LECTURE 41
25-APRIL-2016

PIPE FLOWS

→ Last class we started discussing about Pipe Flows.

→ We also stated, how Osborne Reynolds did several experiments on circular pipes and came up with two different types of flow in pipes

↳ Laminar flow
↳ Turbulent flow

For circular pipes, he came up with a non-dimensional parameter $\frac{\rho V D}{\mu}$, where

$\rho \rightarrow$ density of water ; $V \rightarrow$ velocity of water,
$D \rightarrow$ diameter of pipe ; $\mu \rightarrow$ dynamic viscosity

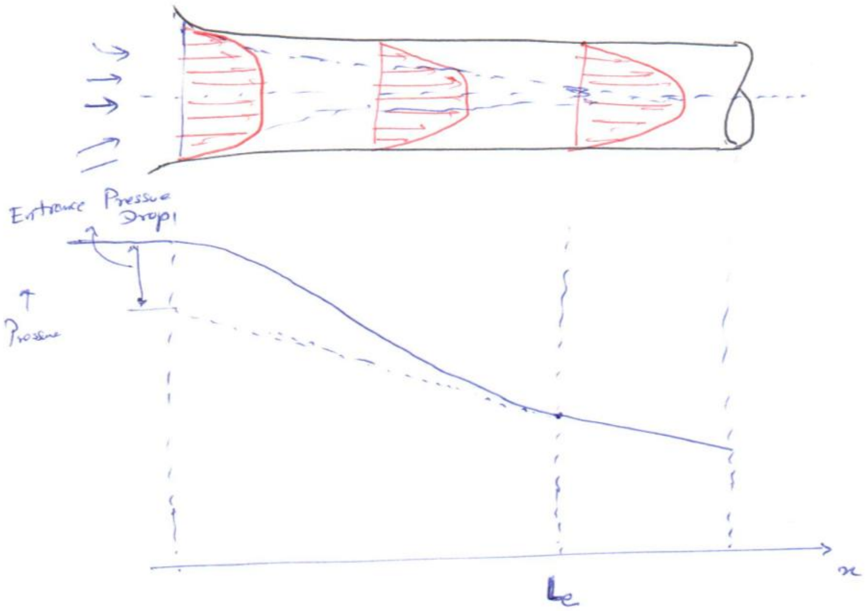
From Hagen's and Reynold's experimental data and interpretation, it was observed that

$Re_{critical} \approx 2300$; beyond which turbulence can start in pipe flow.

⇒ Whenever, now we are talking about pipe flow, we mean Fully-developed pipe flow.

"A fully developed pipe flow is the one, in which the effects of viscosity are fully present

and the pipe entrance effects are not present.
Consider an incompressible flow.



At the entrance there may be no effects of viscosity. The flow is inviscid. However, the boundary layer starts growing and after a length L_e, the boundary layer merges and the flow will be fully viscous. From this portion onwards the pipe flow will be fully-developed pipe flow.

Through dimensional analysis, it is observed that this entrance length L_e is function of Reynold's number.

For Laminar flow; $\frac{L_e}{D} \approx 0.06\, Re$

For turbulent flow; $\frac{L_e}{D} \approx 1.6\, Re^{1/4}$; $Re \leq 10^7$

Example.

A 2-cm diameter pipe is 20 m long and delivers water at 8×10^{-4} m³/s rate at 20°C. What fraction of this pipe is taken as entrance region.

Solution.

$$Q = 4 \times 10^{-4} \ m^3/s$$

$$\therefore V_{avg} = \frac{Q}{A} = \frac{8 \times 10^{-4}}{\frac{\pi}{4} \times (0.02)^2} = \cancel{1.273}\ 2.546 \ m/s$$

For water at 20°C, $\rho = 998 \ kg/m^3$

$$\mu = 0.001 \ \frac{kg}{m\,s}$$

$$\therefore Re = \frac{\rho V D}{\mu} = \frac{998 \times \cancel{1.273}\ 2.546 \times 0.02}{0.001}$$

$$\approx 50820$$

→ The flow is turbulent

$$\therefore \frac{L_e}{D} \approx 1.6\, Re^{1/4} \quad ; \quad Re = 50820$$

$$\therefore L_e \approx 1.6 \times (50820)^{1/4} \times 0.02$$

$$\approx 0.48 \ m$$

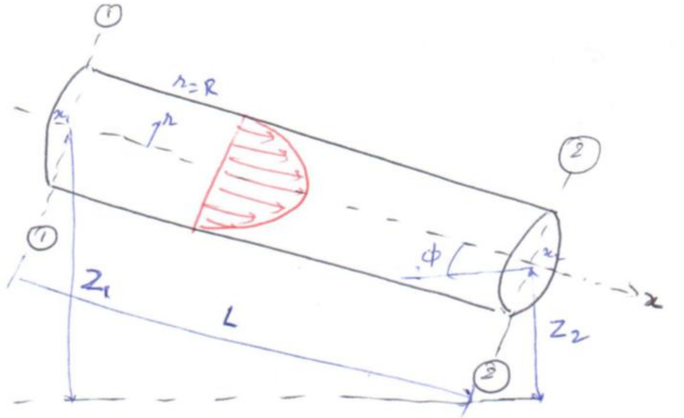
→ This length is much much small compared to overall length of pipe = 20 m.

→ Therefore, we can approximate fully-developed pipe flow for this pipe throughout.

Head Loss Friction Factor

In various pipe flow problems, we need to analyse head losses.

→ For an incompressible steady flow between sections 1 and 2.



$L = x_2 - x_1$

→ We are considering fully-developed pipe flow between sections 1 and 2.

As the flow is steady, $\therefore V_1 = V_2$

$Q_1 = Q_2$

$A_1 = A_2$ (constant dia)

The steady flow energy equation is:

$$\left[\frac{p}{\rho g} + \alpha \frac{V^2}{2g} + z\right]_1 = \left[\frac{p}{\rho g} + \alpha \frac{V^2}{2g} + z\right]_2 + h_f$$

Note there is no pump or turbine between 1 and 2.

As the flow is fully developed velocity profiles at 1 and ② are the same.

$\therefore \alpha_1 = \alpha_2$ Because $V_1 = V_2$.

$$\therefore h_f = (z_1 - z_2) + \left(\frac{p_1 - p_2}{\rho g}\right)$$

$$h_f = \Delta z + \frac{\Delta p}{\rho g}$$

Change in height of hydraulic grade line

→ If we apply momentum equation to the control volume between ① and ②.

Consider pressure, gravity, and shear forces:

$$\Sigma F_x = \Delta p\left(\frac{\pi}{4}D^2\right) + \rho g\left(\frac{\pi D^2}{4}\right) L \sin\phi - \tau_w(2\pi R) L$$
$$= \dot{m}(V_2 - V_1) = 0$$

Because $V_2 = V_1$.

i.e. $$\tau_w (2\pi R) L = \Delta p \, \pi R^2 + \rho g \pi R^2 L \sin\phi$$

You can see that shear stress is related to

$$\tau_w = \frac{R}{2L}\left[\Delta p + \rho g \Delta z\right]$$

So you can see head loss h_f is proportional to shear stress.

⇒ You might have studied about Darcy-Weisbach friction factor 'f' and head loss was defined

$$h_f = f \frac{L}{D}\frac{V^2}{2g}$$

We will utilise the friction factor in pipe flow analysis.

$$f = \text{function}\left(Re, \frac{\varepsilon}{D}, \text{duct shape} \right)$$

where $\varepsilon \rightarrow$ wall roughness height.

Laminar Fully Developed Pipe Flow

Fully developed Poiseuille flow in a round pipe of diameter D, radius R

$$u = u_{max}\left(1 - \frac{r^2}{R^2}\right) ;$$

where $u_{max} = \left(-\frac{dp}{dx}\right)\frac{R^2}{4\mu}$

$$\left(-\frac{dp}{dx}\right) = \frac{\Delta p + \rho g \Delta z}{L}$$

As $V = \frac{Q}{A} = \frac{u_{max}}{2}$ (You can prove yourself)

$$V = \left(\frac{\Delta p + \rho g \Delta z}{L}\right)\frac{R^2}{8\mu}$$

$$Q = \int u \, dA = \pi R^2 V$$

$$= \frac{\pi R^4}{8\mu}\left(\frac{\Delta p + \rho g \Delta z}{L}\right)$$

Since $\tau_w = \left|\mu \frac{du}{dr}\right|_{r=R} = \frac{4\mu V}{R} = \frac{8\mu V}{D} = \frac{R}{2}\left(\frac{\Delta p + \rho g \Delta z}{L}\right)$

You know, $h_f = \Delta z + \frac{\Delta p}{\rho g}$, $\therefore$ $h_f = \frac{32 \mu L V}{\rho g D^2}$