14/03/2017

Lecture 24

Conservation of Energy

In the last class, we were discussing about conservation of angular momentum applicable to fluid mechanics. i.e., for a **non-deformable stationary control volume.**

$$\left.\frac{D\overrightarrow{Ho}}{Dt}\right|_{system} = \sum(\vec{r}\times\vec{F})_0 = \frac{\partial}{\partial t}[\iiint_{cv}(\vec{r}\times\vec{v})\rho dU] + \iint_{cs}(\vec{r}\times\vec{v})\rho(\vec{v}.\hat{n})dA$$

- Thereafter, we worked on couple of example problems.
- Today, we will see how the next conservation principle i.e., **the conservation of energy** principle is applicable in fluid mechanics.
- So again, you have to use Reynolds Transport Theorem to relate the conservation of energy that is applicable in **Lagrangian** form with a fluid control volume, which is in **Eulerian** form.

$$\left.\frac{DB}{Dt}\right|_{system} = \frac{d}{dt}\iiint_{CV}\beta\rho dU + \iint_{CS}\beta\rho(\vec{v}_r.\hat{n})dA$$

- The extensive property B in this case will be the total energy E in the system.
- The intensive property β = $\frac{dE}{dm}$ = e (the energy per unit mass)

So, $\frac{DE}{Dt} = \frac{d}{dt}(\iiint_{CV} e\rho dU) + \iint_{CS} e\rho(\vec{v}_r.\hat{n})dA$

If the control volume is stationary, then

$$\frac{DE}{Dt} = \frac{\partial}{\partial t}\iiint_{CV} e\rho dU + \iint_{CS} e\rho(\vec{v}.\hat{n})dA$$

- Now, recall some of the basic thermodynamics laws:

- If the heat δQ is added to a system or work δW is done by the system, the change in system energy δE is given by:

$$\delta E = \delta Q - \delta W$$

  - By applying differential principle in limiting conditions

$$\left.\frac{dE}{dt}\right|_{system} = \left.\frac{dQ}{dt}\right|_{system} - \left.\frac{dW}{dt}\right|_{system}$$

    This is the first law of thermodynamics.

- As obvious, the conservation of energy principle provides the time rate or material derivative of total energy as a scalar quantity.
- Note: In the notations here,
  Q = +ve → heat added to the system.
  W = +ve → work done by the system.

  So, the RTT becomes:

$$\frac{DE}{Dt} = \left.\frac{dQ}{dt}\right|_{system} - \left.\frac{dW}{dt}\right|_{system} = \frac{d}{dt}\left(\iiint_{CV} e\rho dU\right) + \iint_{CS} e\rho(\vec{v}_r.\hat{n})dA$$

- The system energy per unit mass may constitute of

  $e = e_{internal} + e_{kinetic} + e_{potential} + e_{others}$

  e_{others} include chemical, nuclear, electrostatic, magnetic, etc.

- As you are the beginners in Fluid Mechanics course here, we are not considering chemical, nuclear, electro-magnetic, etc. energies per unit mass.

  So, $e = \hat{u} + \frac{1}{2}v^2 + gz$

  where $\hat{u}$ → is the internal energy per unit mass

  (Note that the symbol '^' on $\hat{u}$ ***does not mean*** unit vector.)

  $\frac{1}{2}v^2$ → is the kinetic energy per unit mass.

  gz → is the potential energy per unit mass.

- In higher thermodynamics courses, you may see that $\frac{dQ}{dt}$ for a system can be associated with respect to convection, conduction, radiation, etc.
- The rate of change of work on/by a system (i.e., $\left.\frac{dW}{dt}\right|_{system}$) can be due to shaft work, pressure force work, and viscous force work.

$$\left.\frac{dW}{dt}\right|_{system} = \left.\frac{dW}{dt}\right|_{shaft} + \left.\frac{dW}{dt}\right|_{pressure} + \left.\frac{dW}{dt}\right|_{viscous}$$

i.e., $= \frac{dW_s}{dt} + \frac{dW_p}{dt} + \frac{dW_v}{dt}$

$= \dot{W}_S + \dot{W}_p + \dot{W}_v$

where the overdot denotes the time derivative.
(The work of gravitational forces is included as potential energy).

Q. What is shaft work?
Ans. It is the portion of work deliberately done by a machine (e.g. pump impellar, fan blade, piston, etc.) that protrudes through the control surface into the control volume ($\dot{W}_S$)

Pressure work: It is the rate of work done by pressure forces (note pressure force occurs only on the surfaces).

- For an elementary surface area dA, the rate of work is:
  $d\dot{W}_p = -p(-\vec{v}.\hat{n})dA$
  where $\vec{v}$ is the normal velocity component into the control volume.

- On a control volume having control surfaces, the total pressure force work rate is :
  $\dot{W}_p = \iint_{CS} p(\vec{v}.\hat{n})dA$

Note: If part of control surface is the surface of machine part, the work will be assigned only to shaft i.e., it is not considered as pressure work force.

Therefore, the RTT becomes (for a non-deformable stationary control volume):

$$\frac{dQ}{dt}-\frac{dW_s}{dt}-\frac{d\dot{W}_p}{dt}-\frac{dW_v}{dt}=\frac{\partial}{\partial t}\iiint_{CV} e\rho dU+\iint_{CS} e\rho(\vec{v}.\hat{n})dA$$

$$\frac{dQ}{dt}-\frac{dW_s}{dt}-\iint_{CS} p(\vec{v}.\hat{n})dA-\frac{dW_v}{dt}=\frac{\partial}{\partial t}\iiint_{CV} e\rho dU+\iint_{CS} e\rho(\vec{v}.\hat{n})dA$$

$$\frac{dQ}{dt}-\frac{dW_s}{dt}-\frac{dW_v}{dt}=\frac{\partial}{\partial t}\iiint_{CV} e\rho dU+\iint_{CS} e\rho(\vec{v}.\hat{n})dA+\iint_{CS} p(\vec{v}.\hat{n})dA$$

Recall $e=\hat{u}+\frac{1}{2}v^2+gz$

$$\frac{dQ}{dt}-\dot{W}_S-\dot{W}_p=\frac{\partial}{\partial t}\iiint_{CV}(\hat{u}+\frac{v^2}{2}+\mathrm{gz})\rho dU+\iint_{CS}\left(e+\frac{p}{\rho}\right)\rho(\vec{v}.\hat{n})dA$$

$$\frac{dQ}{dt}-\dot{W}_S-\dot{W}_p=\frac{\partial}{\partial t}\iiint_{CV}(\hat{u}+\frac{v^2}{2}+\mathrm{gz})\rho dU+\iint_{CS}(\hat{u}+\frac{v^2}{2}+\mathrm{gz}+\frac{p}{\rho})\rho(\vec{v}.\hat{n})dA$$

⇨ For steady flow conditions, having one-dimensional inlet and one-dimensional outlet:

$$\frac{dQ}{dt}-\dot{W}_S-\dot{W}_p=-\dot{m}_1[\hat{u}+\frac{v_1^{\,2}}{2}+\mathrm{gz}_1]+\dot{m}_2[\hat{u}+\frac{v_2^{\,2}}{2}+\mathrm{gz}_2]$$

As $\dot{m}_1=\dot{m}_2=\dot{m}$

Also using enthalpy $\hat{h}=\hat{u}+\frac{p}{\rho}$

$$\hat{h}_1+\frac{v_1^2}{2}+gz_1=(\hat{h}_2+\frac{v_2^2}{2}+gz_2)-q-w_s+w_v$$

where

$$q=\frac{\frac{dQ}{dt}}{\dot{m}}=\frac{\dot{Q}}{\dot{m}}=\frac{dQ}{dm}$$

$$w_s=\frac{\dot{W}_s}{\dot{m}}=\frac{dW_s}{dm}$$

$$w_v=\frac{\dot{W}_v}{\dot{m}}=\frac{dW_v}{dm}$$