LECTURE 3
12 - JAN - 2015

VELOCITY FIELD PROPERTIES OF FLUID

Yesterday, we discussed on the concept of fluid. Subsequently discussed about dimensions and units.

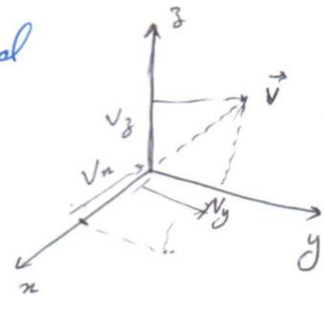
→ The particle that is representing a fluid can now be actually correlated to exist at a mathematical point in space and time.

Therefore, while solving fluid mechanics problems, the respective properties like – velocity, pressure, head, etc. can be identified for this particle and will be function of space and time.

e.g. $\vec{V}(x, y, z, t)$, $\vec{a}(x, y, z, t)$, etc.

→ As told yesterday, velocity and acceleration, etc. are vectors.

i.e. In three-dimensional orthogonal cartesian co-ordinate system, it has three components.



→ The velocity is the most important property in fluid mechanics.

⇒ There are two different approach through which we can study mechanics of continuous objects.

(i) We can take individual particles in the continuum and then track the changes in fluid properties like velocity, acceleration, pressure, etc. of that particle during its motion.

This is <u>Lagrangian</u> approach of solving mechanics problems. (Car Example)

You should note that, it need not be a single particle. It can be a collection of particles like rigid-body and all.

(Duster example)

Lagrangian approach is mostly suitable in solid mechanics.

(ii) We can also analyse motion, rather than taking individual particles, by taking the whole domain or field and then find the properties like velocity, acceleration, pressure, etc. with respect to space and time.

$V(x, y, z, t)$, $a(x, y, z, t)$, $p(x, y, z, t)$, etc.

This way of solving is called Eulerian approach.

→ For fluid mechanics, Eulerian description is more useful.

e.g. A pressure probe measures pressure at a location and not the pressure variation in an individual particle (as the probe cannot move).

→ We will adhere to Eulerian approach in our classes.

→ Velocity field, as told earlier, is the most important property in fluid mechanics.

$$\vec{V}(x, y, z, t) = u(x, y, z, t)\,\hat{i} + v(x, y, z, t)\,\hat{j} + w(x, y, z, t)\,\hat{k}$$

→ The acceleration $\vec{a}$, while using Eulerian approach will be.

$$\vec{a} = \frac{d\vec{V}}{dt}$$

(As we need to follow the particle of Lagrangian approach for evaluating acceleration, this has to be subsequently converted to Eulerian approach).

We need to introduce the Total derivative.

i.e. $$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

Thermodynamic Properties of Fluid

(i) Pressure, p (N/m^2)

(ii) Density, ρ (kg/m^3)

(iii) Temperature, T (K)

(iv) Specific Weight, $\gamma = \rho g$

(v) Specific Gravity, $SG_{gas} = \dfrac{\rho_{gas}}{\rho_{air}}$

$$SG_{liquid} = \frac{\rho_{liquid}}{\rho_{water}}$$

(vi) Potential Energy

→ The work required to move the system of mass 'm' from origin to position vector $\vec{r} = x\hat{i} + y\hat{j} + \hat{k}z$ against the gravity field $\vec{g}$.

i.e. $$P.E = -m\vec{g}\cdot\vec{r}$$

(vii) Kinetic Energy ⇒ Work required to change speed of mass 'm' from zero to velocity V

$$K.E = \frac{1}{2}mV^2$$

→ Please note that in thermo-statics, the only energy in a substance is that stored due to molecular activity and molecular bonding forces. This ~~entire~~ energy is subsequently represented at continuum level as internal energy, $\hat{u}$.

→ The total energy stored per unit mass in fluid is therefore given as:

$$e = \hat{u} + \frac{1}{2}V^2 + (-\vec{g}\cdot\vec{r})$$

→ If the coordinates x, y, z are such that

$$\vec{g} = 0\,\hat{i} + 0\,\hat{j} - g\,\hat{k}$$

Then $\vec{g}\cdot\vec{r} = -gz$

$$\therefore \quad e = \hat{u} + \frac{1}{2}V^2 + gz \quad ; \quad \hat{u}\,(T, p)$$

State Relation for Gases

You know the atmosphere consists of air. Air is actually mixture of several gases. However, the constituents of gases in air remain at almost same percentage throughout and, therefore, we can consider air as a substance.

⇒ From the perfect gas law, $p = \rho R T$

where $R = C_p - C_v$ = gas constant.

c_p, c_v → specific heats

$$c_v = \left(\frac{\partial \hat{u}}{\partial T}\right)_\rho = \frac{d\hat{u}}{dT} = c_v(T)$$

~~c_p~~

Note enthalpy, $h = \hat{u} + \dfrac{p}{\rho}$

$$= \hat{u} + RT$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = \frac{dh}{dT} = c_p(T)$$

→ Each gas has its own gas constant.

Secondary Thermodynamic Properties

We have seen pressure, temperature, density as primary thermodynamic variables.

There are secondary variables like

→ Viscosity
→ Thermal Conductivity
→ Surface Tension
→ Vapor Pressure, etc.

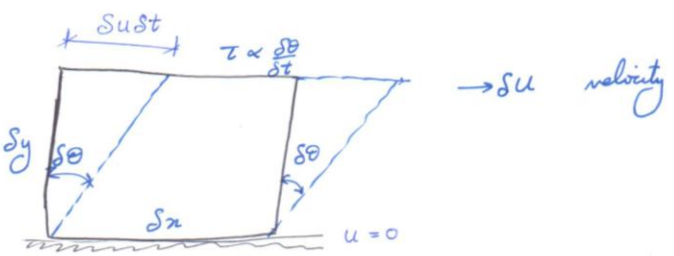
Viscosity

* Viscosity is an important property that appear while fluid is considered as continuum.

* It is the quantitative measure of fluid's resistance to flow.

* Theoretically it determines the rate of strain for a given applied shear stress.

→ Consider the fluid element (approximated in to a two-dimensional vertical plane) as below



The top part is pulled with a velocity, say δu.
The top part gets deformed by changing the vertical δy into an inclined arm, over a period of time, say δt.

→ Now the distance moved by horizontal ~~arm~~ arm having length δx is $= \delta u \, \delta t$

→ The bottom part in contact with solid is having velocity <u>zero</u>.

→ The shear stress, due to this pulling is τ

Shear strain angle $= \delta\theta$

This shear strain will continuously increase with time.

→ For liquids $\therefore \quad \tau \propto \frac{\delta\theta}{\delta t}$ (Strain Rate)

From the figure, $\tan \delta\theta = \dfrac{\delta u \delta t}{\delta y}$

If $\delta\theta \to 0$,

then $\lim_{\delta\theta \to 0} \tan\delta\theta = \delta\theta$

$$\therefore \quad \frac{\delta\theta}{\delta t} = \frac{\delta u}{\delta y}$$

Again applying differential terms by limiting $\delta t \to 0$, $\delta y \to 0$

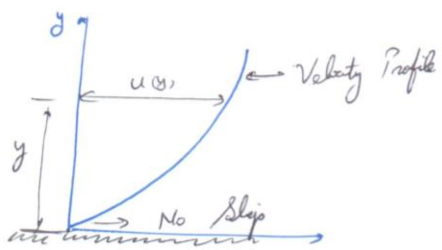
We get $$\frac{d\theta}{dt} = \frac{du}{dy}$$

$$\therefore \quad \tau \propto \frac{du}{dy}$$

i.e. $$\boxed{\tau = \mu \frac{du}{dy}}$$

where $\mu \to$ constant of proportionality

= viscosity coefficient μ.



At the wall, the velocity of fluid is zero. That is the fluid will remain static and contact with solid wall.
This is called NO-SLIP condition.

{ No-slip is characteristic of all viscous flow. }