass & linear momentum, you get now

$$\frac{dQ}{dt}-\frac{dW_v}{dt}=[\frac{\partial(\rho e)}{\partial t}+\frac{\partial}{\partial x}\{\rho u(e+\frac{p}{\rho})\}+\frac{\partial}{\partial y}\{\rho v(e+\frac{p}{\rho})\}+\frac{\partial}{\partial z}\{\rho w(e+\frac{p}{\rho})\}]\Delta\mathrm{x}\,\Delta\mathrm{y}\,\Delta\mathrm{z}$$

i.e. $$\frac{dQ}{dt}-\frac{dW_v}{dt}=[\rho\frac{\partial e}{\partial t}+\rho u\frac{\partial e}{\partial x}+\rho v\frac{\partial e}{\partial y}+\rho w\frac{\partial e}{\partial z}+\mathrm{u}\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+w\frac{\partial p}{\partial z}$$

$$+\mathrm{e}\frac{\partial p}{\partial t}+(\mathrm{e}+\frac{p}{\rho})\frac{\partial(\rho u)}{\partial x}+(\mathrm{e}+\frac{p}{\rho})\frac{\partial(\rho v)}{\partial y}+(\mathrm{e}+\frac{p}{\rho})\frac{\partial(\rho w)}{\partial z}]\Delta\mathrm{x}\,\Delta\mathrm{y}\,\Delta\mathrm{z}$$

$$=[\rho\{\frac{\partial e}{\partial t}+u\frac{\partial e}{\partial x}+v\frac{\partial e}{\partial y}+w\frac{\partial e}{\partial z}\}+(\vec{v}.\nabla)\,\mathrm{p}+\mathrm{e}\{\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\}$$

$$+\mathrm{p}\{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\}]\Delta\mathrm{x}\,\Delta\mathrm{y}\,\Delta\mathrm{z}$$

$$\frac{dQ}{dt}-\frac{dW_v}{dt}=[\rho\frac{de}{dt}+(\vec{v}.\nabla)p+p\nabla.\vec{v}]\Delta\mathrm{x}\,\Delta\mathrm{y}\,\Delta\mathrm{z} \qquad \rightarrow(3)$$

In equation (3), now you need to describe what all constitutes $\frac{dQ}{dt}$ & $\frac{dW_v}{dt}$ for the elementary volume.

To describe $\frac{dQ}{dt}$

For this elementary volume, we assume that the heat is transferred only through conduction, i.e, radiation is omitted.

The heat transfer through the conduction process occurs on the surfaces of the volume.

We adopt ***Fourier's Law of heat transfer***.

The heat flux or vector heat transfer per unit area as:

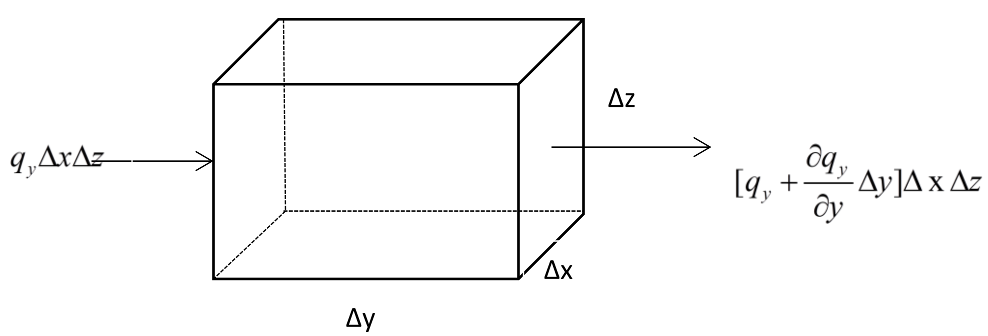
$$\vec{q} = -k\nabla T$$

where, k is thermal conductivity (another fluid property)

You get,

$$q_x = -k\frac{\partial T}{\partial x}, q_y = -k\frac{\partial T}{\partial y}, q_z = -k\frac{\partial T}{\partial z}$$

The negative sign suggests that the heat flux is positive in the direction of decreasing temperature.



For the six faces of the elementary rectangular volume,

The inlet fluxes per unit area are: q_x , q_y & q_z

The outlet heat fluxes per unit area are: $q_x + \frac{\partial q_x}{\partial x}\Delta x, q_y + \frac{\partial q_y}{\partial y}\Delta y, q_z + \frac{\partial q_z}{\partial z}\Delta z$

Therefore, the net heat out flux form the volume $=[\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}]\Delta x\Delta y\Delta z$

However, $\frac{dQ}{dt}$ is Rate of heat added to the system.

$$\frac{dQ}{dt}=-[\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}]\Delta x\Delta y\Delta z$$

Therefore, $i.e. \frac{dQ}{dt}=-\nabla.\vec{q}\Delta x\Delta y\Delta z$

$$i.e. \frac{dQ}{dt}=-\nabla.(\mathrm{k}\,\nabla\,\mathrm{T})\Delta x\Delta y\Delta z$$

To describe $\dot{W}_v$

The work done by the viscous forces have to be described as work done by the system due to viscous forces.

Rate of change of viscous work component:

= Viscous stress component * Velocity component *Area of elemental force