LECTURE 19

23-FEB-2016

Local and Convective Rates

Recall, yesterday, we worked on two example problems using linear momentum principles. The first problem was to find the force on the fixed jet vane holder due to impingement of water jet. The second problem was to find the force ~~on~~ required to hold a sluice gate.

→ In all of the above cases, we were dealing with situations having inertial reference frame or non-accelerating co-ordinates.

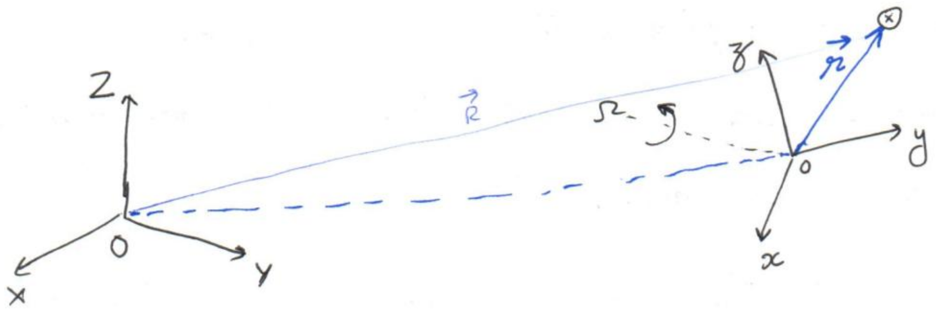
Q: What happens if the co-ordinates of reference are moving or accelerating?

⇒ In reality, all the co-ordinates we are dealing with are moving and may be with acceleration. Because our earth itself is rotating and revolving.

⇒ Similarly, there may be many other situations, where we may have to deal with accelerating co-ordinate system.

⇒ Let us consider a fluid particle, whose position vector, with respect to, a moving accelerating co-ordinate system be $\vec{r}$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



→ This co-ordinate initially might have been in a position as shown by the inertial co-ordinates OXYZ.

→ Therefore OXYZ is fixed inertial co-ordinate and Oxyz is moving non-inertial co-ordinate.

→ The time rate of change of any property with respect to specific particles (or system) can be called as material derivative of that property.

⇒ We can now suggest the instantaneous position of the particle $\vec{r}$ as a property of the particle.

The material derivative of the instantaneous position $\vec{r}$ will give us the instantaneous velocity $\vec{v}$

$\therefore$ According to definition, $\frac{d}{dt}\vec{r} = \vec{v}$

(Material Derivative)

Sometimes we may write $\frac{D}{Dt}(\ )$

$\Rightarrow$ We can also suggest that the position vector $\vec{r} \rightarrow \vec{r}(x, y, z, t)$, where '$t$' is time

This position ~~can~~ of that particle can also be defined through inertial co-ordinate (i.e. ~~Eulerian description~~)

as $\vec{R} \rightarrow \vec{R}(X, Y, Z, t)$; $\vec{R} = X\hat{I} + Y\hat{J} + Z\hat{K}$

Similarly, any property 'f' (it can be scalar, vector, tensor, etc.) can be defined in ~~Lagrangian~~ Material coordinate or ~~Eulerian form~~ fixed co-ordinate

Suppose, if $f \rightarrow f(X, Y, Z, t)$

i.e. Lagrangian description, then

$$\frac{d}{dt}\left[f(X, Y, Z, t)\right] = \frac{\partial}{\partial t}\left[f(X, Y, Z, t)\right]$$

This is because the co-ordinate OXYZ is fixed and is not changing with time.

However, if we describe the property f as:

$$f \rightarrow f(x, y, z, t)$$

The position of co-ordinate oxyz changes with respect to time.

$$\therefore \frac{d}{dt}\left[f(x,y,z,t)\right] = \frac{\partial}{\partial t}\left[f(x,y,z,t)\right] + \frac{\partial}{\partial x}\left[f(x,y,z,t)\right]\frac{dx}{\partial t}$$
$$+ \frac{\partial}{\partial y}\left[f(x,y,z,t)\right]\frac{dy}{dt} + \frac{\partial}{\partial z} f(x,y,z,t)\frac{dz}{dt}$$

This is because co-ordinate is changing position with respect to time.

When we defined instantaneous particle velocity, $\vec{v} = \frac{d\vec{r}}{dt}$

$$\text{i.e.} \quad \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$\Rightarrow$ If the property f is a scalar, then the terms.

$$\frac{\partial}{\partial x}\left[f(x,y,z,t)\right]\frac{dx}{dt} + \frac{\partial}{\partial y}\left[f(x,y,z,t)\right]\frac{dy}{dt} + \frac{\partial}{\partial z}\left[f(x,y,z,t)\right]\frac{dz}{dt}$$

$$= (\vec{v}\cdot\nabla)f \quad ; \quad \nabla \rightarrow \hat{i}\frac{\partial}{\partial x}(\,) + \hat{j}\frac{\partial}{\partial y}(\,) + \hat{k}\frac{\partial}{\partial z}(\,)$$

$$\therefore \frac{d}{dt}\left[f(x,y,z,t)\right] = \underbrace{\frac{\partial}{\partial t}\left[f(x,y,z,t)\right]}_{\text{Local Rate of Change}} + \underbrace{(\vec{v}\cdot\nabla)f}_{\text{Convective Rate of change.}}$$

So if we incorporate $f(x,y,z,t)$ as velocity $\vec{v}(x,y,z,t)$.

Then the Material derivative will give

$$\frac{d}{dt}\left[\vec{v}(x,y,z,t)\right] = \underline{\underline{\vec{a}(x,y,z,t) = \frac{\partial}{\partial t}\vec{v} + (\vec{v}\cdot\nabla)\vec{v}}}$$

That is, your acceleration will consist of local acceleration term and a convective acceleration term.

<u>Some basic Terminologies</u>

<u>Streamlines</u> : A streamline is a line, which is tangent everywhere to the velocity vector at a given <u>instant</u>.

<u>Pathline</u> : It is the actual path traced by a fluid particle.

<u>Streakline</u> : It is the locus of particles that have earlier passed through a prescribed point

<u>Timeline</u> : Timeline is a set of fluid particles that form a line at a given instant.