15/03/2017

LECTURE – 25

Conservation of Energy (Contd……)

Yesterday we saw the expression for conservation of energy using RTT.

$$\frac{dQ}{dt}-\frac{dW_S}{dt}-\frac{dW_p}{dt}=\frac{\partial}{\partial t}[\iiint_{CV}(\hat{u}+\frac{v^2}{2}+\text{gz})\rho dU]+\iint_{CS}(\hat{h}+\frac{v^2}{2}+\text{gz}+)\rho(\vec{v}.\hat{n})dA$$

(Of course, the above expression is for stationary control volume)

Subsequently for one dimensional inlet & outlet having steady fluid flow, the energy equation becomes

$$\frac{dQ}{dt}-\dot{W}_S-\dot{W}_p=-\dot{m}_{in}[\hat{h}_{in}+\frac{{v_{in}}^2}{2}+\text{gz}_{in}]+\dot{m}_{out}[\hat{h}_{out}+\frac{{v_{out}}^2}{2}+\text{gz}_{out}]$$

As for steady flow, $\dot{m}_{in}=\dot{m}_{out}=\dot{m}$

$$q=\frac{\frac{dQ}{dt}}{\dot{m}}=\frac{\dot{Q}}{\dot{m}}=\frac{dQ}{dm}$$

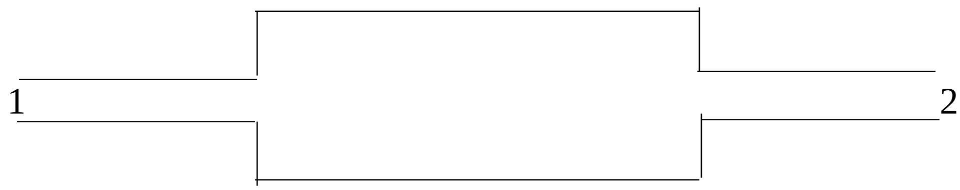
$$w_s=\frac{\dot{W}_s}{\dot{m}}=\frac{dW_s}{dm}$$

$$w_v=\frac{\dot{W}_v}{\dot{m}}=\frac{dW_v}{dm}$$

$$\hat{h}_1+\frac{v_1^2}{2}+gz_1=(\hat{h}_2+\frac{v_2^2}{2}+gz_2)-q-w_s+w_v$$

Quick Example (Adopted from FM White's fluid Mechanics)

Air with gas contact R = 287 J/kg-K & specific heat C_p, flows steadily as shown , through a turbine that produces 521.85 kJ of energy. For the inlet & exit conditions shown in figure, estimate a) the velocity at the outlet V_2 b) the heat transfer Q



Solution:

Given, D_1 = 15 cm, T_1 = 423 K, $p_1 = 1.034\times10^6$ Pa, V_1 = 30 m/sec;

D_2 = 15 cm, T_2 = 275 K, $p_1 = 2.758\times10^5$ Pa, V_2 = ?

Also given, R = 287 J/kg-K; C_p = 1.005 kJ/kg-K = 1005 J/kg-K

As the flow is steady & one dimensional,

From perfect gas law: $p = \rho RT$

where, p = gas pressure (P_a)

ρ= density of gas

R = universal gas constant

T = temperature in Kelvin

$p_1 = \rho_1 R T_1$

&

$p_2 = \rho_2 R T_2$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1.034*10^6}{287*423} = 8.52\,\text{Kg/m}^3$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.758*10^5}{287*275} = 3.50\,\text{Kg/m}^3$$

Mass flow rate, $\dot{m} = \dot{m}_1 = \dot{m}_2$ (as, flow is steady)

$$\dot{m}_1 = \rho_1 A_1 v_1 = 8.52 * \frac{\pi}{4} * (0.15)^2 * 30 = 4.514 \text{ Kg/s}$$

$$\dot{m}_1 = \dot{m}_2 = \rho_2 A_2 v_2 = 3.50 * \frac{\pi}{4} * (0.15)^2 * v_2$$

$$4.514 = 3.50 * \frac{\pi}{4} * (0.15)^2 * v_2$$

$$v_2 = 73 \text{ m/s}$$

As, the flow is steady, the energy equation from RTT

$$\frac{dQ}{dt} - \frac{dW_S}{dt} - = \dot{m}[\hat{h}_2 + \frac{{v_2}^2}{2} + \text{g}\text{z}_2] - \dot{m}[\hat{h}_1 + \frac{{v_1}^2}{2} + \text{g}\text{z}_1]$$

Given the turbine produces 700 horsepower = 514.5 kJ/sec

So, $\frac{dQ}{dt} - 514500 = 4.514[1005 * 275 + \frac{73^2}{2} + 0] - 4.514[1005 * 423 + \frac{30^2}{2}]$ =-150kW

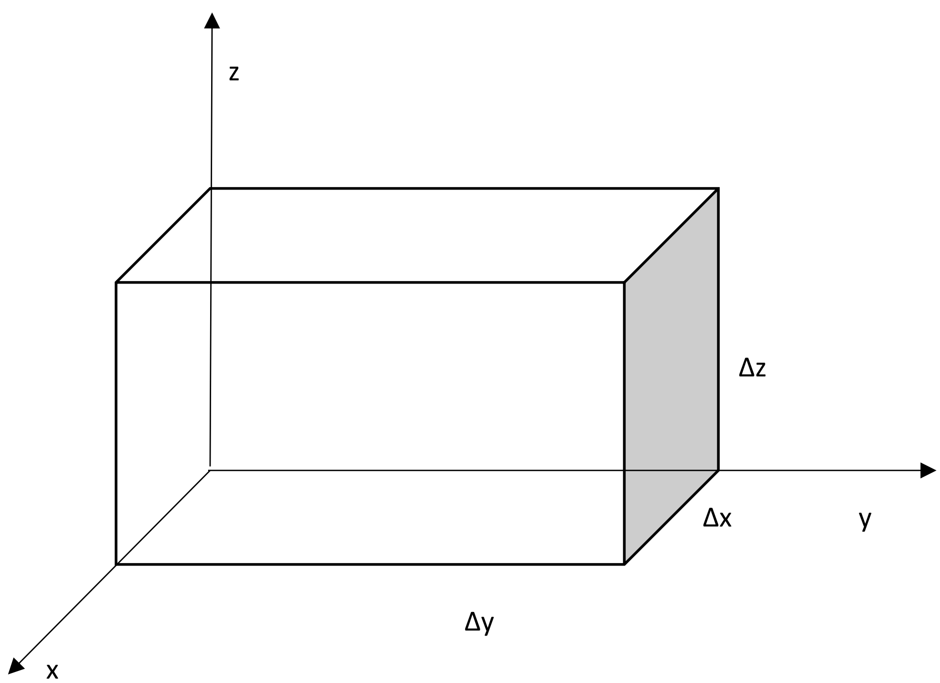
The potential quantity is cancelled as z_1 & z_2 are at same elevation.

DIFFERENTIAL RELATIONS FOR FLUID FLOW

- Till now we were discussing on the integral or control volume relations for fluid flow.
- We used Reynold's Transport Theorem for the control volume analysis.
- We have also seen through several examples, the control volumes were of different sizes according to the problem concerned.
- In fluid mechanics, to analyze fluid flow, one can also adopt analysis that involve infinitesimally small volume rather than the large control volume.
- This infinitesimally small volume may be associated with mathematical point in spaces.
- Today, we will start discussing about the Differential Approach or Infinitesimal Approach.
- We will be using the Reynold's Transport Theorem for an infinitesimally small control volume.

Differential Equation of Mass Conservation

Let us consider a small rectangular prism shaped element of fluid that is flowing.



As seen in the figure, the size of the element is given by ΔxΔyΔz & corresponding coordinate axis is also suggested.

Let velocity vector be $\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$

In the y direction, let the mass influx on the left side be = Density×Area ×Velocity

$$= \rho v \Delta x \Delta z$$

And the outflux in y direction

$$= [\rho v + \frac{\partial(\rho v)}{\partial y}\Delta y]\Delta x \Delta z$$

Similarly, we assign mass fluxes in x & z direction also,

Direction	**Mass Influx**	**Mass Outflux**
x	$\rho u \Delta y \Delta z$	$= [\rho u + \frac{\partial(\rho u)}{\partial x}\Delta x]\Delta y \Delta z$
z	$\rho w \Delta x \Delta y$	$= [\rho w + \frac{\partial(\rho w)}{\partial z}\Delta z]\Delta x \Delta y$

Applying the Reynold's Transport Theorem (conservation of mass) on this rectangular prism for fluid

$$\left.\frac{dB}{dt}\right|_{system} = \frac{Dm}{Dt} = 0 = \frac{\partial}{\partial t}\iiint_{CV} \rho dU + \iint_{CS} \rho(\vec{v}.\hat{n})\,\mathrm{dA}$$

Since one dimensional inputs & outputs are present, & the rectangular volume is not shrinking or expanding;

$$0 = \iiint_{cv} \frac{\partial \rho}{\partial t} dU + \Sigma[\rho A v]_{outlet} - \Sigma[\rho A v]_{inlet}$$

As the volume is elemental;

$$\iiint_{cv} \frac{\partial \rho}{\partial t} dU = \frac{\partial \rho}{\partial t}\Delta x \Delta y \Delta z$$

Again, $\Sigma(\rho A v)_{outlet} = [\rho u + \frac{\partial(\rho u)}{\partial x}\Delta x]\Delta y \Delta z + [\rho v + \frac{\partial(\rho v)}{\partial y}\Delta y]\Delta x \Delta z + [\rho w + \frac{\partial(\rho w)}{\partial z}\Delta z]\Delta x \Delta y$

Therefore, the RTT becomes,

$$0 = \frac{\partial \rho}{\partial t}\Delta x \Delta y \Delta z + \frac{\partial(\rho u)}{\partial x}\Delta x \Delta y \Delta z + \frac{\partial(\rho v)}{\partial y}\Delta x \Delta y \Delta z + \frac{\partial(\rho w)}{\partial z}\Delta x \Delta y \Delta z$$

As the volume is arbitrary & non-zero quantity,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

This is the **Differential Equation of Mass Conservation**.