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# Automatic Recognition of FEC Code and Interleaver Parameters in a Robust Environment

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# Outline of the Presentation

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- Blind Reconstruction of Reed-Solomon Encoder and Interleavers Over Noisy Environment
  - Introduction
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  - Simulation Results

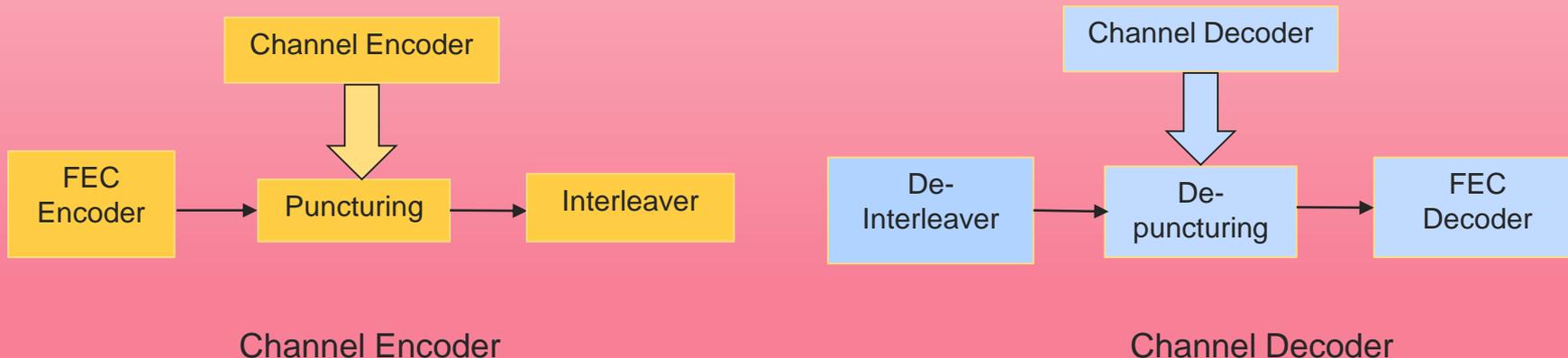
# Part 1: Classification of Error Correcting Codes and Estimation of Interleaver Parameters

Swaminathan R and A.S.Madhukumar, "Classification of error correction codes and estimation of interleaver parameters in a robust environment", IEEE Transactions on Broadcasting, vol. 63, no. 3, pp. 463 - 478, Sept. 2017.



# Introduction

- Channel encoding/decoding have become an **integral part** of modern digital communication systems
- Efficient encoding and decoding methods have been proposed to **control and correct the errors** introduced by the noisy channel
- **Interleaver** plays a vital role in communication and storage systems **to distribute the burst errors**



# Introduction

- Reconstructing an unknown code from the observation of noisy codewords problem is related to **cryptanalysis**
- This is called the **code reconstruction** problem in the literature
- An observer wants to extract information from a noisy data stream where the **error correcting code used is unknown**
- This problem arises in a **non-cooperative context** where observing a binary sequence originating from an unknown source
- Accurate information about the parameters of encoding scheme is required at the receiver to decode FEC codes
- **Non-cooperative scenario**: Parameters are either not known or only partially known at the receiver
- Applications:
  - **Military and spectrum surveillance system,**
  - **Signal intelligence (SIGINT) (intelligence gathered by interception of signals)**
  - **Adaptive modulation and coding (AMC)**
  - **Cognitive radio**

# Introduction

- **Military surveillance** may involve decoding an adversary's received data when the underlying channel code is not known
- **AMC communication systems**: Control channel will signal the AMC parameters to the receiver
- Blind recognition lead to **conservation of channel resources**
- AMC Wireless sensor networks (WSNs) : Reduces transmission overheads and total energy consumption of WSNs
- **Designing separate decoder** for every application is a **costly and a tedious process**
- It is essential to design an **intelligent receiver system** which adapts itself to any applications

# Motivations

The main motivations are given as follows:

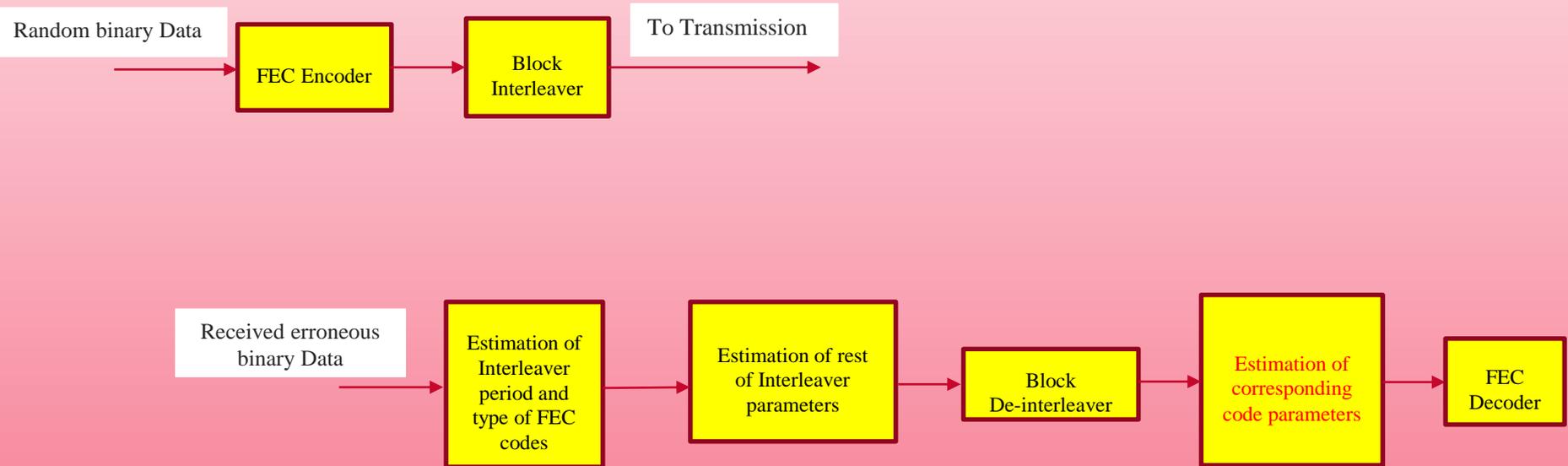
- It is essential to **blindly reconstruct channel encoder** using the **intercepted sequences** acquired from **remote sensing through aircraft and satellite**
- Code classification techniques were proposed only for **non-erroneous scenario**
- Code classification algorithm to classify among block, convolutional coded and uncoded has not been proposed
- Previously proposed **block interleaver** parameter estimation algorithms were **restricted only to estimation of interleaver period**

# Contributions

The main contributions are as follows:

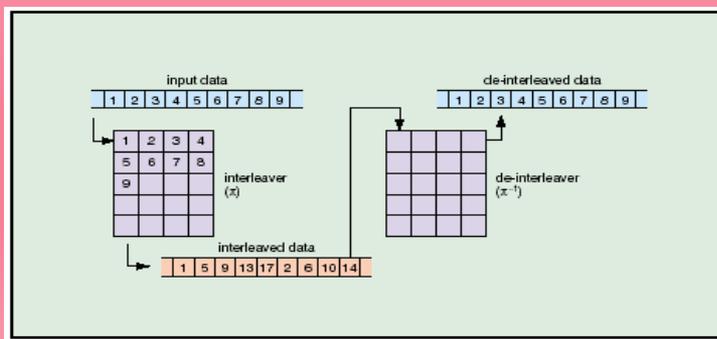
- Automatic recognition of type of FEC codes
- To differentiate among **block** coded, **convolutionally** coded, and **uncoded** data, algorithm is proposed for noisy scenario
- Algorithm is also given for estimating code and **Interleaver parameters**
- **Code dimension  $k$**  and **Codeword length  $n$**  are the estimated code parameters of block and convolutional codes
- **Interleaver period  $\beta$** , **Number of columns ( $N_c$ )**, and **Number of rows ( $N_r$ )** are the estimated matrix-based block interleaver parameters in the presence of bit errors
- Discussion is restricted to matrix-based block interleaver

# Generic Block Diagram



# Block Interleaver

- Error correcting codes provide protection against random errors and interleaver provides protection against error bursts
- A block interleaver receives a block of symbols rearranges them without removing any of the symbols
- Matrix-based block interleaver stores each block of data symbols row-wise and sends column-wise for transmission
- Interleaver period information alone is not sufficient to de-interleave the data stream
- **The size of the interleaver matrix or interleaver period is given by**  
 $\beta = N_r \times N_c$ ,  $N_r$  - Number of rows and  $N_c$  - Number of Columns



Block Interleaver/de-interleaver

# Parameter Estimation: Non-erroneous

Code classification (with interleaver) without the presence of bit errors:

- This can be done by using Rank-based methodology
- Reshape the column wise intercepted data stream into a matrix form of size  $a \times b$ , where  $a = 2 \times b$
- The rank of the data matrix  $S$  in GF (2) is computed by varying the number of columns  $b$  using Gauss elimination process

Convolutional Code:

- If  $\beta = \gamma \cdot n$  and while varying  $b$ , if  $b = \alpha \cdot \beta$ , where  $\alpha$  and  $\gamma$  are positive integers, then rank deficiency will be observed and Rank  $\rho(S) = \alpha \cdot \gamma \cdot k + m$  and rank ratio  $p = \frac{\rho(S)}{b} = r + \delta$ , where  $\delta \rightarrow 0$  as  $b \rightarrow \infty$
- If  $\beta \neq \gamma \cdot n$  and while varying  $b$ , if  $b = \alpha \cdot \text{lcm}(n, \beta)$ , then rank deficiency will be observed and  $\rho(S) = \alpha \cdot \gamma \cdot k + m$
- For the case when  $b \neq \alpha \cdot \beta$  and  $b \neq \alpha \cdot \text{lcm}(n, \beta)$ , the data matrix will have **full rank** i.e.  $\rho(S) = b$  and  $p = 1$

# Parameter Estimation: Non-erroneous

Block Code :

- If  $\beta = \gamma \cdot n$  and while varying  $b$ , if  $b = \alpha \cdot \beta$ , then rank deficiency will be observed and  $\rho(S) = \alpha \cdot \gamma \cdot k$  and rank ratio  $p = \frac{\rho(S)}{b} = r$
- If  $\beta \neq \gamma \cdot n$  and while varying  $b$ , if  $b = \alpha \cdot \text{lcm}(n, \beta)$ , then rank deficiency will be observed and  $\rho(S) = \alpha \cdot \gamma \cdot k$
- For the case when  $b \neq \alpha \cdot \beta$  and  $b \neq \alpha \cdot \text{lcm}(n, \beta)$ , the data matrix will have **full rank** i.e.  $\rho(S) = b$  and  $p = 1$ .

Uncoded:

- Irrespective of the value of  $b$ , full rank will be obtained and rank ratio will be unity.

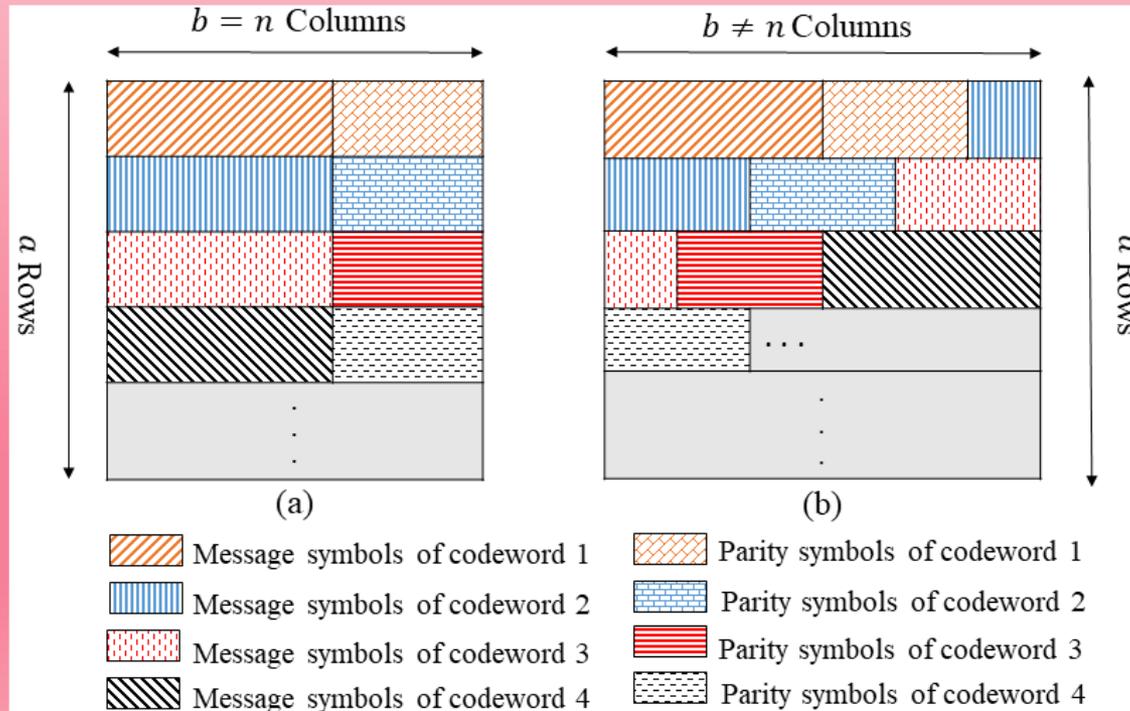
# Parameter Estimation: Non-erroneous

Reason for rank deficiency:

- $n$  output convolutionally coded data symbols depend on  $k$  present and  $m$  previous input uncoded data symbols
- $\alpha.n$  output coded data symbols depend on  $\alpha.k$  present and  $m$  previous input uncoded symbols (i.e.  $\alpha.k + m$  symbols)
- If convolutionally coded data is block interleaved and if  $b = \alpha \times \beta$  with  $\beta = \gamma \times n$ , then  $\alpha.\gamma$  codewords in a particular row will depend on  $\alpha.\gamma.k + m$  symbols
- It is also applicable to all other rows of  $S$
- For block codes, it is to be noted that  $n$  output coded data symbols depend only on  $k$  input uncoded data symbols unlike convolutional codes, since  $m = 0$
- The message and parity bits of  $\alpha.\gamma$  codewords in all the rows will be properly aligned in the same column

# Parameter Estimation: Non-erroneous

- After converting  $S$  into  $F$  through Gauss elimination process, when  $b = \alpha \times \beta$ , there will be only  $\alpha \cdot \gamma \cdot k + m$  non-zero or independent columns out of  $b$  columns
- Hence, the deficient rank value is obtained



# Parameter Estimation: Non-erroneous

## Convolutional Code

- The inequality  $\alpha.\gamma.k + m < b$  holds true or the rank deficiency can be obtained only when  $\alpha \geq \alpha_{\min}$  or  $b \geq b_{\min}$
- The expressions for  $\alpha_{\min}$  and  $b_{\min}$  are derived as follows: After substituting  $b = \alpha.\gamma.n$ , the inequality can be written as

$$\alpha.\gamma.k + m < \alpha.\gamma.n$$

After rearranging,

$$\alpha.\gamma.n. \left(1 - \frac{k}{n}\right) > m$$

$$\alpha.\gamma.n > \frac{n}{n-k}m$$

$$\alpha > \frac{m}{\gamma.(n-k)}$$

The minimum value of  $\alpha$ , denoted by  $\alpha_{\min}$ , is given by

$$\alpha_{\min} = \text{floor} \left( \frac{m}{\gamma.(n-k)} \right) + 1$$

$b_{\min}$  is given by

$$b_{\min} = \alpha_{\min}.\beta$$

## Block codes:

- The inequality  $\alpha.\gamma.k < b$ , where  $b = \alpha.\gamma.n$ , remains true irrespective of  $\alpha$ , since  $k < n$
- Hence,  $\alpha_{\min} = 1$  and  $b_{\min} = \beta$  for block codes

# Parameter Estimation: Non-erroneous

## Estimation of code rate $r$ and interleaver period $\beta$ :

- Interleaver period  $\beta$  (i.e. for the case when  $\beta$  is a multiple of  $n$ ) or  $\text{lcm}(n, \beta)$  (i.e. for the case when  $\beta$  is not a multiple of  $n$ ) can be identified by observing the difference between the successive number of columns with rank deficiency
- Let  $b = \alpha.\gamma.n$  and  $b' = (\alpha + 1).\gamma.n$  indicate the two rank deficient columns and their difference is given by

$$\begin{aligned}b' - b &= (\alpha + 1).\gamma.n - \alpha.\gamma.n \\ &= \gamma.n \\ &= \beta \text{ or } \text{lcm}(n, \beta)\end{aligned}$$

- The difference between two successive rank deficient values  $\rho(S)$  and  $\rho'(S)$  is given by

$$\begin{aligned}\rho'(S) - \rho(S) &= ((\alpha + 1).\gamma.k + m) - (\alpha.\gamma.k + m) \\ &= \gamma.k\end{aligned}$$

- The code rate  $r$  can be estimated as follows:

$$\frac{\rho'(S) - \rho(S)}{b' - b} = r$$

- For block codes, the above steps are applicable for estimating  $r$  by assuming  $m = 0$ .

# Parameter Estimation: Non-erroneous

## Convolutional Code (without interleaver):

- While varying  $b$ , if  $b = \alpha \cdot n$  and  $b > b_{min}$ , then rank deficiency will be observed and Rank  $\rho(S) = \alpha \cdot k + m$  and rank ratio  $p = \frac{\rho(S)}{b} = r + \delta$
- If  $b \neq \alpha \cdot n$  or  $b < b_{min}$ , the data matrix will have **full rank** i.e.  $\rho(S) = b$  and  $p = 1$

## Block Code (without interleaver)

- If  $b = \alpha \cdot n$ , then  $\rho(S) = \alpha \cdot k$  and rank ratio  $p = \frac{\rho(S)}{b} = r$
- If  $b \neq \alpha \cdot n$ , then  $\rho(S) = b$  and  $p = 1$
- $b = \alpha \times n$  and  $b' = (\alpha + 1) \times n$  indicate the two rank deficient columns and the difference  $b' - b$  gives the value of codeword length  $n$ .
- The difference between rank values corresponding to rank deficient columns gives the estimate of code dimension  $k$ .

## Uncoded

- While varying  $b$ , full rank will be obtained irrespective of  $b$ , as the incoming uncoded symbols are independent of each other

# Parameter Estimation: Non-erroneous

TABLE I  
MINIMUM NUMBER OF COLUMNS REQUIRED TO OBTAIN THE FIRST RANK DEFICIENT MATRIX FOR DIFFERENT CONVOLUTIONAL CODES

No.	Code rate ( $r$ )	$n$	$k$	$Y$	$b_{\min}$	$g_i^j$
1	1/2	2	1	3	6	[5,7]
2				4	8	[15,17]
3				5	10	[23,35]
4				6	12	[65,57]
5				6	12	[75,53]
6				7	14	[133,171]
7				8	16	[345,237]
8				9	18	[561,753]
9				10	20	[1167,1545]
10				11	22	[2335,3661]
11	1/3	3	1	4	6	[13,15,17]
12				7	12	[133,165,171]
13				10	15	[1117,1365,1633]
14	1/4	4	1	7	12	[133,171,117,165]

# Code Classification

- The incoming data symbols with or without interleaver can be classified easily from the **rank ratio equations**

## Convolutional Code:

- The deficient rank ratio will be much greater than  $r$  for lower values of  $b$ .
- As  $b$  increases, deficient rank ratio will tend to remain constant slightly above  $r$
- Deficient rank ratio will decay rapidly for smaller values of  $b$
- For larger values of  $b$ , it will **approximately remain constant slightly above  $r$**

## Block Code:

- Deficient rank ratio will remain constant at  $r$

## Uncoded:

- Rank ratio will remain constant at unity for all values of  $b$

# Parameter Estimation: Erroneous

- The **all-zero-column-based** rank evaluation is limited to **non-erroneous scenario**
- Due to erroneous bits, dependent columns will not get converted into all-zero-columns
- This would result in full rank for both convolutional and block codes and code classification cannot be performed
- **Erroneous scenario:** Rank calculation will be performed based on the number of zeros in each columns
- Dependent and independent columns (rank) can be segregated
- **Reason:** If  $c^{th}$  column is dependent, then the number of zeros in that particular column will be smaller compared to the independent columns.

# Parameter Estimation: Erroneous

- **Notations:** Let us denote the column echelon form of data matrix  $S_j$  as  $F_j$ ,  $j$  denotes the iteration number,  $N$  denotes the number of iterations,  $\omega_j(c)$  denotes the zero-ratio in  $c^{\text{th}}$  column of  $F_j$ ,  $\gamma(c)$  denotes the average of  $\omega_j(c)$  over  $N$  iterations, and  $data(\cdot)$  refers to an array of binary data symbols.
- $R_j = data(1 + (j - 1)b : ba + (j - 1)b)$
- Reshape  $R_j$  into a matrix  $S_j$  of size  $a \times b$
- Convert  $S_j$  into  $F_j$  using Gauss elimination process
- Compute  $\omega_j(c)$  in each column of  $F_j$ , where  $c \in \{1, 2, \dots, b\}$
- Form a row vector  $A_j = [\omega_j(1) : \omega_j(b)]$
- Repeat the above steps for  $N$  iterations
- Accumulate all the row matrices into a single matrix  $A$  of size  $N \times b$ , where  $A = [A_1 ; A_2 ; A_3 ; \dots ; A_N]$
- Compute  $B = mean(A)$ , where  $B = [\gamma(1) : \gamma(b)]$  and  $\gamma(c) = \frac{\sum_{j=1}^N \omega_j(c)}{N}$
- Evaluate  $\rho(b)$  as follows:  $\rho(b) = \mathbf{card} \left( c \in \{1, 2, \dots, b\} \mid \gamma(c) < \Gamma^{\text{th}} \right)$  and  $p(b) = \frac{\rho(b)}{b}$

# Parameter Estimation: Erroneous

## Matrix Size Estimation:

Step 1: **Requires**  $\partial = \text{lcm}(n, \beta)$  or  $\beta$  and **assumes**  $N_r' \text{ and } N_c' > 1$

Step 2:            *for*  $i = 1:n\_max$   
                         *Get possible combinations of*  
                                 *two factors  $N_r'$  and  $N_c'$*   
                                 *that satisfy  $N_r' \times N_c' = \frac{\partial}{i}$*   
  
                         *end*

Step 3: **Fix Number of columns as a multiple of interleaver period**  
**(i.e.  $N_{col} = \alpha \times \partial$ , where  $\alpha > 1$ )**

Step 4: **De-interleave and evaluate mean of  $\gamma(c)$  for all possible values of  $[N_r' N_c']$**

Step 5:  $[N_r^{est}, N_c^{est}] = \underset{N_r', N_c'}{\text{argmax}}(\mu'(b))$ , where  $\mu'(b) = \frac{\sum_{c=1}^b \gamma(c)}{b}$

# Fixing Threshold: Analytical approach

- Data matrix  $S$  is converted into a column echelon form  $F$  using modified GJETP as follows:  $S \cdot \chi = F$ , where  $\chi$  is a permutation matrix
- Columns in  $\chi$  corresponding to all-zero columns in  $F$  is known as kernel of  $S$  i.e.  $S \cdot d = 0$ , where  $d = [d_1 d_2 \cdots d_b]^T$
- XOR of the column elements in  $S$  with column index  $i$  corresponding to  $d_i = 1$  is equal to 0
- Otherwise, there should be even number of ones in the column positions of  $S$  corresponding to  $d_i = 1$ , such that  $S \cdot d = 0$
- **Erroneous scenario:** XOR of the column elements in  $S$  with index  $i$  corresponding to  $d_i = 1$  will not be equal to zero due to odd number of ones
- The number of ones  $\phi_j(c)$  in  $c^{\text{th}}$  dependent column of  $F_j$  is a RV denoted by  $B_c^j$
- $\phi_j(c)$  follows a binomial distribution with parameters  $a$  and  $P_c^j$  (i.e.  $B_c^j \sim \mathcal{B}(a, P_c^j)$ ), where  $P_c^j$  denotes the probability of getting ones in  $c^{\text{th}}$  dependent column of  $F_j$

# Fixing Threshold: Analytical approach

- The probability of getting ones in  $c^{\text{th}}$  dependent column of  $F_j$  is given by

$$\begin{aligned}
 P_c^j &= 1 - \sum_{l=0}^{\text{floor}\left(\frac{z_j^c}{2}\right)} \binom{z_j^c}{2l} p_e^{2l} (1 - p_e)^{z_j^c - 2l} \\
 &= \frac{1 - (1 - 2p_e)^{z_j^c}}{2},
 \end{aligned}$$

where  $z_j^c$  denotes the hamming weight of  $c^{\text{th}}$  column in column permutation matrix  $\chi_j$  and  $p_e$  denotes the BER value

- Binary symmetric channel (BSC) model is assumed for calculating the threshold value
- The number of ones  $\phi_j(k)$  in  $k^{\text{th}}$  independent column of  $F_j$  is also a RV denoted by  $D_k^j$  and it follows  $\mathcal{B}(a, P_k'')$
- $P_k'' = 0.5$ . **Reason:** Probability of ones and zeros appearing in an independent column is equally likely
- For larger values of  $a$ , binomial RVs  $B_c^j$  and  $D_k^j$  can be approximated as normal RVs
- $B_c^j \sim \mathcal{N}(a.P_c^j, a.P_c^j.(1 - P_c^j))$  and  $D_k^j \sim \mathcal{N}\left(\frac{a}{2}, \frac{a}{4}\right)$ , where  $\mathcal{N}(\mu, \sigma^2)$  denotes the normal distribution with mean  $\mu$  and variance  $\sigma^2$ .
- The mean value of  $N$  different normal RVs  $B_c^j$  is also a RV denoted by  $\mu_c$
- $\mu_c$  also follows normal distribution with mean and variance  $\frac{a.q}{N}$  and  $\frac{a.r}{N^2}$ , where  $q = \sum_{j=1}^N P_c^j$  and  $r = \sum_{j=1}^N P_c^j.(1 - P_c^j)$
- Mean value of  $N$  different normal RVs  $D_k^j$  is denoted by  $\mu'_k$  also follows normal distribution with mean and variance  $\frac{a}{2}$  and  $\frac{a}{4N}$

# Fixing Threshold: Analytical approach

- The probability of mis-detection of either dependent or independent column is given by

$$P_{\text{md}} = Pr\left(\frac{\mu_c}{a} > \Gamma^{\text{th}} \mid c^{\text{th}} \text{ column is dependent}\right) + Pr\left(\frac{\mu'_k}{a} < \Gamma^{\text{th}} \mid k^{\text{th}} \text{ column is independent}\right)$$

- By substituting the normal PDF

$$P_{\text{md}} = \int_{a\Gamma^{\text{th}}}^{\infty} \frac{N}{\sqrt{2\pi}ar} \exp\left(-\frac{N^2(\mu_c - \frac{aq}{N})^2}{2ar}\right) d\mu_c + \int_{-\infty}^{a\Gamma^{\text{th}}} \frac{\sqrt{2N}}{\sqrt{\pi}a} \exp\left(-\frac{2N(\mu'_k - \frac{a}{2})^2}{a}\right) d\mu'_k$$

- The expression for  $\Gamma_{\text{opt}}^{\text{th}}$ , which minimizes  $P_{\text{md}}$ , can be derived by differentiating  $P_{\text{md}}$  and equating it to zero
- After simplification,  $\Gamma^{\text{th}}$  is given by

$$\Gamma_{\text{opt}}^{\text{th}} = \frac{-\gamma - \sqrt{\gamma^2 - (\alpha \Delta)}}{\alpha}$$

where  $\alpha = Na(4r - N)$ ,  $\gamma = Na(q - 2r)$ , and  $\Delta = raN - aq^2 - r \ln\left(\frac{4r}{N}\right)$

- Exact BER value  $p_e$  need not be known to evaluate  $\Gamma_{\text{opt}}^{\text{th}}$
- We can fix a maximum value for BER  $p_e^{\text{max}}$  depending upon the environment and calculate  $\Gamma_{\text{opt}}^{\text{th}} \rho(S)$
- The same optimal threshold values work well for the case when  $p_e \leq p_e^{\text{max}}$ .

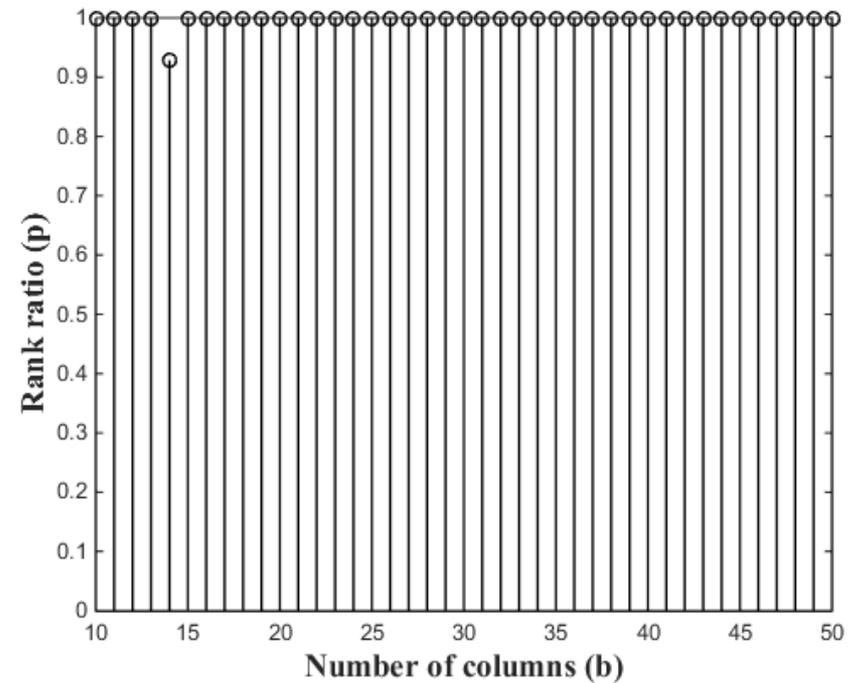
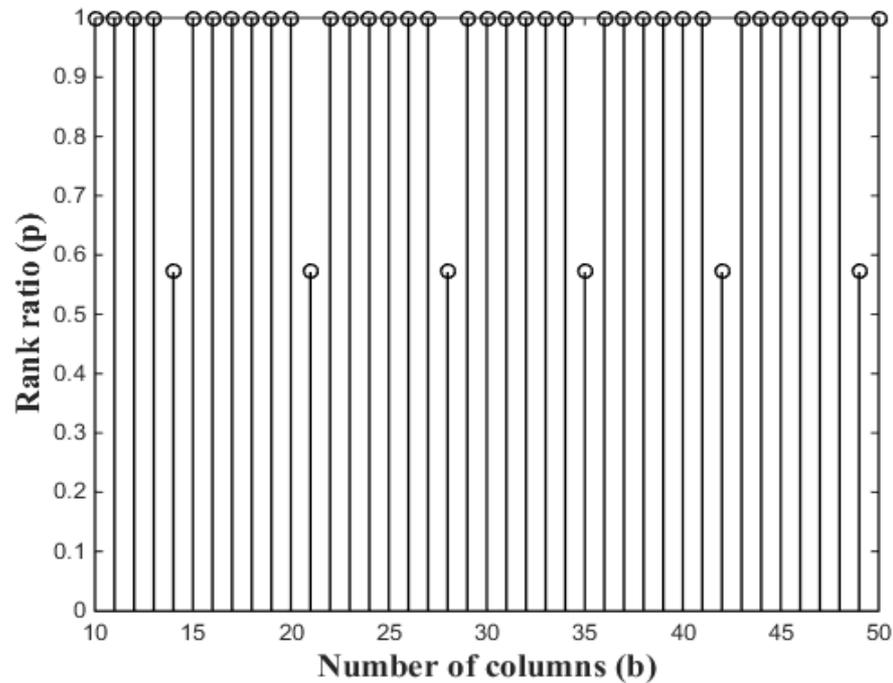
# Fixing Threshold: Histogram approach

- The optimal threshold value can also be fixed by plotting the **histogram for zero-ratio or non-zero-ratio**
- A range of possible threshold values, which segregate the dependent and independent columns, is obtained
- From the range of possible values, **a safe optimal threshold value is fixed**
- The same optimal threshold value is applicable for all the other values of  $b$  to classify the dependent and independent columns

# Simulation Results

- Various transmission standards specify the allowable BER for a given quality of service (QOS)
- For example, the **post-FEC BER** requirement for desirable operation of **DVB receiver** is  $2 \times 10^{-4}$
- Considering the BER values together with the allowances of coding gain, the **pre-FEC BER values** for acceptable performance will be usually greater than  $10^{-3}$
- Considering these factors, we have taken a **safe value of  $10^{-2}$**  as the BER threshold **to account the worst case scenario**
- The overall performance of the algorithm is extensively tested within the range of  $5 \times 10^{-3}$  to  $6 \times 10^{-2}$

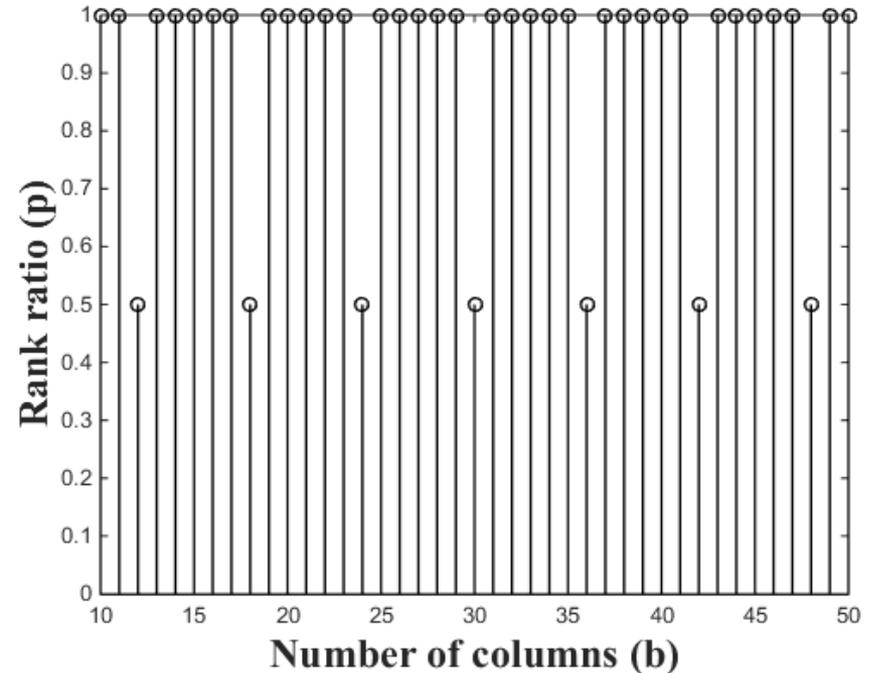
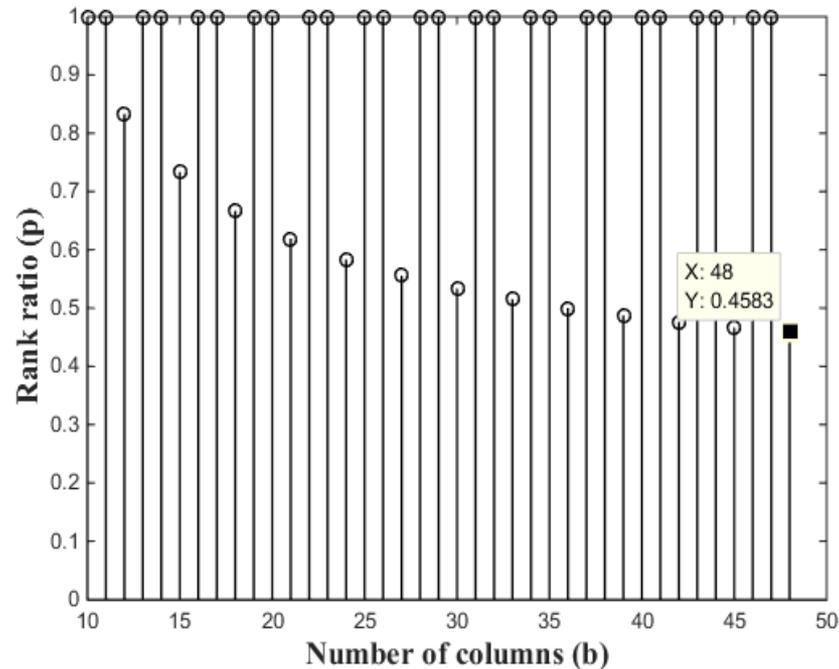
# Simulation Results



- Variation of rank ratio  $p$  with  $b$  for  $B(7, 4)$  assuming  $\text{BER} = 0$
- Variation of rank ratio  $p$  with  $b$  for  $B(7, 4)$  assuming  $\text{BER} = 5 \times 10^{-3}$

Rank values are obtained using algorithm proposed for non-erroneous scenario

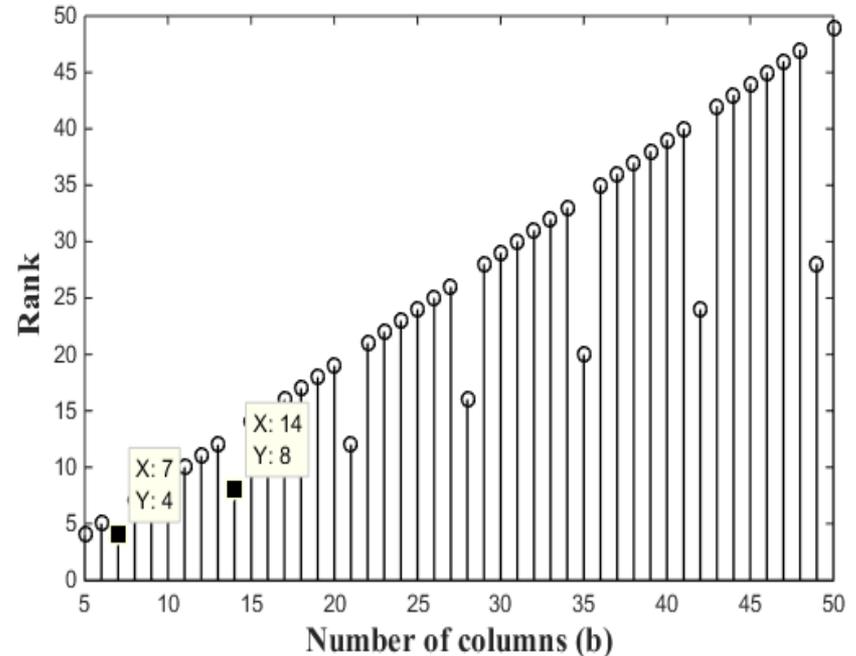
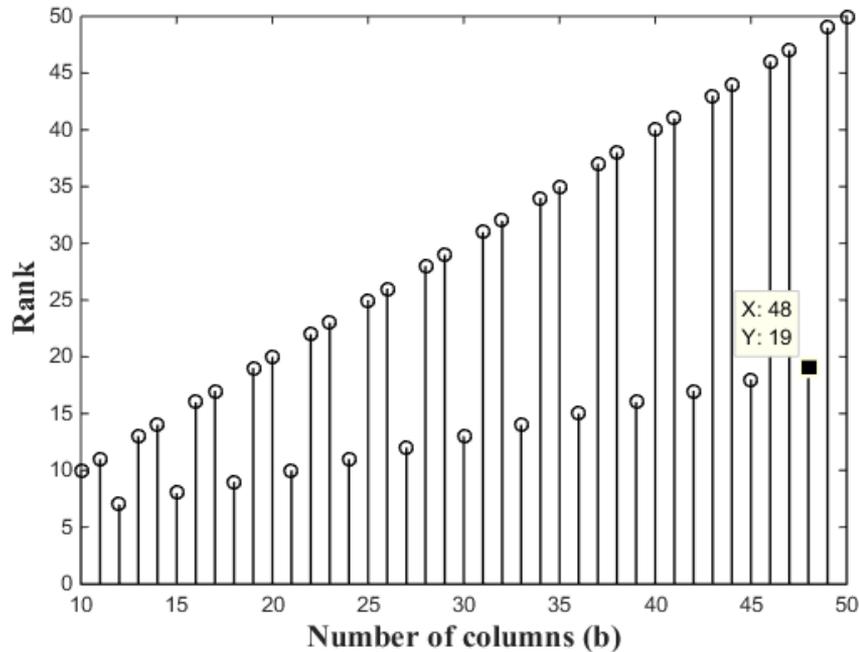
# Simulation Results



- Rank ratio  $p$  versus number of columns  $b$  for  $C(3, 1, 7)[133, 165, 171]$  with  $BER = 2 \times 10^{-2}$ .
- Rank ratio  $p$  versus number of columns  $b$  for  $B(6, 3)$  with  $BER = 10^{-2}$

Rank values are obtained using algorithm proposed for erroneous scenario: Histogram approach

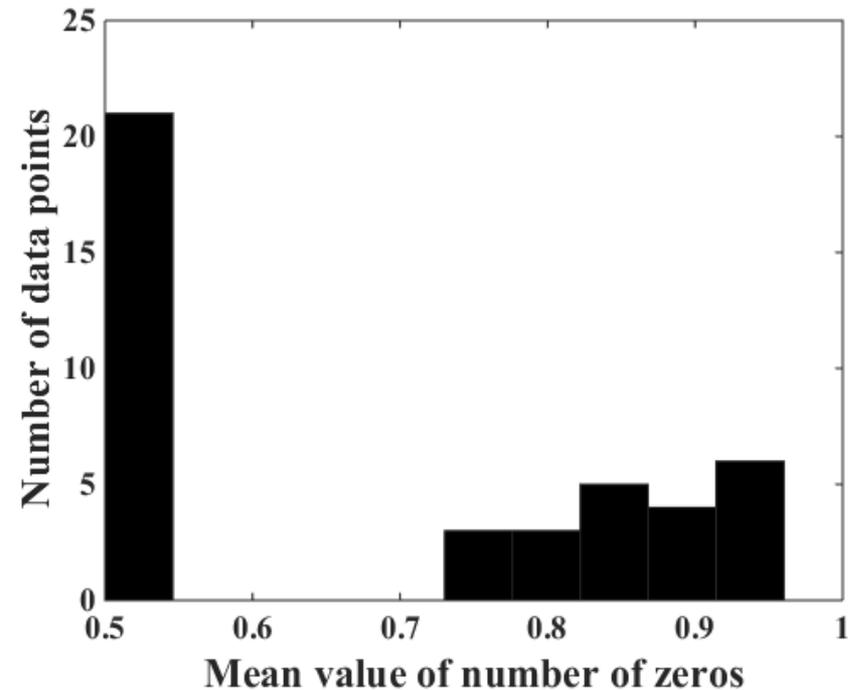
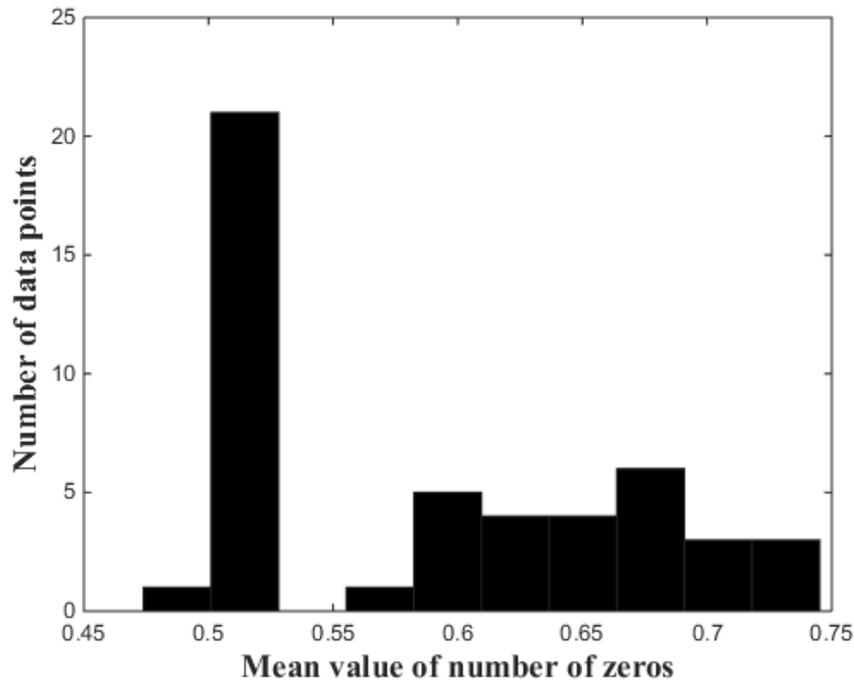
# Simulation Results



- Rank  $\rho(S)$  versus number of columns  $b$  for  $C(3, 1, 4)[13, 15, 17]$  with  $\text{BER} = 2 \times 10^{-2}$ .
- Rank  $\rho(S)$  versus number of columns  $b$  for  $B(7, 4)$  with  $\text{BER} = 10^{-2}$

By plotting rank ratio versus number of columns, code classification can be performed and by plotting rank versus number of columns,  $n$  and  $k$  can be identified

# Simulation Results

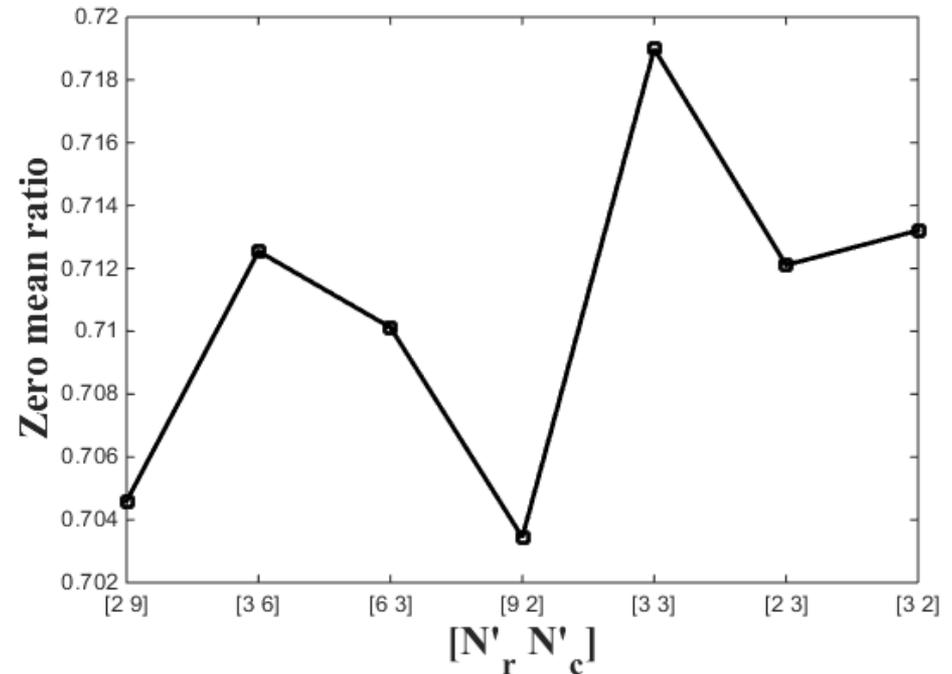
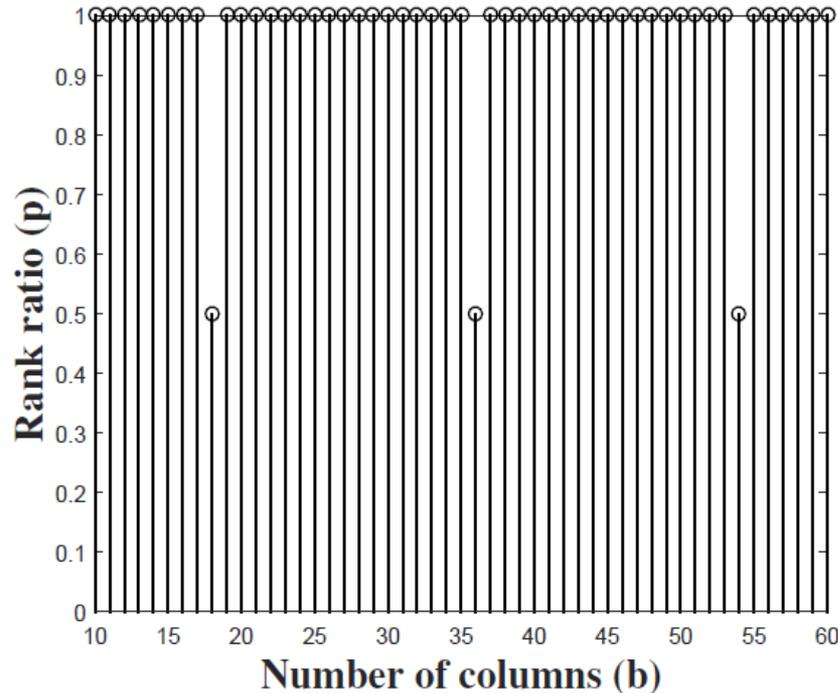


- Histogram plot for  $mean(A)$  considering  $b = 48$ ,  $C(3, 1, 7)[133, 165, 171]$ , and  $BER = 2 \times 10^{-2}$ .
- Histogram plot for  $mean(A)$  considering  $b = 42$ ,  $B(6, 3)$ , and  $BER = 10^{-2}$

(a) Fig. 1 :  $\Gamma_{opt}^{th}$  can be fixed between 0.53 to 0.56

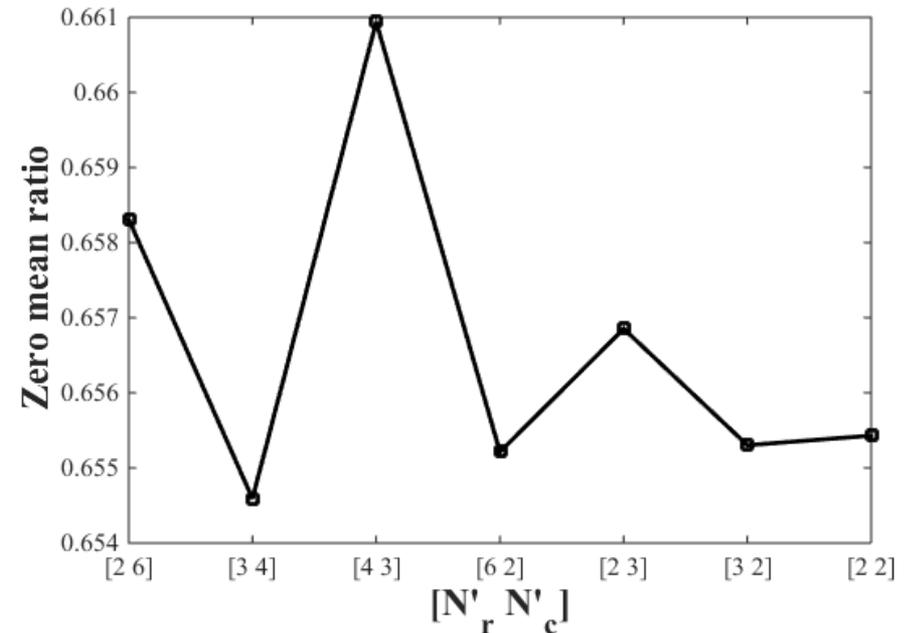
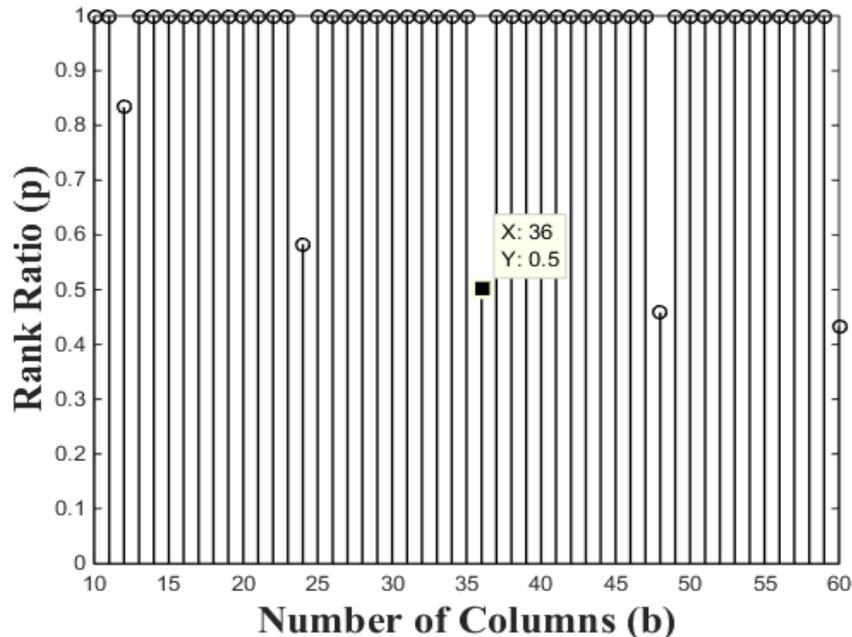
(b) Fig. 2 :  $\Gamma_{opt}^{th}$  can be fixed between 0.55 to 0.57

# Simulation Results



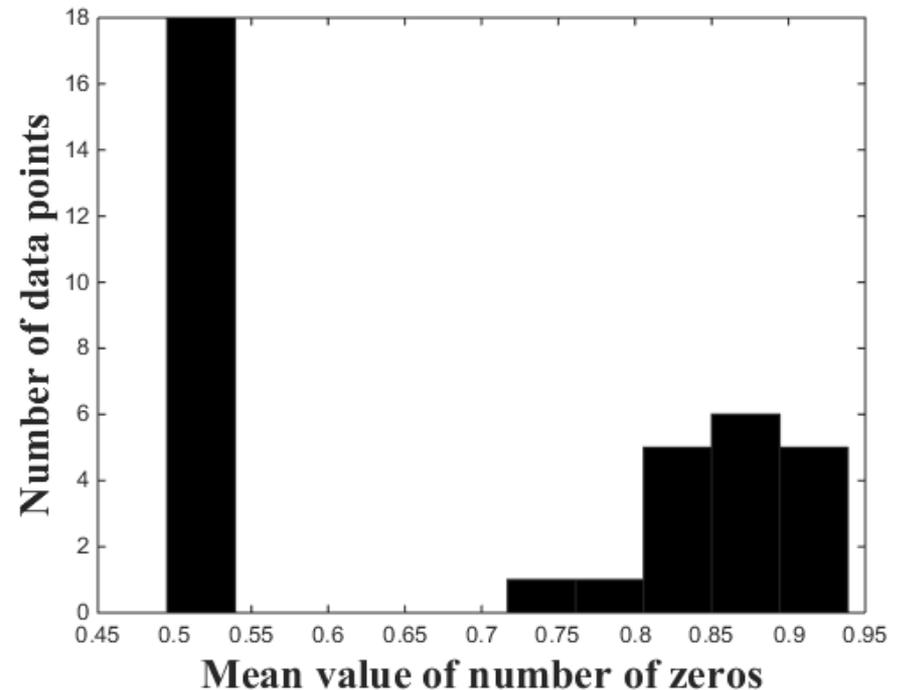
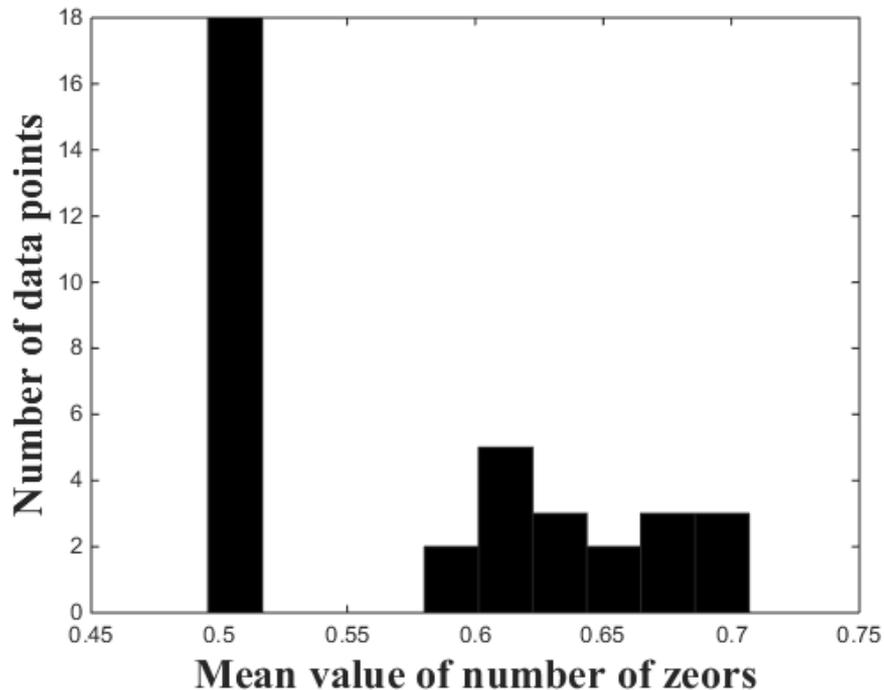
- Variation of rank ratio  $p$  with  $b$  for  $B(6, 3)$  considering block interleaver assuming  $N_r = 3$ ,  $N_c = 3$ , and  $\text{BER} = 10^{-2}$ .
- Variation of zero mean ratio  $\delta'(b)$  with all possible values of  $[N'_r \ N'_c]$  for  $B(6, 3)$  considering block interleaver assuming  $N_r = 3$ ,  $N_c = 3$ , and  $\text{BER} = 10^{-2}$ .

# Simulation Results



- Variation of rank ratio  $p$  with  $b$  for  $C(3, 1, 7)[133, 165, 171]$  considering block interleaver assuming  $N_r=4$ ,  $N_c=3$ , and  $\text{BER} = 2 \times 10^{-2}$ .
- Variation of zero mean ratio with all possible values of  $[N'_r N'_c]$  for  $C(3, 1, 7)[133, 165, 171]$  considering block interleaver assuming  $N_r=4$ ,  $N_c=3$ , and  $\text{BER} = 2 \times 10^{-2}$

# Simulation Results



- Histogram plot for  $mean(A)$  assuming  $b=36$ ,  $N_r=4$ ,  $N_c=3$ ,  $C(3, 1, 7)[133, 165, 171]$ , and  $BER = 2 \times 10^{-2}$ .
- Histogram plot for  $mean(A)$  assuming  $b=36$ ,  $N_r=3$ ,  $N_c=3$ ,  $B(6, 3)$ , and  $BER = 10^{-2}$ 
  - Fig. 1 :  $\Gamma_{opt}^{th}$  can be fixed between 0.52 to 0.59
  - Fig. 2 :  $\Gamma_{opt}^{th}$  can be fixed between 0.54 to 0.7

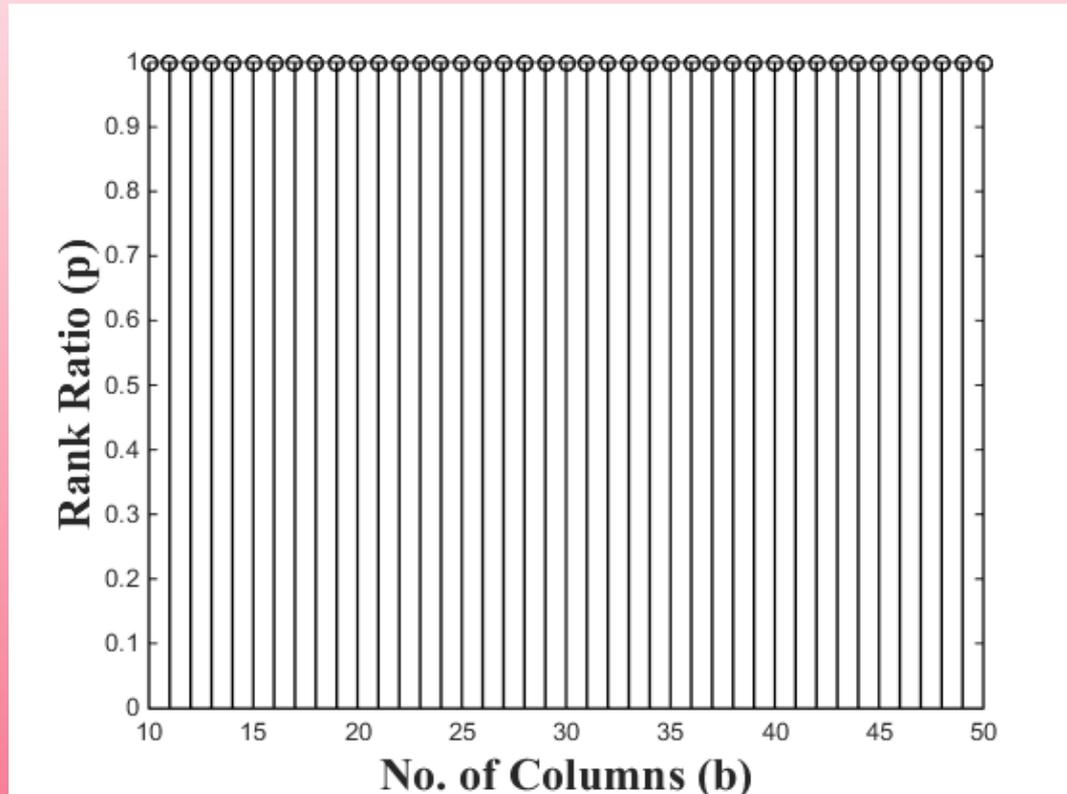
# Simulation Results

- As the BER increases, the range for choosing threshold value decreases and incorrect value will change the rank ratio characteristics of the block and convolutional codes

TABLE II  
THRESHOLD VALUES  $\Gamma_{\text{opt}}^{\text{th}}$  FOR VARIOUS TEST CASES

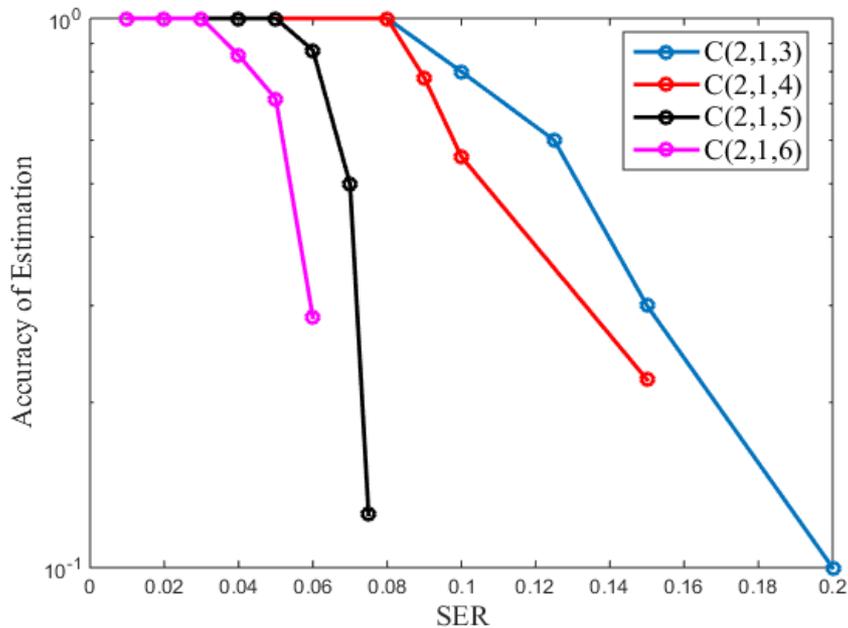
BER	FEC codes	Number of rows	$\Gamma^{\text{th}}$
$10^{-2}$	C[2,1,3], C[2,1,4] C[2,1,5], C[2,1,6] C[2,1,7], C[3,1,4] C[3,1,7], C[3,1,11] C[4,1,7], C[4,1,10] B(8,5), B(7,4) B(6,3), B(3,2)	$a = 20 \times b$	0.55
$2 \times 10^{-2}$	C[2,1,3], C[2,1,4] C[2,1,5], C[3,1,4] C[3,1,7], C[4,1,7] C[4,1,10] B(8,5), B(7,4) B(6,3), B(3,2)	$a = 20 \times b$	0.55
	C[2,1,6], C[3,1,11]	$a = 50 \times b$	0.52
	C[2,1,7]	$a = 70 \times b$	0.51

# Simulation Results

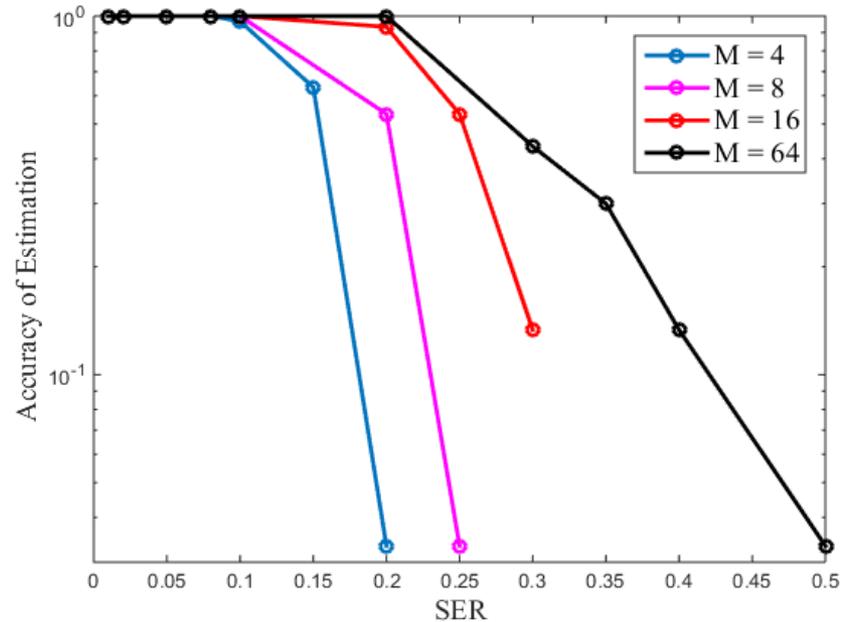


Variation of Rank ratio with respect to number of columns for uncoded data symbols assuming  $BER = 10^{-2}$

# Simulation Results



(a)



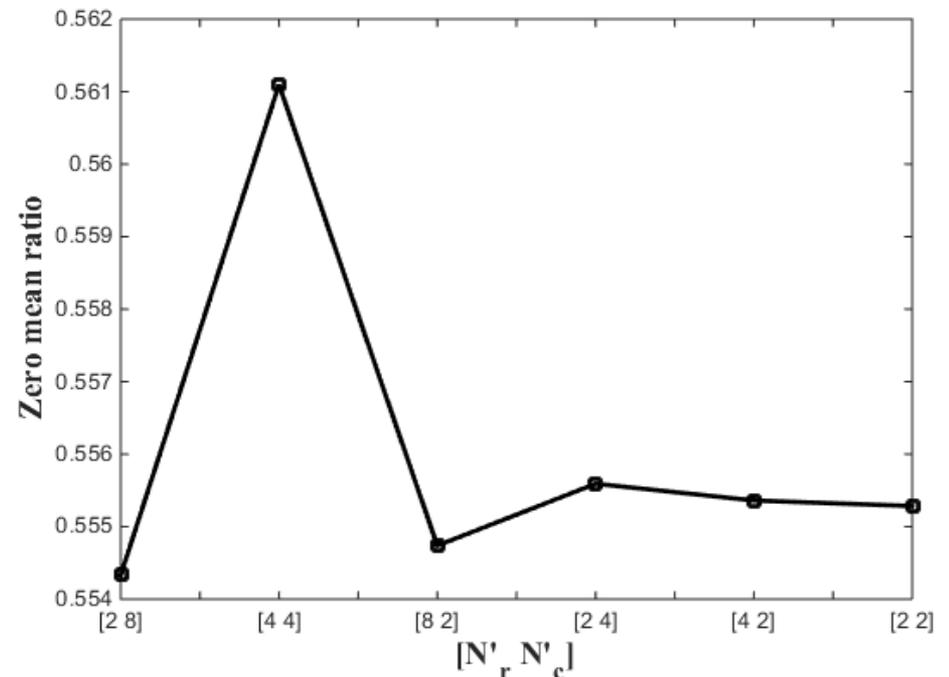
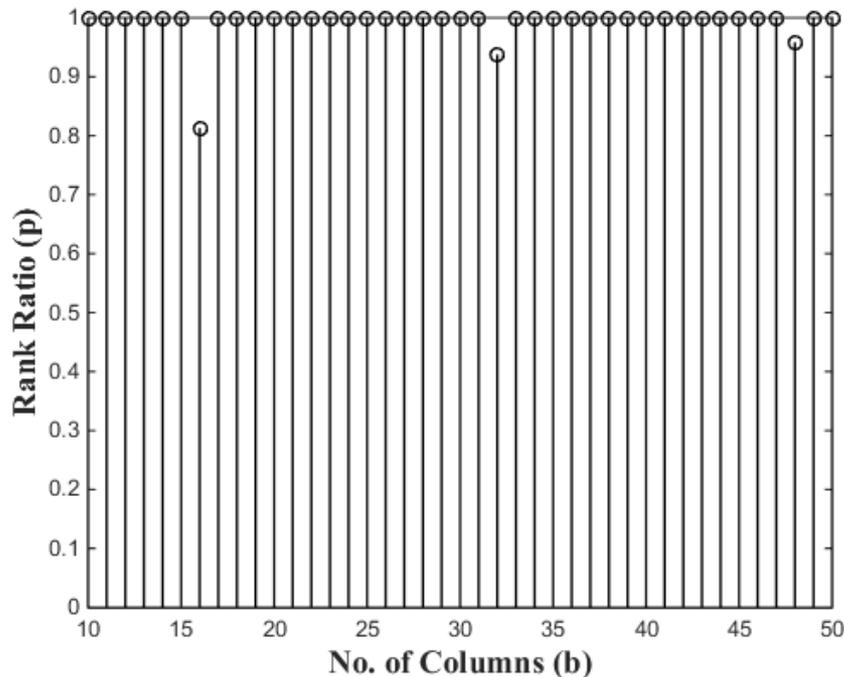
(b)

- (a) Accuracy of estimation of rate 1/2 convolutional codes considering QPSK constellation by varying SER values
- (b) Accuracy of estimation of block interleaver parameters for different M-QAM modulation schemes by varying SER values assuming  $N_r = 4$ ,  $N_c = 3$ , and  $C(3, 1, 4)$ [13, 15, 17]

# Discussions

- The code classification in the presence of interleaver can be performed with **100 % accuracy until  $BER \leq 2 \times 10^{-2}$**  based on **histogram approach**
- When  **$BER > 2 \times 10^{-2}$** , the proposed methodology **fails** to classify the incoming data symbols
- Reason: Unique rank ratio characteristics will change drastically due to more number of erroneous bits
- However, the **estimation of interleaver parameters** are observed to be successful until  **$BER$  of  $6 \times 10^{-2}$**
- For  **$BER > 6 \times 10^{-2}$** , the proposed algorithm (**histogram approach**) **fails** to recognize the interleaver parameters
- For the case **without interleaver**, the **histogram approach fails** to recognize the type of FEC codes **for  $BER > 4 \times 10^{-2}$**
- If optimal threshold value is fixed based on **analytical approach**, then code classification can be performed with **100% accuracy until  $BER$  of  $5 \times 10^{-3}$**
- For  **$BER > 5 \times 10^{-3}$** , optimal threshold based on the **analytical approach fails** to classify the incoming symbols
- However, interleaver parameters can be estimated correctly until  **$BER$  of  $2 \times 10^{-2}$**

# Simulation Results



- Variation of rank ratio  $p$  with  $b$  for  $B(8, 5)$  considering block interleaver assuming  $N_r = 4$ ,  $N_c = 4$ , and  $\text{BER} = 6 \times 10^{-2}$
- Variation of zero mean ratio  $\delta'(b)$  with all possible values of  $[N'_r \ N'_c]$  for  $B(8, 5)$  considering block interleaver with  $N_r = 4$ ,  $N_c = 4$ , and  $\text{BER} = 6 \times 10^{-2}$

# Simulation Results

TABLE III  
COMPARISON OF ACCURACY OF ESTIMATION OF  
DIFFERENT METHODOLOGIES

Test case	Actual No. of dependent columns for $b=48$	Estimated number of dependent columns for $b=48$ (% of Accuracy)		
		Fixing $\Gamma_{opt}^{th}$ based on (25)	Fixing $\Gamma_{opt}^{th}$ based on [6, eq.(A.4)]	Fixing $\Gamma_{opt}^{th}$ based on histogram approach
C(2,1,3)	22	19 (86.4%)	15 (68.2%)	22 (100%)
C(2,1,4)	21	17 (81%)	17 (81%)	21 (100%)
C(3,1,4)	29	28 (96.6%)	26 (89.7%)	29 (100%)
C(3,1,7)	26	23 (88.5%)	23 (88.5%)	26 (100%)
C(3,1,10)	23	16 (82.6%)	16 (82.6%)	23 (100%)
C(4,1,7)	30	29 (96.7%)	26 (86.7%)	30 (100%)

# Observations

- **Histogram approach:** Distribution of mean value of number of zeros in each columns has been predicted accurately.
- **Analytical approach:** Approximated the binomial distribution of mean value of number of zeros to normal distribution
- The performance degradation is mainly due to the approximation of binomial to normal distribution
- All the three methodologies are compared by keeping the computation time constant

# Conclusions

- Innovative algorithms for joint estimation of type of FEC codes and block interleaver parameters have been proposed
- Firstly, estimation of interleaver period along with code classification among block, convolutional coded, and uncoded data symbols is performed
- After that while de-interleaving, rest of the block interleaver parameters are estimated
- It can be concluded that the deficient rank ratio remains constant at  $r$  for block codes
- For convolutional codes, the deficient rank ratio decays rapidly and remain approximately constant slightly above  $r$
- Moreover, irrespective of the number of columns, full rank is obtained for uncoded data stream
- To justify the proposed claims, simulation results for recognizing the type of FEC codes and interleaver parameters are shown

# Part 2: Blind Reconstruction of Reed-Solomon Encoder and Interleavers Over Noisy Environment

Swaminathan R, A.S.Madhukumar, Wang Guohua, and Ting Shang Kee, "Blind reconstruction of Reed-Solomon encoder and interleavers over noisy environment", IEEE Transactions on Broadcasting, vol. 99, no. PP, pp. 1 - 16, Early Access, 2018



# Motivations

The main motivations are given as follows:

- **Non-cooperative scenario:** It is mandatory to recognize the code and interleaver parameters at the receiver
- To propose an intelligent receiver system which adapts itself to any specific applications
- **Previously proposed algorithm** in [R1] for blind reconstruction of RS encoder **can recognize only codeword length**
- **LLR-based technique** [R2]: Assumes a **predefined candidate set** of RS encoders at transmitter and receiver
- The bit position adjustment parameter to achieve time synchronization is not recognized in [R1] and [R2]

[R1] Y. Zrelli, M. Marazin, R. Gautier, E. Rannou, and E. Radoi, "Blind identification of code word length for non-binary error-correcting codes in noisy transmission," *EURASIP J. Wireless Commun. Netw.*, vol. 2015, no. 43, pp. 1–16, 2015

[R2] H. Zhang, H.-C. Wu, and H. Jiang, "Novel blind encoder identification of Reed-Solomon codes with low computational complexity," in *Proc. IEEE GLOBECOM*, Atlanta, GA, USA, 2013, pp. 3294–3299

# Motivations

- Block interleaver parameter estimation algorithms were restricted to convolutional encoded data.
- Only **Interleaver period was estimated** for non-binary RS codes [R3]
- **Algorithms** are not proposed for **estimating all block interleaver parameters** for non-binary codes
- **Bit/symbol position adjustment** parameter to achieve time synchronization is **not estimated**
- What is the **probability of correct detection** of RS code and block interleaver parameters using blind estimation algorithms ?

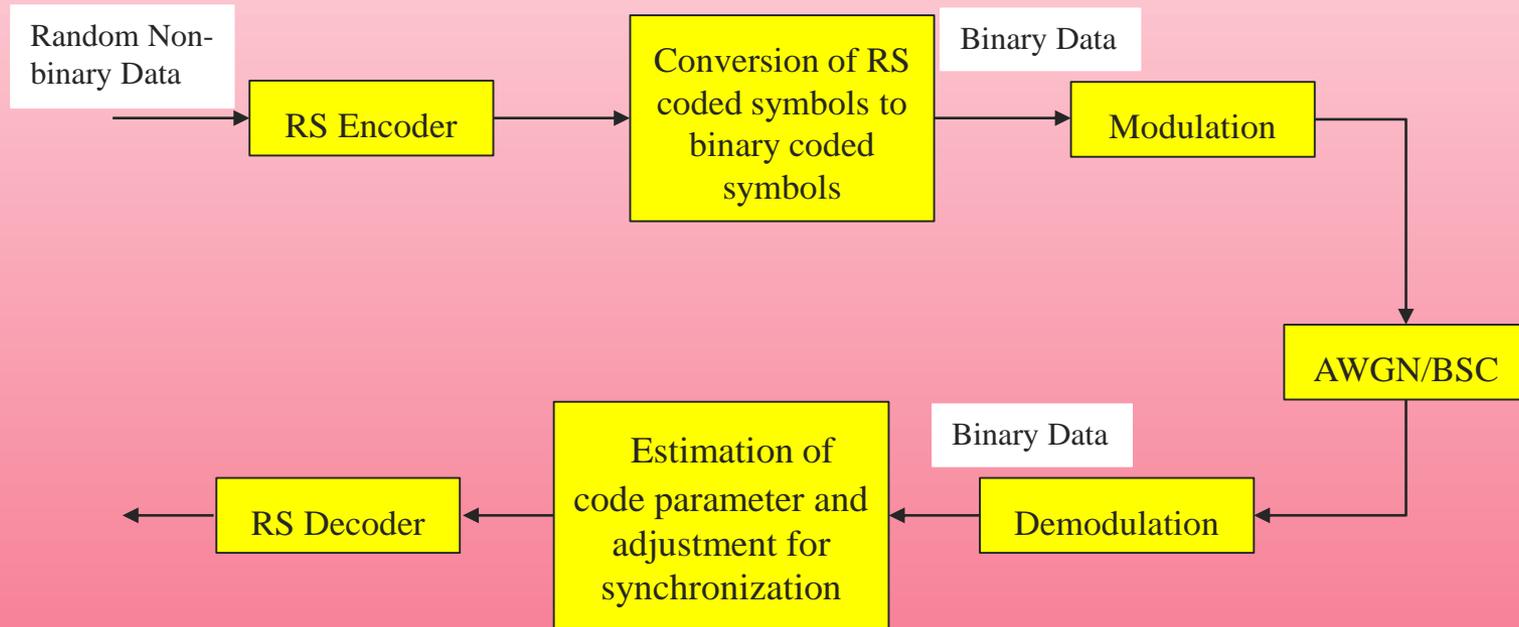
[R3] L. Lu, K. H. Li, and Y. L. Guan, "Blind detection of interleaver parameters for non-binary coded data streams," in *Proc. IEEE ICC*, Dresden, Germany, 2009, pp. 1–4.

# Contributions

The main contributions are given as follows:

- Innovative algorithms are proposed for the **blind recognition of RS encoder (with and without block interleaver)**
- Estimated RS code parameters: **codeword length  $n$ , code dimension  $k$ , number of bits per symbol  $m$ , primitive polynomial  $p$ , and generator polynomial  $g(x)$**
- Estimated interleaver parameters: Interleaver period  $\beta$  and number of rows  $N_r$  and columns  $N_c$  of block interleaver matrix
- An innovative approach for **synchronization** compensation through appropriate bit/symbol positioning is discussed
- **Simulation results** are given for different test cases validating the proposed algorithms
- Performance of the algorithm in terms of **accuracy of estimation** is given and **compared with the prior works**

# Generic Block Diagram



Blind reconstruction of RS encoder

# Reed-Solomon Codes

- RS codes are different from binary linear block codes and hence, the parameter estimation is slightly different
- The code symbols generated from RS codes belong to  $GF(q)$ , where  $q = 2^m$  and  $m \geq 3$
- Let  $\alpha$  be a primitive element of  $GF(q)$  such that  $\alpha^{q-1} = 1$
- In the case of  $t'$  error correcting  $(n, k)$  RS codes,  $\alpha, \alpha^2, \dots, \alpha^{2t}$  are the roots of  $g(X)$  with degree  $n - k$ , which is given by

$$g(X) = (X - \alpha) (X - \alpha^2) \dots (X - \alpha^{2t})$$

- For RS codes,  $n = q - 1$  and  $n - k = 2t$
- Parameters to be estimated are  $n, k, m$ , primitive polynomial used for generating the Galois field (GF), and  $g(X)$
- $g(X)$  can be estimated by recognizing  $n$  and  $k$ , since  $n - k = 2t$  and  $\alpha, \alpha^2, \dots, \alpha^{2t}$  are the roots of  $g(X)$

# Algorithm 1: Estimation of RS code parameters - Noiseless case

- **Notations:** Let  $\phi$  denotes the adjustment of bit position to achieve synchronization.  $a$ ,  $b$ ,  $\rho(m, p, \phi)$ , and  $\rho'(m, p, \phi)$  denote the number of rows, columns, rank, and rank ratio of data matrix  $S$ , respectively
- **Assumptions:**  $a \geq 2b$ ,  $m \in [m_{\min}, m_{\max}]$ ,  $p \in [p_{\min}, p_{\max}]$ , and  $\phi \in [0, ((q - 1)m) - 1]$
- $p = \text{primpoly}(m, 'all')$
- Shift the binary data symbols by  $\phi$  bit positions and convert the same into non-binary symbols
- Create a GF array from the non-binary data symbols using primitive polynomial  $p$
- Reshape the RS encoded GF array elements into a data matrix  $S$  of size  $a \times b$ , where  $b = q - 1$
- Convert  $S$  into  $F$  using finite-field Gauss elimination process
- Compute  $\rho(m, p, \phi)$  from the number of non-zero columns in  $F$
- Compute  $\rho'(m, p, \phi) = \rho(m, p, \phi) / b$
- Obtain  $[m_{\text{est}}, p_{\text{est}}, \phi_{\text{est}}] = \underset{m, p, \phi}{\text{argmin}}(\rho'(m, p, \phi))$ ,  $n_{\text{est}} = 2^{m_{\text{est}}} - 1$ , and  $k_{\text{est}} = \rho(m_{\text{est}}, p_{\text{est}}, \phi_{\text{est}})$

# Observations

- If  $b = n_{\text{est}} = 2^{m_{\text{est}}} - 1$ , then rank deficiency will be obtained.  $\rho(m_{\text{est}}, p_{\text{est}}, \phi_{\text{est}})$  and  $\rho'(m_{\text{est}}, p_{\text{est}}, \phi_{\text{est}})$  for  $b = n_{\text{est}}$  are, respectively, given by

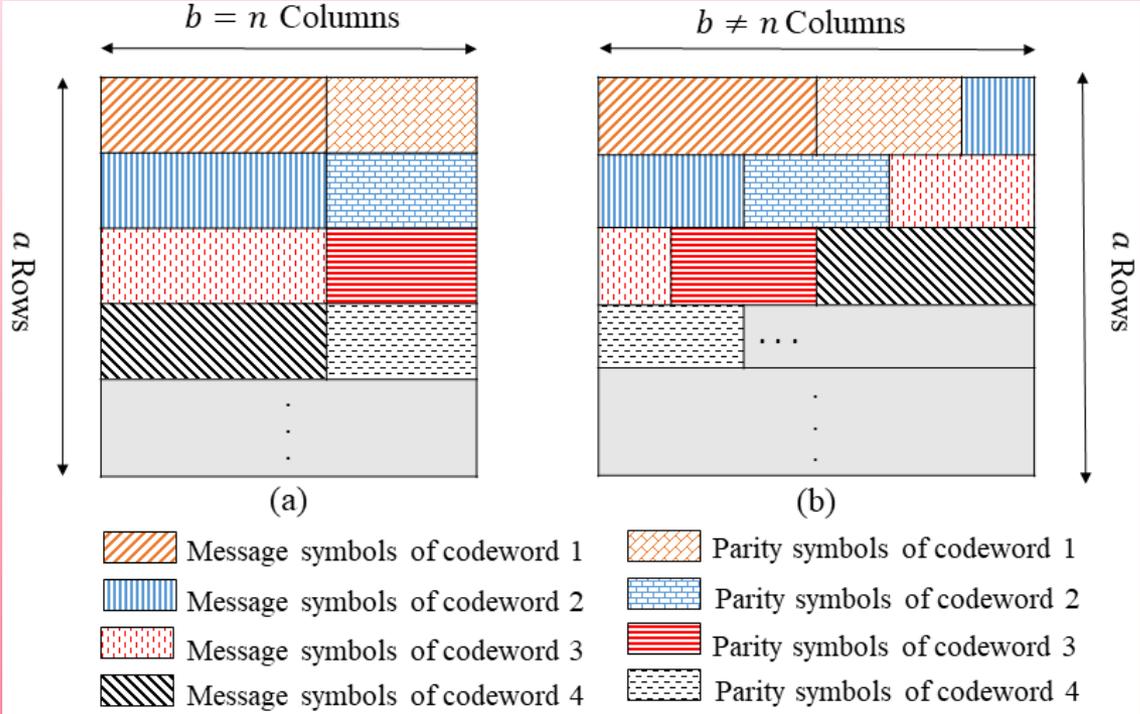
$$\begin{aligned}\rho(m_{\text{est}}, p_{\text{est}}, \phi_{\text{est}}) &= k_{\text{est}}, \\ \rho'(m_{\text{est}}, p_{\text{est}}, \phi_{\text{est}}) &= \frac{k_{\text{est}}}{b} = r,\end{aligned}$$

- If  $b$  is a multiple of  $n_{\text{est}}$  (i.e.  $b = \alpha \cdot n_{\text{est}}$ ), then the deficient rank value is given by  $\alpha \cdot k_{\text{est}}$ . However, if  $b \neq \alpha \cdot n_{\text{est}}$ , then full rank will be obtained.

## Explanation:

- The output  $n$  data symbols depend only on  $k$  input symbols in the case of RS codes
- Therefore,  $\alpha \cdot n_{\text{est}}$  output symbols of  $S$  depend on  $\alpha \cdot k_{\text{est}}$  input symbols
- When  $b = \alpha \cdot n_{\text{est}}$ , then  $\alpha$  codewords in all the rows will be aligned properly in the same column
- If the data and parity symbols in all the rows are aligned properly in the same column, linear relation is satisfied in all the rows
- There will exist linear relations between columns in  $S$
- After converting  $S$  into  $F$  through finite field Gauss elimination process, all  $\alpha \cdot (n_{\text{est}} - k_{\text{est}})$  dependent columns will be eliminated, which will give rise to rank deficiency

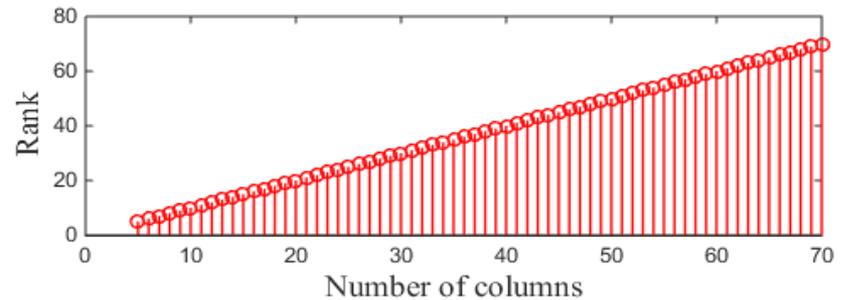
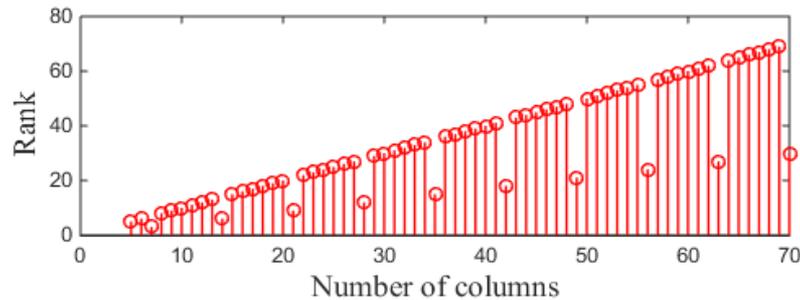
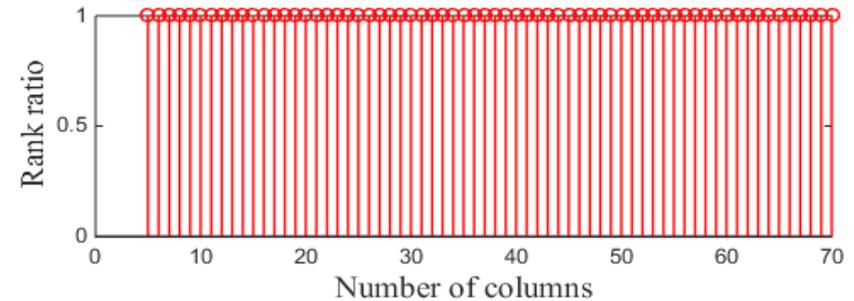
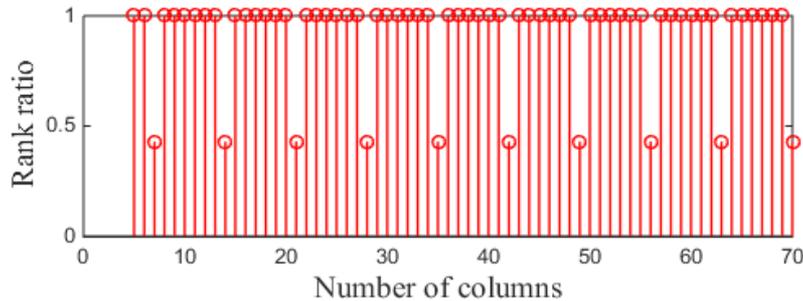
# Observations



Structure of data matrix for the case when (a)  $b = n$  and (b)  $b \neq n$

- If data and parity symbols are segregated in different rows and are not aligned properly in the same column, then the linear relation will be affected
- $S$  will behave like a random matrix and will not have any dependent columns
- After converting  $S$  into  $F$ , no dependent columns will be eliminated and full rank will be obtained.

# Observations



- Variation of rank ratio and rank versus number of columns for RS(7, 3, 3, 11) considering non-erroneous case
- Variation of rank ratio and rank versus number of columns for RS(7, 3, 3, 11) considering  $SER=3 \times 10^{-2}$

# Parameter Estimation: Noisy case

- The **dependent columns** in  $S$  will be converted into **all-zero-columns** in  $F$  using **finite-field Gauss elimination process**
- The **Algorithm 1** proposed for non-erroneous scenario **fails** for erroneous scenario, **since full rank** will be obtained
- **Rank-deficient matrix** under erroneous channel conditions will have **less number of non-zero elements** compared to the **full-rank matrix**
- Therefore, the rank-deficient data matrix is identified based on evaluating the **non-zero-mean-ratio** in the case of **erroneous scenario**

# Algorithm 2: Estimation of RS code parameters - Noisy case

- **Notations:** The mean and normalized mean values of number of non-zero elements in  $c^{\text{th}}$  column of  $F$  are denoted as  $\sigma(c, m, p, \phi)$  and  $\sigma'(c, m, p, \phi)$ , respectively
- $p = \text{primpoly}(m, 'all')$
- Shift the binary data symbols by  $\phi$  bit positions and convert the same into non-binary symbols
- Create a GF array from the non-binary data symbols using primitive polynomial  $p$
- GF array is created using primitive polynomial  $p$  from the non-binary data symbols
- RS encoded GF array elements are reshaped into  $S$  of size  $a \times b$ , where  $b = q - 1$
- Using finite-field Gauss elimination process,  $S$  is converted into  $F$
- Evaluate  $\sigma(c, m, p, \phi)$ , where  $c \in \{1, 2, \dots, b\}$  and obtained values are normalized with respect to maximum value
- Calculate the normalized non-zero-mean-ratio, which is given by  $\mu'(m, p, \phi) = \frac{\sum_{c=1}^b \sigma'(c, m, p, \phi)}{b}$
- Obtain  $[m_{\text{est}}, p_{\text{est}}, \phi_{\text{est}}] = \underset{m, p, \phi}{\text{argmin}}(\mu'(m, p, \phi))$  and  $n_{\text{est}} = 2^{m_{\text{est}}} - 1$

# Discussions

- The rank deficiency will be observed for correct values of  $m$ ,  $p$ , and  $\phi$
- Since rank-deficient data matrix will have less number of non-zero elements,  $\mu'(m_{\text{est}}, p_{\text{est}}, \phi_{\text{est}})$  will be smaller compared to other possible combinations of  $[m, p, \phi]$
- Code and generator polynomials of RS codes have equal number of roots and is given by  $n - k = 2t$
- By finding the number of roots of code polynomial,  $n_{\text{est}} - k_{\text{est}}$  is identified
- $k_{\text{est}}$  can be recognized from  $n_{\text{est}} - k_{\text{est}}$
- After recognizing the number of roots of code polynomial,  $g(x)$  is obtained

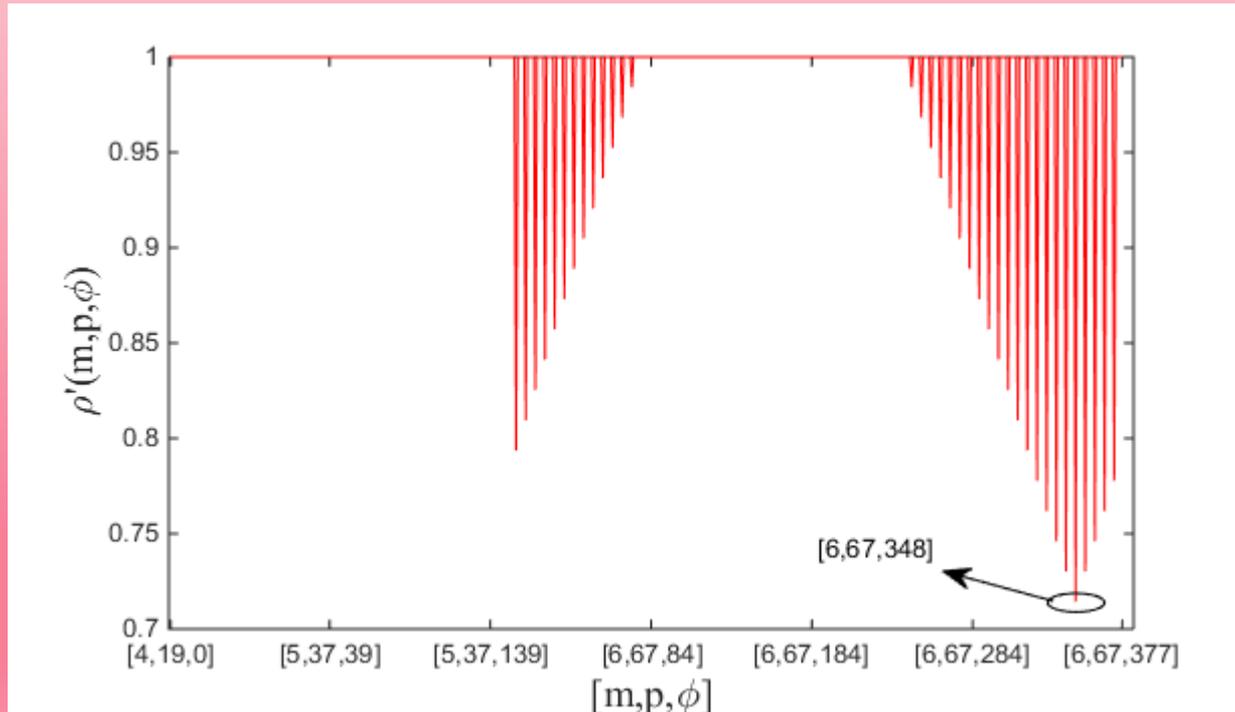
# Simulation parameters

Table 1: Simulation parameters

Modulation schemes	BPSK, QPSK, 8-PSK, 8-QAM 16-PSK, 16-QAM, 32-QAM, 64-QAM, 256-QAM
Symbol error rate (SER)	0.001 to 0.1
Signal-to-noise ratio (SNR)	$\geq 5$ dB
Number of rows	$a=2b$ (for non-erroneous case) and $a > 2b$ (for erroneous case)
RS Codes tested	RS(7, 3, 3, 11), RS(15, 7, 4, 19), RS(15, 9, 4, 19), RS(15, 11, 4, 19), RS(31, 15, 5, 37), RS(31, 19, 5, 37), RS(31, 23, 5, 37), RS(63, 45, 6, 67), RS(255, 127, 8, 285)
Block interleaver parameters	(a) $N_r = 15$ and $N_c = 7$ (b) $N_r = 5$ , $N_c = 2$ , and $d = 4$ (c) $N_r = 5$ and $N_c = 6$

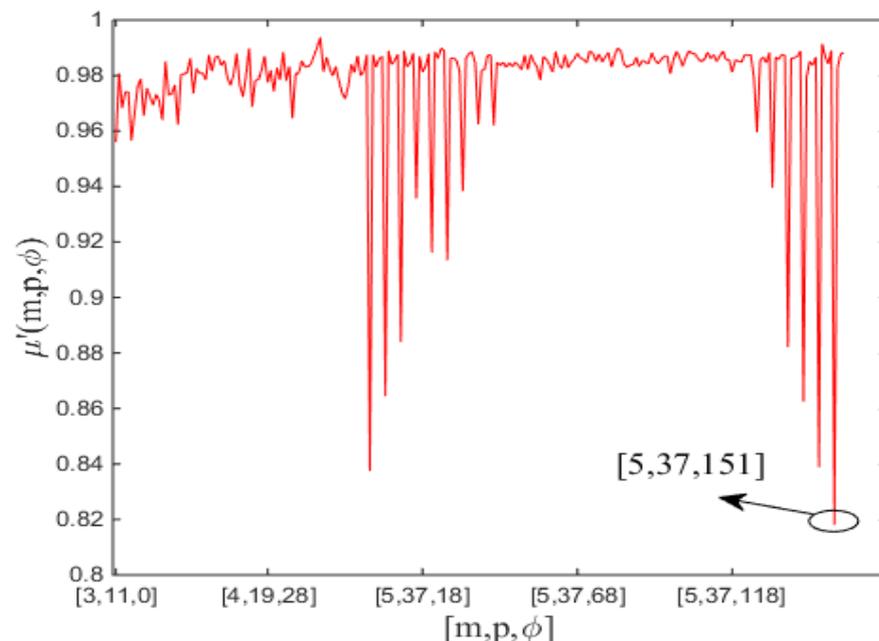
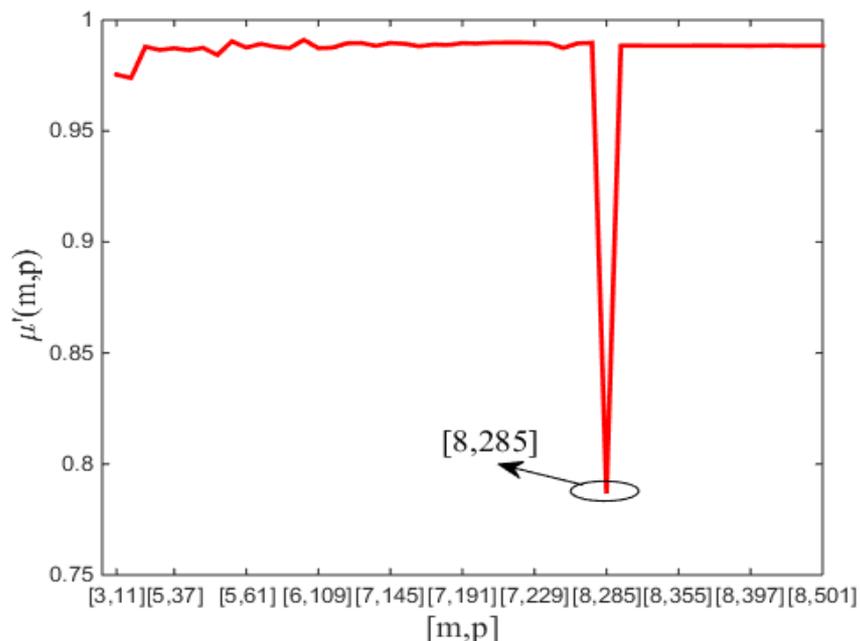
# Simulation Results

- We assume  $a = 2b$  for non-erroneous case and  $a > 2b$  for erroneous case
- Higher the number of rows, better is the accuracy for erroneous case
- If the receiver starts at  $\Delta^{\text{th}}$  position of  $t^{\text{th}}$  RS code word, then frame synchronization is achieved by shifting  $\phi = (tm(q-1)+1) - (\Delta M_1 - M_1 + 1)$  bit positions, where  $M_1 = \log_2 M$



Variation of  $\rho'(m, p, \phi)$  with respect to  $[m, p, \phi]$  for RS(63, 45, 6, 67) assuming 64-QAM scheme,  $\Delta = 6$ , and non-erroneous scenario

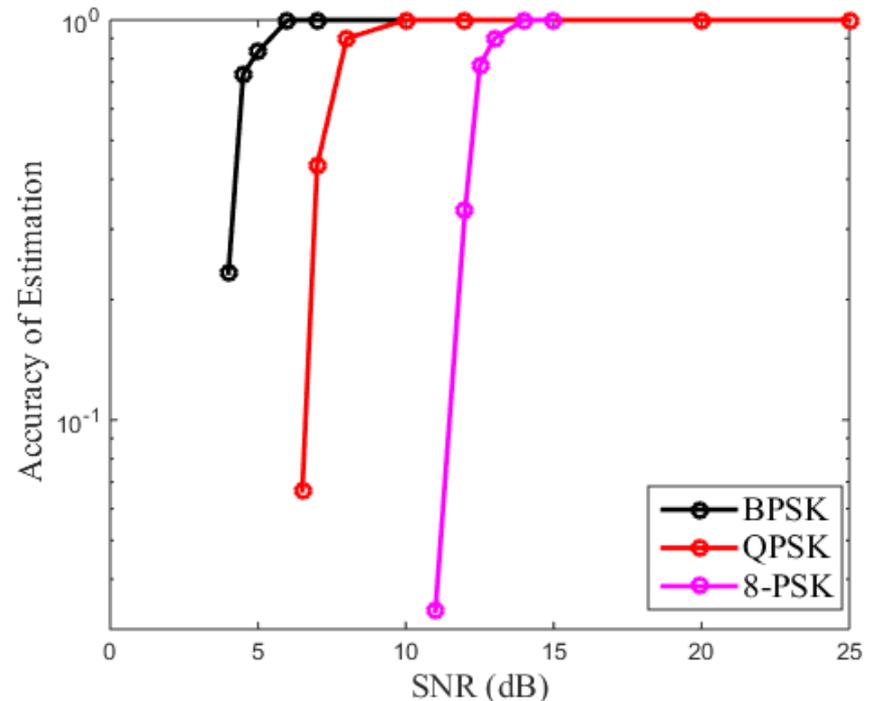
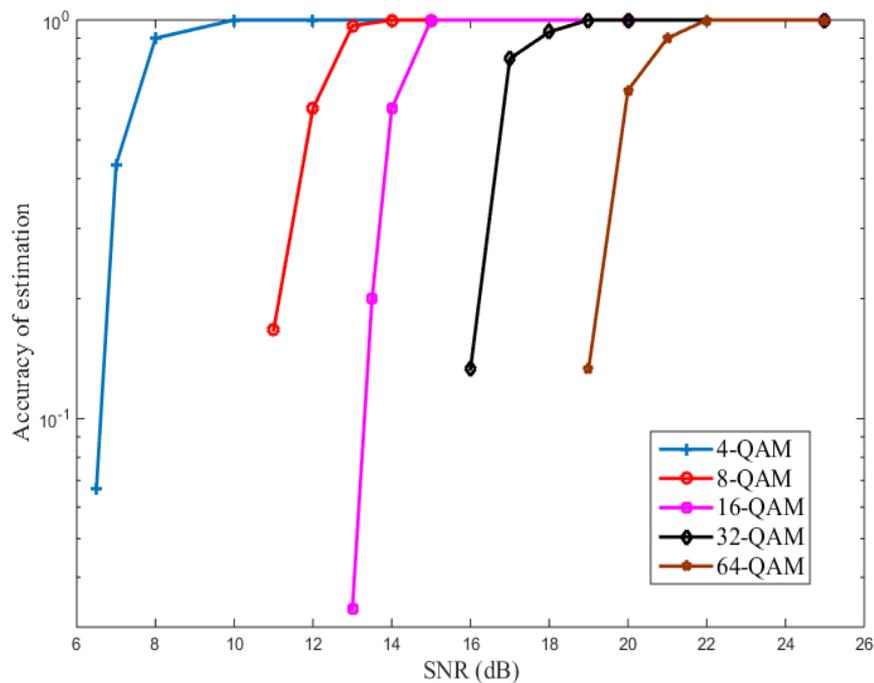
# Simulation Results



(a) Variation of normalized non-zero-mean-ratio  $\mu'(m,p)$  with  $[m,p]$  for RS(255, 127, 8, 285) coded data symbols assuming 256-QAM scheme and  $\text{SER}=2 \times 10^{-3}$

(b) Variation of normalized non-zero-mean-ratio  $\mu'(m,p,\phi)$  with  $[m,p,\phi]$  for RS(31, 15, 5, 37) coded data symbols assuming 16-QAM scheme,  $\Delta = 2$ , and  $\text{SER}=10^{-2}$

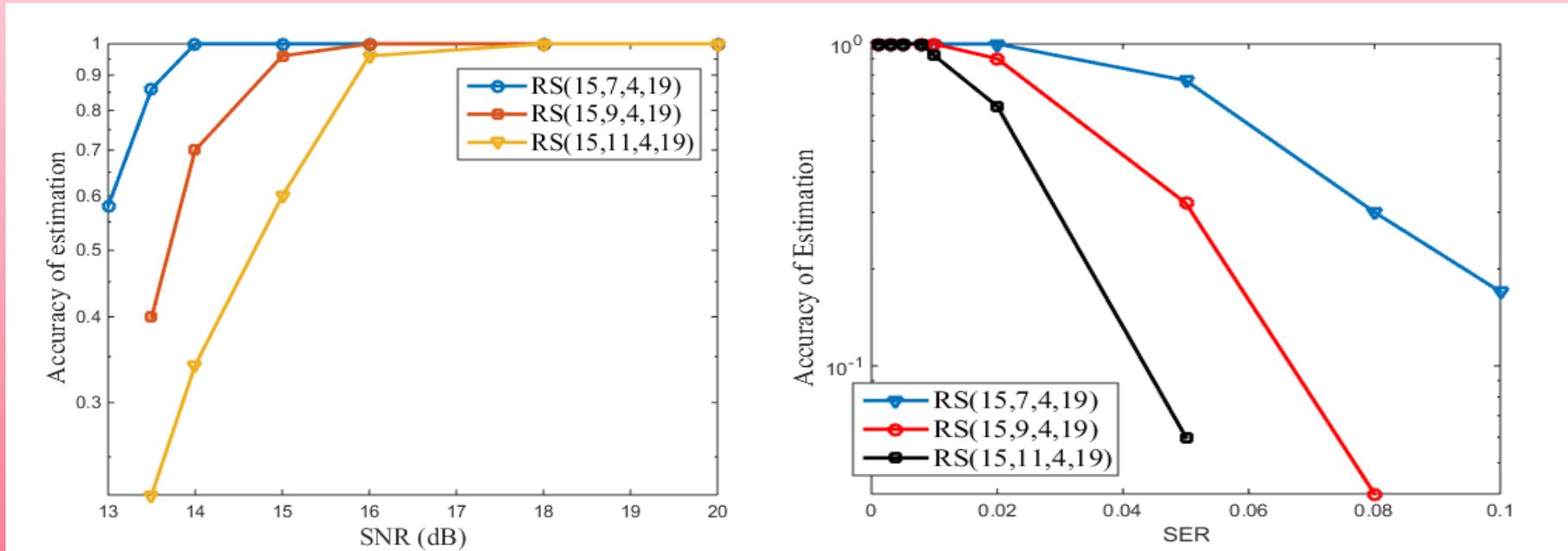
# Simulation Results



(a) Accuracy of estimation of RS codes RS(15, 9, 4, 19) for different  $M$ -QAM schemes

(b) Accuracy of estimation of RS codes RS(15, 9, 4, 19) for different  $M$ -PSK schemes

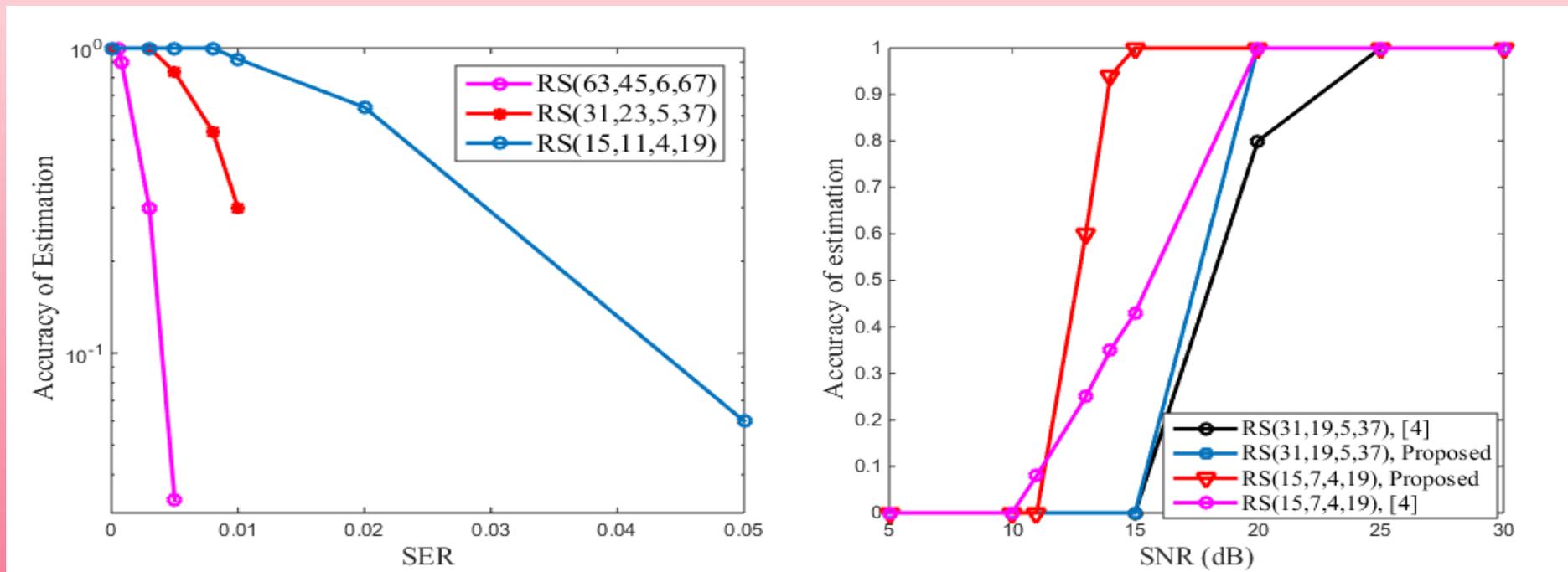
# Simulation Results



(a) Accuracy of estimation of RS codes assuming  $n = 15$ ,  $m = 4$ ,  $p = 19$ , and 16-QAM scheme for different values of code dimension  $k$

(b) Accuracy of estimation of RS codes assuming  $n = 15$ ,  $m = 4$ ,  $p = 19$ , and 16-QAM scheme for different values of code dimension  $k$

# Simulation Results



(a) Accuracy of estimation of RS codes with 16-QAM scheme for different values of codeword length  $n$

(b) Performance comparison of the proposed algorithm with the algorithm proposed in [4]

# Simulation Results

Table 1: Comparison of probability of correct detection of RS code RS(15, 7, 4, 19)

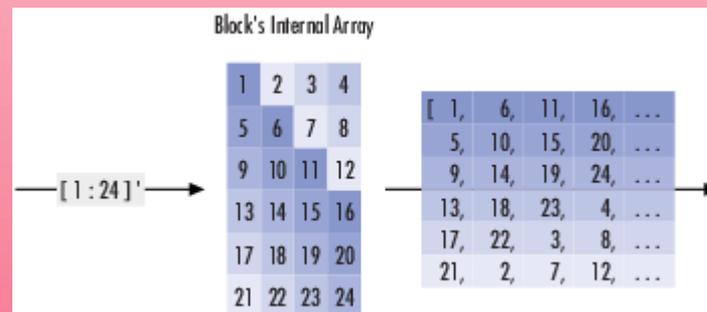
SER	Probability of detection [2]	Probability of detection (proposed algorithm)
0.001	1	1
0.01	1	1
0.05	1	1
0.075	0.52	0.92
0.1	0	0.40

**Proposed algorithm outperforms existing algorithm**

[2] A. Zahedi and G-R. Mohammad-Khani, "Reconstruction of a non-binary block code from an intercepted sequence with application to Reed-Solomon codes," IEICE Transactions on Fundamentals of Electronics Communications and Computer Sciences, VOL.E95-A, no. 11, pp. 1873--1880, Nov. 2012.

# Helical Scan Interleaver

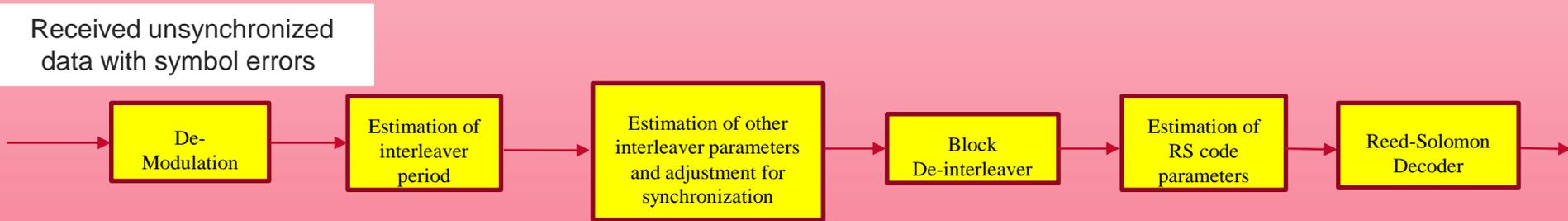
- **Helical Scan Interleaver** uses a fixed size matrix, arranges input symbols across rows, and outputs all the symbols without using default value or values from previous call
- Interleaver parameters (similar to block interleaver): **Number of columns ( $N_c$ )**, **Number of rows ( $N_r$ )**, **Helical array step size ( $d$ )**, **Interleaver period ( $\beta$ )**



Helical Scan Interleaver  $[1] N_r = 6, N_c = 4, d = 1, \beta = 24$

# Generic Block Diagram

- The generic block diagram for blind recognition of interleaver and RS code parameters is given as follows:



# Algorithm 3: Estimation of interleaver period - Noiseless case

- **Notations:** The rank and rank ratio of  $S$  are denoted by  $\rho(m, p, b)$  and  $\rho'(m, p, b)$ , respectively
- **Assumptions:**  $a \geq 2b$ ,  $b \in [b_{\min}, b_{\max}]$ ,  $m \in [m_{\min}, m_{\max}]$ , and the incoming bit stream is RS encoded and block interleaved
- $p = \text{primpoly}(m, 'all')$
- Convert the incoming RS coded and block interleaved binary data symbols into the respective elements of GF using  $p$
- Reshape the RS encoded GF elements into a data matrix  $S$  of size  $a \times b$
- Convert  $S$  into  $F$  using finite-field Gauss elimination process
- Compute  $\rho(m, p, b)$  from the number of non-zero columns in  $F$
- Compute  $\rho'(m, p, b) = \rho(m, p, b)/b$
- Obtain  $[m_{\text{est}}, p_{\text{est}}, b_{\text{est}}] = \underset{m, p, b}{\text{argmin}}(\rho'(m, p, b))$  and  $n_{\text{est}} = 2^{m_{\text{est}}} - 1$

# Observations

If  $\beta$  is a multiple of  $n$  i.e.  $\beta = \gamma \cdot n$  and  $b = \alpha' \cdot \beta$ , then the rank deficiency will be obtained. The deficient rank and rank ratio are, respectively, given by

$$\begin{aligned}\rho(m_{\text{est}}, p_{\text{est}}, b_{\text{est}}) &= \alpha' \cdot \gamma \cdot k_{\text{est}}, \\ \rho'(m_{\text{est}}, p_{\text{est}}, b_{\text{est}}) &= \frac{\rho(m_{\text{est}}, p_{\text{est}}, b_{\text{est}})}{b} = r.\end{aligned}\quad (1)$$

However, if  $b \neq \alpha' \cdot \beta$ , then full rank will be obtained.

If  $\beta$  is not a multiple of  $n$ , then rank deficiency will be obtained for  $b = \alpha' \cdot \text{lcm}(n, \beta)$ . Assuming  $\text{lcm}(n, \beta) = \Gamma \cdot n$ , the deficient rank and rank ratio values are, respectively, given by

$$\begin{aligned}\rho(m_{\text{est}}, p_{\text{est}}, b_{\text{est}}) &= \alpha' \cdot \Gamma \cdot k_{\text{est}}, \\ \rho'(m_{\text{est}}, p_{\text{est}}, b_{\text{est}}) &= \frac{\rho(m_{\text{est}}, p_{\text{est}}, b_{\text{est}})}{b} = r.\end{aligned}\quad (2)$$

However, if  $b \neq \alpha' \cdot \text{lcm}(n, \beta)$ , then full rank will be obtained.

# Observations

- From Algorithm 3,  $\beta_{\text{est}} = b_{\text{est}}$  for the case when  $\beta$  is a multiple of  $n$
- For the case when  $\beta$  is not a multiple of  $n$ ,  $\text{lcm}(n_{\text{est}}, \beta_{\text{est}}) = b_{\text{est}}$
- $\beta_{\text{est}}$  or  $\text{lcm}(n_{\text{est}}, \beta_{\text{est}}) = b_{\text{est}}$  is applicable when  $\rho'(m_{\text{est}}, p_{\text{est}}, b_{\text{est}})$  is the only minimum value in the search space
- If there are multiple values of  $b$  for which  $\rho'(m, p, b)$  is minimum, then the difference between successive number of columns with rank deficiency gives the estimate of  $\beta_{\text{est}}$  or  $\text{lcm}(n_{\text{est}}, \beta_{\text{est}})$
- Let  $b = \alpha' \cdot \beta$  and  $b' = (\alpha' + 1) \cdot \beta$  denote two successive columns with deficient rank values for the case when  $\beta = \gamma \cdot n$
- From  $b' - b$ , the interleaver period  $\beta$  is identified
- $\text{lcm}(n, \beta)$  is identified from  $b' - b$  for the case when  $\beta \neq \gamma \cdot n$  and  $\text{lcm}(n, \beta) = \Gamma \cdot n$

# Algorithm 4: Estimation of interleaver period - Noiseless case

- **Notations:** Mean value and normalized mean value of number of non-zero elements in  $c^{\text{th}}$  column of  $F$  -  $\sigma(c, m, p)$  and  $\sigma'(c, m, p)$ . Normalized non-zero-mean-ratio -  $\mu'(m, p, b)$
- **Assumptions:**  $a \geq t b$ ,  $b \in [b_{\min}, b_{\max}]$ ,  $m \in [m_{\min}, m_{\max}]$
- $p = \text{primpoly}(m, 'all')$
- Convert the incoming RS coded and block interleaved binary data symbols into the respective elements of GF using  $p$
- Reshape the RS encoded GF array elements into a data matrix  $S$  of size  $a \times b$
- Convert  $S$  into  $F$  using finite-field Gauss elimination process
- Evaluate  $\sigma(c, m, p)$ , where  $c \in \{1, 2, \dots, b\}$ , and normalize the obtained values with respect to the maximum value
- Calculate normalized non-zero-mean-ratio  $\mu'(m, p, b)$ , where  $\mu'(m, p, b) = \frac{\sum_{c=1}^b \sigma'(c, m, p)}{b}$
- Obtain  $[m_{\text{est}}, p_{\text{est}}, b_{\text{est}}] = \underset{m, p, b}{\text{argmin}}(\mu'(m, p, b))$  and  $n_{\text{est}} = 2^{m_{\text{est}}} - 1$

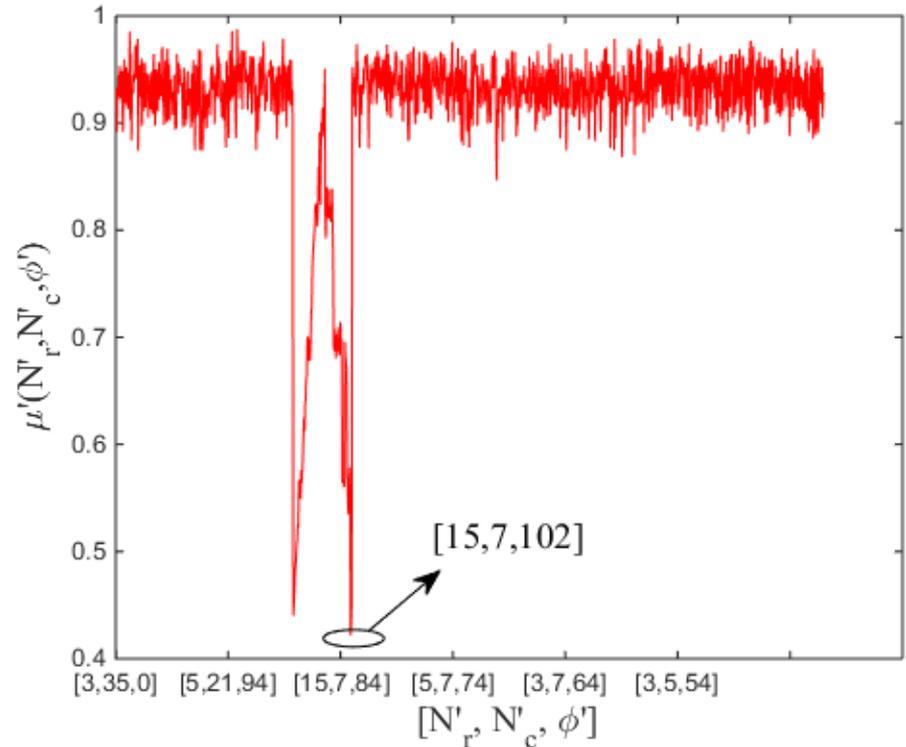
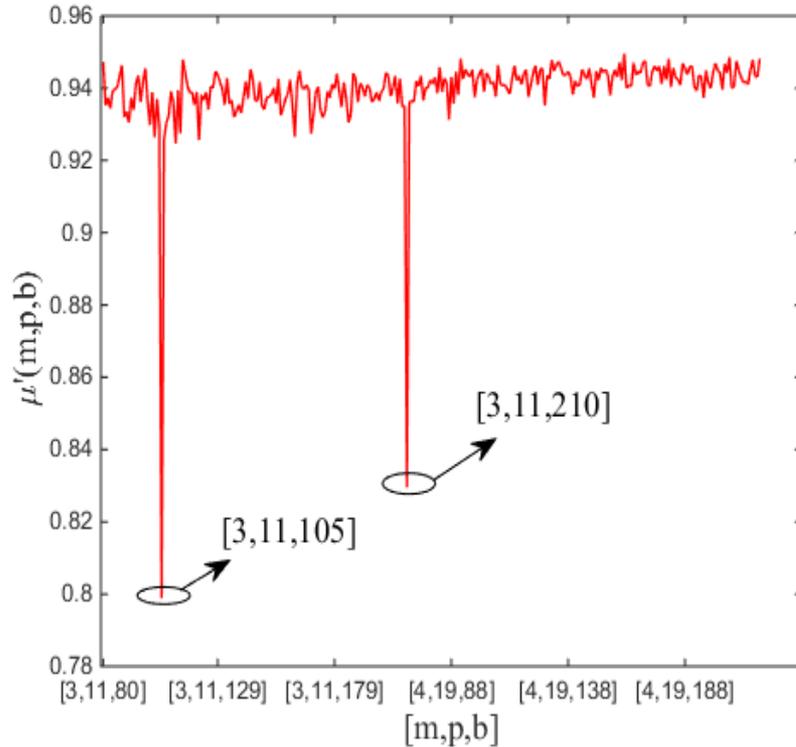
# Algorithm 5: Estimation of rest of interleaver parameters

- **Notations** :  $\zeta_{\text{est}} = \text{lcm}(n_{\text{est}}, \beta_{\text{est}})$ ,  $\phi'$  - symbol position adjustment to achieve synchronization,  $d$  - helical array step size, and Normalized non-zero-mean-ratio of  $F$  for matrix-based and helical scan interleavers -  $\mu'(N'_r, N'_c, \phi')$  and  $\mu'(N'_r, N'_c, d, \phi')$
- **Assumptions** :  $a \geq t b$ ,  $\phi' \in [0, \zeta_{\text{est}} - 1]$ ,  $d \in [1, N'_r - 1]$
- Convert the incoming RS coded and block interleaved binary data symbols into the respective elements of GF using  $p_{\text{est}}$
- Get all possible values of  $\delta'$  that satisfy  $\text{lcm}(n_{\text{est}}, \delta') = \zeta_{\text{est}}$
- Get all possible combinations of two factors  $N'_r$  and  $N'_c$  that satisfy  $N'_r N'_c = \delta'$
- Shift the coded and interleaved non-binary symbols by  $\phi'$  symbol positions
- De-interleave using  $N'_r$  and  $N'_c$  in the case of matrix-based block interleaver
- De-interleave using  $N'_r$ ,  $N'_c$ , and  $d$  in the case of helical scan interleaver
- Fix  $b$  as a multiple of  $n_{\text{est}}$
- Reshape the RS encoded GF elements into a data matrix  $S$  of size  $a \times b$
- Convert  $S$  into  $F$  using finite-field Gauss elimination process

# Algorithm 5 – Contd.

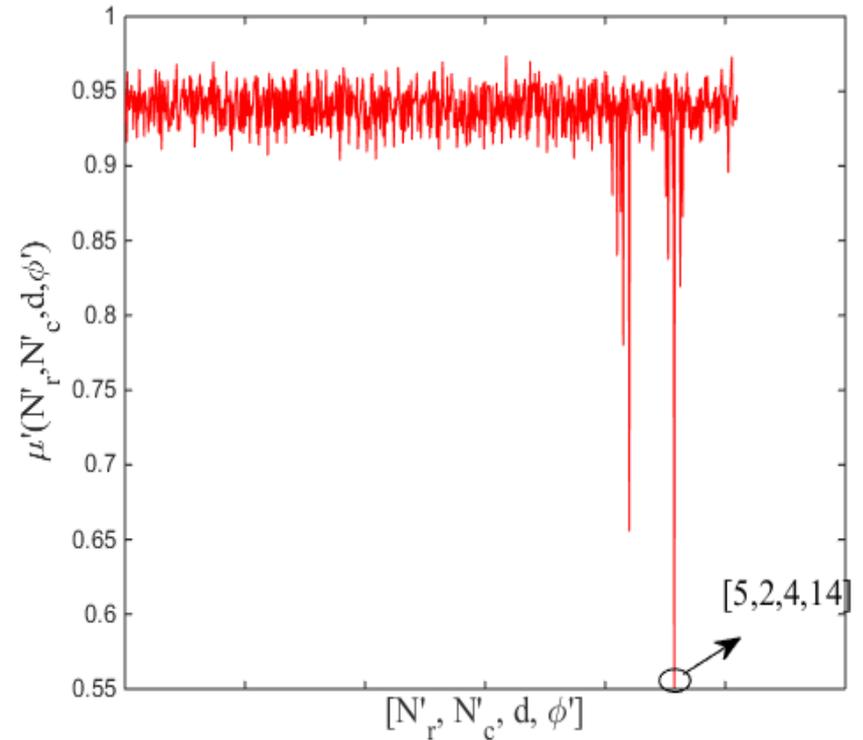
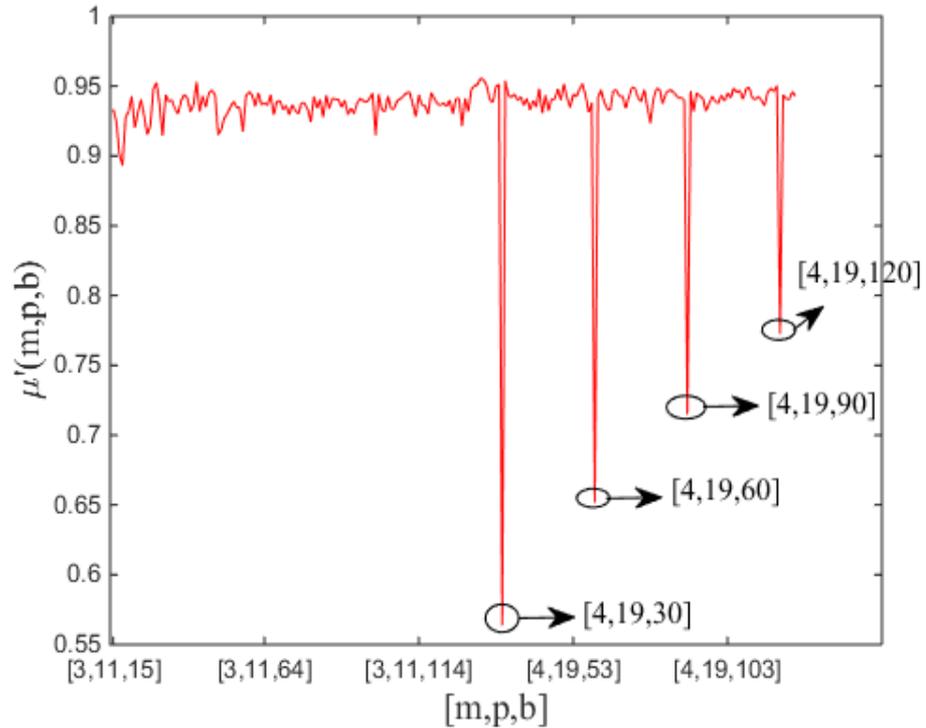
- Evaluate  $\mu'(N'_r, N'_c, \phi')$  for all possible values of  $N'_r$  and  $N'_c$  in the case of matrix interleaver
- Evaluate  $\mu'(N'_r, N'_c, d, \phi')$  for all possible values of  $N'_r$ ,  $N'_c$ , and  $d$  in the case of helical scan interleaver
- **Matrix-based block interleaver:** Obtain  $[N_r^{\text{est}}, N_c^{\text{est}}, \phi_1^{\text{est}}] = \underset{N'_r, N'_c, \phi'}{\text{argmin}} (\mu'(N'_r, N'_c, \phi'))$
- **Helical scan interleaver:** Obtain  $[N_r^{\text{est}}, N_c^{\text{est}}, d^{\text{est}}, \phi_1^{\text{est}}] = \underset{N'_r, N'_c, d, \phi'}{\text{argmin}} (\mu'(N'_r, N'_c, d, \phi'))$
- **Matrix-based block interleaver:** Shift  $\phi_1^{\text{est}}$  symbol positions and de-interleave using  $N_r^{\text{est}}$  and  $N_c^{\text{est}}$
- **Helical scan interleaver:** Shift  $\phi_1^{\text{est}}$  symbol positions and de-interleave using  $N_r^{\text{est}}$ ,  $N_c^{\text{est}}$ , and  $d^{\text{est}}$
- Identify the number of roots of generator polynomial and estimate  $n - k$
- Obtain  $k_{\text{est}}$  from  $n - k$
- Obtain the generator polynomial  $g(x)$

# Simulation Results



- Variation of  $\mu'(m, p, b)$  with  $[m, p, b]$  for RS(7, 3, 3, 11) and matrix-based block interleaver assuming  $N_r = 15$ ,  $N_c = 7$ , 8-PSK scheme,  $\Delta = 4$ , and  $\text{SER} = 8 \times 10^{-3}$ .
- Variation of  $\mu'(N'_r, N'_c, \phi')$  with  $[N'_r, N'_c, \phi']$  for RS(7, 3, 3, 11) assuming  $N_r = 15$ ,  $N_c = 7$ , 8-PSK scheme,  $\Delta = 4$ , and  $\text{SER} = 8 \times 10^{-3}$

# Simulation Results



- Variation of  $\mu'(m, p, b)$  with  $[m, p, b]$  for RS(15, 7, 4, 19) and helical scan interleaver assuming  $N_r = 5$ ,  $N_c = 2$ ,  $d = 4$ , 16-PSK scheme,  $\Delta = 17$ , and  $\text{SER} = 10^{-2}$
- Variation of  $\mu'(N'_r, N'_c, d, \phi)$  with  $[N'_r, N'_c, d, \phi]$  for RS(15, 7, 4, 19) and helical scan interleaver assuming  $N_r = 5$ ,  $N_c = 2$ ,  $d = 4$ , 16-PSK scheme,  $\Delta = 17$ , and  $\text{SER} = 10^{-2}$

# Simulation Results

Table 1: Comparison of probability of correct detection of interleaver period for RS code RS(7, 3, 3, 11)

BER	Probability of correct detection [1]	Probability of correct detection (proposed algorithm)
0.006	0.85	1
0.009	0.37	1
0.015	0.1	1

Proposed algorithm outperforms existing algorithm

[1] L. Lu, K. H. Li, and Y. L. Guan, "Blind detection of interleaver parameters for non-binary coded data streams," in Proc. IEEE ICC, 2009, pp. 1--4.

# Conclusions

- **Blind estimation algorithms** have been proposed for estimating RS code and block interleaver parameters based **on rank deficiency and normalized non-zero-mean-ratio values**
- The **bit/symbol positioning adjustment** is also integrated with the proposed code parameter estimation algorithms
- The simulation studies show that the proposed algorithms can successfully estimate RS code and block interleaver parameters for various test cases
- Accuracy of estimation plots are shown for different  $M$ -QAM and  $M$ -PSK schemes, code dimension, and codeword length values

## Observations:

- It has been inferred that the **accuracy of parameter estimation improves** with **decrease** in **code dimension** and codeword length values
- The lower modulation order schemes perform better than the higher modulation order schemes
- The **proposed algorithm** for noisy environment consistently **outperforms** the **algorithms proposed in the prior works**.

Thank you