LECTURE 17
20-02-2016

Application of RTT to Linear Momentum Principle

Recall from your mechanics,
linear Momentum = Mass × Velocity

So, in our control volume approach, the extensive property will be: $B = m\vec{V}$

$\therefore$ Intensive property, $\beta = \frac{dB}{dm} = \vec{V}$

Therefore, in Reynolds Transport Theorem,

$$\frac{d}{dt}(m\vec{V})_{system} = \frac{d}{dt}\left(\int_{cv} \vec{V}\rho\, dV\right) + \int_{cs} \vec{V}\rho(\vec{V}_r \cdot \hat{n})\, dA$$

→ From Newton's second law,
⇔ The rate of change of linear momentum in a system is the net force acting on it.

$$\therefore \frac{d}{dt}(m\vec{V})_{system} = \Sigma\vec{F} = \frac{d}{dt}\left(\int_{cv} \vec{V}\rho\, dV\right) + \int_{cs} \vec{V}\rho(\vec{V}_r \cdot \hat{n})\, dA$$

→ We should note here that:

- ★ $\vec{V}$ is fluid velocity relative to an inertial, non-accelerating co-ordinate system.
- ★ $\Sigma\vec{F}$ is the vector sum of all body and surface forces acting on the control volume

→ $\Sigma\vec{F}$ is a vector. Therefore, in three dimensional orthogonal co-ordinate system, it will be having three components.

i.e.

$$\Sigma F_x = \frac{d}{dt}\left(\int_{cv} v_x \rho \, dU\right) + \int_{cs} v_x \rho (\vec{V_r} \cdot \hat{n}) \, dA$$

$$\Sigma F_y = \frac{d}{dt}\left(\int_{cv} v_y \rho \, dU\right) + \int_{cs} v_y \rho (\vec{V_r} \cdot \hat{n}) \, dA$$

$$\Sigma F_z = \frac{d}{dt}\left(\int_{cv} v_z \rho \, dU\right) + \int_{cs} v_z \rho (\vec{V_r} \cdot \hat{n}) \, dA$$

⇒ One-dimensional Momentum flux

If your control volume has one-dimensional ~~inflow~~ inlets and outlets (The meaning is $\int (\vec{v} \cdot \hat{n}) \, dA = vA$)

Then momentum flux $= \int_{cs} \vec{v} \rho (\vec{v} \cdot \hat{n}) \, dA$

(Ofcourse for a fixed control volume).

$\vec{v}$ and ρ will be uniform over the cross sectional areas of inlet and outlet.

∴ If ① is inlet and ② is outlet

Momentum flux $= -\dot{m}_1 v_1 + \dot{m}_2 v_2$

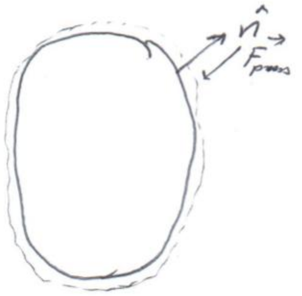
Forces acting in a control volume

The quantity $\Sigma \vec{F}$ describes the net force.

There are
→ Body forces (gravity, electro-magnetic, etc.)
→ Surface forces (pressure, viscous, etc.)

Net Pressure force on a Closed Surface

If we have a control volume of the shape shown,



→ Pressure force on a surface is normal to the surface and inward.

Pressure force is $\vec{F}_{pre} = \int_{cs} p(-\hat{n})\, dA$

→ On a <u>closed surface</u>, if pressure has uniform value p_a, then the net pressure force will be zero. $\vec{F}_{press} = -p_a \int \hat{n}\, dA = 0$

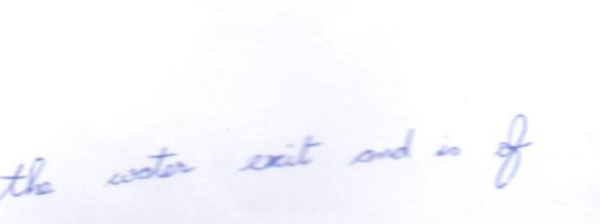
(Please Note that this result is independent of the shape of the control surface)

→ If you are using gage pressure $p_{gage} = p - p_{atm}$

Then $\vec{F}_{pre} = \int_{cs} p_{gage} (-\hat{n})\, dA.$

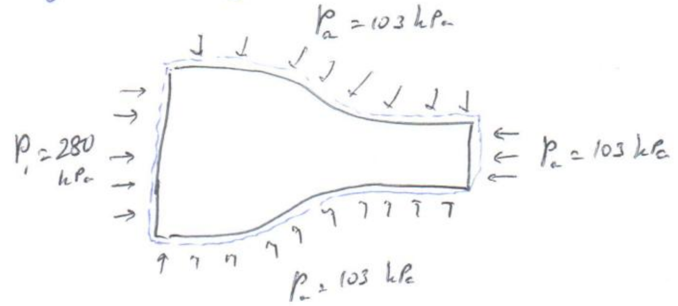
<u>Example</u>.

A nozzle is used to control the water exit and is of following shape. The ~~surface~~ water pressure at entrance of nozzle is 280 kPa. The atmosphere pressure is 103 kPa.



Diameter at section ① is $D_1 = 7$ cm, and $D_2 = 2.5$ cm.

Form a suitable control volume and find the net pressure force acting on the control volume.



$\Rightarrow$ The control volume is as shown.

$\Rightarrow$ We have inlet section (1) and outlet section (2).

The atmospheric pressure of 103 kPa acts throughout all surfaces of this control volume.
Therefore, the pressure force due to that cancels off.

Remaining $p_{gg} = 280 - 103 = 177$ kPa acts at section (1).

$\therefore$ The net pressure force $\vec{F}_{press} = 177 * 10^3 * \frac{\pi}{4} * (0.07)^2 \hat{i}$

$$= \underline{681 \hat{i} \text{ N}}$$

$$\underline{\underline{\vec{F}_{press} = 681 \hat{i} + 0 \hat{j} + 0 \hat{k}}}$$