An Archived-Based Stochastic Ranking Evolutionary Algorithm (ASREA) for Multi-Objective Optimization

Deepak Sharma
LogXlabs Research Center, Paris, France
deepak.sharma@logxlab.com

Pierre Collet
FDBT, LSIIT
Université de Strasbourg, France
pierre.collet@unistra.fr

ABSTRACT

In this paper, we propose a new multi-objective optimization algorithm called Archived-based Stochastic Ranking Evolutionary Algorithm (ASREA) that ranks the population by comparing individuals with members of an archive. The stochastic comparison breaks the usual $O(mn^2)$ complexity into $O(man)$ (m being the number of objectives, a the size of the archive and n the population size), whereas updating the archive with distinct and well-spread non-dominated solutions and developed selection strategy retain the quality of state of the art deterministic multi-objective evolutionary algorithms (MOEAs).

Comparison on ZDT and 3-objective DTLZ functions shows that ASREA converges on the Pareto-optimal front at least as well as NSGA-II and SPEA2 while reaching it much faster, and being cheaper on ranking comparisons.

Categories and Subject Descriptors

F.m [**Theory of Computation**]: Miscellaneous

General Terms

Algorithms, Performance

Keywords

Stochastic Ranking, Evolutionary Algorithms, Multiobjective Optimization, Performance Assessment

1. INTRODUCTION

Standard evolutionary algorithms (EAs) need a unique evaluation function to guide the search, meaning that they cannot cope with problems where several objectives must be optimized at the same time. The first idea that comes to mind is to aggregate the different objectives (but this does not make much sense when objectives are antagonistic) or to rank them (lexicographic method, [10]) but none of these methods yields really good results.

Things changed in 1989, when Goldberg suggested to rank individuals based on a Pareto-based dominance criterion, while preserving diversity [11]. This idea opened the way to current state of the art MOEAs such as NSGA-II [4] or SPEA2 [21], that work really well out of the box on a wide range of problems with deterministic ranking[1] and diversity preserving procedures. These deterministic features are costly sometimes when higher objective problems with large populations size are solved.

The fact that EAs are stochastic by nature poses the question whether it is necessary to feed them with an individual rank that is deterministic and sometimes expensive to produce: using a non-deterministic ranking may not be that detrimental, as slightly blurring the evaluation may help the EAs to escape from local minima and make it less prone to premature convergence.

In the remaining part of the paper, the proposed MOEA is described with the relevant literature in section 2. Thereafter, ASREA is tested on ZDT and 3-objective DTLZ functions in section 3 and further compared with NSGA-II and SPEA2 algorithms on performance assessment indicators. Finally, the paper is concluded in section 4 with future works.

2. ARCHIVED-BASED STOCHASTIC RANKING EVOLUTIONARY ALGORITHM (ASREA)

ASREA is an attempt at creating a more homogeneous MOEA by using a stochastic ranking operator that allows to cut the usual $O(mn^2)$ complexity into $O(man)$, where m is the number of objectives, a is the size of archive and n is the population size. Finding the appropriate size of archive that not only accommodates well and evenly spread non-dominated solutions but also guarantees the convergence to the approximate Pareto-solutions, is a difficult task [15] because it depends on many factors such as number of objectives, population size etc. ASREA has not yet been tested on many objectives problems, but preliminary tests on various test functions of $m = 2$ and $m = 3$ objectives suggest that an archive size of $(a =)10 \times m$ allows ASREA to obtain good results.

2.1 Description of the algorithm

[1] $0(mn^2)$ in the worst case for NSGA-II, with m the number of objectives and n the size of the population. SPEA2 does better by using a smaller archive (so that in the $O(mn^2)$ complexity of SPEA2, n refers to the size of the archive) but with worse results in some cases (cf. ZDT4 in fig. 2).

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.
GECCO'10, July 7–11, 2010, Portland, Oregon, USA.
Copyright 2010 ACM 978-1-4503-0072-8/10/07 ...$10.00.

ASREA starts with evaluating a random initial population (**ini_pop**). All distinct non-dominated individuals are copied into the archive. If the number of non-dominated individuals is larger than the size of archive, then the diverse non-dominated individuals (according to some clustering techniques) are copied into the archive until it is full. In this paper, NSGA-II's crowding distance (CD) operator [4] is used to select the diverse non-dominated individuals, but another clustering technique could be used.

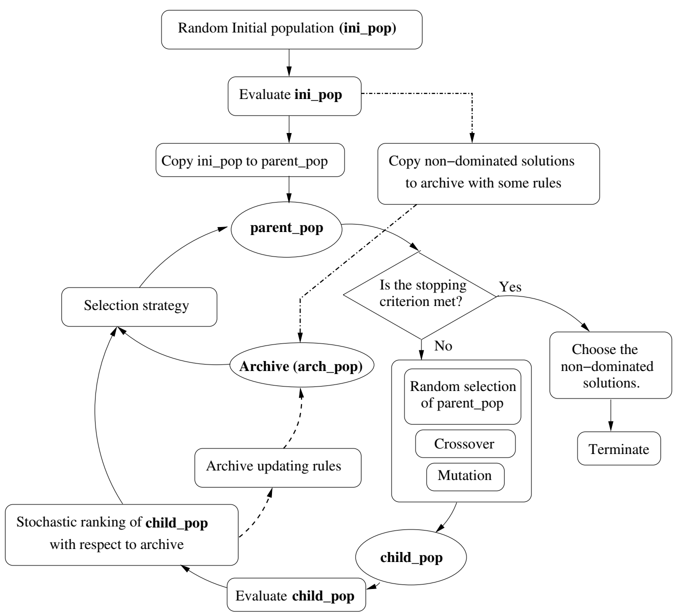


Figure 1: ASREA's flowchart.

As shown in fig. 1, **ini_pop** is copied into **parent_pop** and a relatively standard EA loop starts, by checking whether a stopping criterion is met.

If this is not the case, a **child_pop** is created by repeatedly selecting *randomly* two parents from the **parent_pop**, and creating a child through a crossover followed by a mutation (in this paper, a standard SBX crossover is used, followed by a polynomial mutation [3]) after which, the **child_pop** is evaluated.

Now comes the main contribution of this paper, as it is time to assign a rank and propagate good individuals to the next generation. Where most of the existing MOEAs such as NSGA-II, SPEA2 etc., have deterministic dominance-based ranking operators, we choose to use a ranking operator inspired from the EP-tournament operator [6] and close to the working principle of niched Pareto genetic algorithm (NPGA [13]). In both studies, the rank of an individual is assigned while comparing it with only a few individuals randomly selected in the population.

In ASREA, the same concept is used, but the comparison for the rank evaluation of **child_pop** is done with respect to an archive of distinct non-dominated solutions. This feature of ASREA not only reduces the ranking complexity of the algorithm but allows to deterministically propagate the good individuals to the next generation. In this algorithm, the rank of individual (A) of **child_pop** is calculated on a dominance criterion which is given below:

$$\text{rank}(A) \quad = \quad 1 + \textit{number of } \textbf{\textit{arch\_pop}} \textit{ members that dominate } A \qquad (1)$$

Note that in ASREA, the lower rank is better, with best rank = 1. The ranking procedure discussed above is one of the differences in the working principle of ASREA from other MOEAs, and specially the archived-based ones, in which the archive, parent and current offspring populations are mixed, after which the rank is assigned deterministically.

Overview of Archived Based MOEAs.

The idea of using off-line population, or so called *'archive'*, to store good individuals or non-dominated solutions has been explored earlier [18]. This introduces elitism in MOEAs for better convergence. There are some notable archived based MOEAs such as, PESA [2] and PESA2 [1] in which the archive is filled from the current solution by checking the dominance. If the archive is full, then the selection is done on a squeeze factor of a hyperbox filled with individuals in PESA or region-based hyperbox selection in PESA2. Thereafter, the archive is used for making the mating pool for crossover using a binary tournament selection operator. In SPEA2 [21], the archive is updated by selecting the good individuals from the current child population and previous archive, based on rank assignment scheme and $k-th$ nearest neighbor clustering method. The archive then propagates to the next iteration for mating pool and other genetic operators.The $\epsilon-$dominance and $\epsilon-$Pareto optimality based archive selection strategy also exists in literature [17]. Overall, the archive or off-line population discussed in literature is used for storing the good individuals and propagate them to the next iteration whereas in ASREA, we use archive to evaluate the rank of the child population and partially fill the parent population for the next iteration (discussed in following paragraphs).

The archive also gets updated during this ranking process if individual A gets rank 1. In this scenario, A will join the archive depending on two cases:

1. When A dominates one or several members of archive and is non-dominated with respect to the rest of them, then A replaces all the dominated individuals of the archive.

2. When A is non-dominated with respect to all the members of the archive and distinct from them, then A may still join the archive:

    - If the archive is not full (size of current **arch_pop** $< a$), then A goes in the archive.
    - If the archive is already full, then, the crowding distance (CD) is computed for all members of the archive including individual A and all individuals are sorted based on CD. The size of the archive is now $a+1$. The extreme solutions in each objective are kept and the worst individual CD-wise is dropped, so as to come back to a size of a.

Note that the m extreme individuals of the current non-dominated front are always stored in the archive so as to keep the Pareto front as wide as possible.

This is done for all individuals of **child_pop** (that now have a rank), and the archive is updated with a subset of current non-dominated solutions that are distinct and well-spread in the explored objective space.

The next task for ASREA is to select good individuals to create the **parent_pop** for the next generation. As mentioned in the study [18], the performance of algorithms can substantially improve through the use of strongly elitist selection strategies making extensive use of the archive and high selection pressures in the multi-objective optimization that naturally maintains the diversity in the population. Therefore, a selection strategy is developed for ASREA in which 50% of **parent_pop** is filled from the archive and the rest from the current **child_pop**, using a binary tournament selection based on calculated rank and crowding distance. The strategy is described in algorithm 1.

```
Copy the extreme solutions of arch_ pop to the parent
population.
Using arch_pop, fill first half of parent_pop in the
following way:
while (first half of parent_pop not filled) do
    Pick randomly two individuals of arch_pop;
    if Crowding distances are different then
        Copy the individual with larger crowding
        distance into parent_pop;
    else
        Copy any individual randomly;
    end
end
Using child_pop, fill second half of parent_pop in
the following way:
while (second half of parent_pop not filled) do
    Pick randomly two individuals of child_pop;
    if Ranks are different then
        Copy the individual with smaller rank into
        parent_pop;
    else
        Copy the individual with larger crowding
        distance into parent_pop;
    end
end
```

Algorithm 1: *Selection strategy to fill the parent pop of the next generation.*

The EA loop is then finished and can start again, by checking whether a termination criterion is met (e.g. number of generations in this paper) as in fig. 1.

2.2 Salient Features of the ASREA algorithm

The first important feature of ASREA is the reduced computational complexity of the ranking operator, that is, $O(man)$ for the worst case, where m is the number of objectives, a is the size of the archive, and n is the population size. As mentioned in the beginning of section 2, the appropriate size of a depends on many factors such as number of objectives, population size etc. But, preliminary tests on two- and three-objectives test functions suggest that a should be linearly proportional to m, that is, ($a = 10m$).

A second important feature of ASREA is its inherent ability to preserve diversity in the population. This comes when the not-so-good individuals of **child_pop** may nevertheless obtain a good rank (including rank = 1) because they are only compared with the archive of limited size. Had the ranking method been deterministic, then these not-so-good individuals would not have made it into the next generation and it may have been needed to implement a diversity preserving scheme in order to avoid premature convergence. Therefore, we refer to the ranking procedure as *stochastic*.

Another feature of ASREA is its drastic but subtle selection strategy that incorporates strong elitism and high selection pressure to fill the parent population for next generation. Finally, the spread of the present non-dominated front is preserved by keeping the extreme solutions of all m objectives in the new population.

3. EXPERIMENTS

3.1 Test Suites and Performance Assessment Tools of MOEAs

In this paper, three Zitzler-Deb-Thiele functions (ZDT3, ZDT4 and ZDT6) and three Deb-Thiele-Laumanns-Zitzler functions (three-objective DTLZ1, DTLZ2 and DTLZ3) are chosen to assess the performance of the proposed algorithm to compare it with two other well-known existing MOEAs: NSGA-II and SPEA2. These two algorithms use a deterministic ranking procedure but in different ways: NSGA-II[2] sorts the non-dominated fronts from the child and parent populations whereas SPEA2 uses the fitness assignment scheme to rank the *combined population* of archive and child populations.

3.2 Performance Assessment Tools

In the field of multi-objective optimization (MOO), the strengths and weaknesses of any algorithm are based on the quality of the evolved solution such as proximity to the reference set, spread and evenness of the non-dominated solutions in the objective space, etc. Several tools are available to indicate the performance of MOO algorithms which independently explore different features of the algorithm [7, 23, 16]. In this study, we choose two indicators and attainment surface plots (only for two-objective functions) for the assessment, whose source codes are available at [19].

R-indicator ($\mathbf{I}_{R2}$): [12]

$$I_{R2} = \frac{\sum_{\lambda \in \Lambda} u^\star(\lambda, A) - u^\star(\lambda, R)}{|\Lambda|} \quad (2)$$

where R is a reference set, $u^\star$ is the maximum value reached by the utility function u with weight vector λ on an approximate set A, that is, $u^\star = \max_{z \in A} u_\lambda(z)$. The augmented Tchebycheff function is used as the utility function. The second order R-indicator gives the idea of proximity with respect to the reference set A [23, 16].

Hypervolume indicator ($\mathbf{I}_{\bar{H}}$): [22]

The hypervolume indicator I_H measures the hypervolume of that portion of the objective space that is weakly dominated by an approximate set A. This indicator gives the idea of spread quality and has to be maximized. As recommended in the study [16], the difference in values of hypervolume indicator between the approximate set A and the reference set R is calculated in this paper, that is, $I_{\bar{H}} = I_H(R) - I_H(A)$. The smaller value suggests good spread [23, 16].

[2]Source codes for NSGA-II and SPEA2 are available at [14] and [19].

Attainment surface: [9, 8]

An approximate set A is called the $k\% - approximate$ *set* of the empirical attainment function (EAD) $\alpha_r(z)$, iff it weakly dominates exactly those objective vectors that have been attained in at least k percentage of the r runs. Formally, $\forall \in Z : \alpha_r(z) >= k/100 \leftrightarrow A \preceq \{z\}$ where $\alpha_r(z) = \frac{1}{r}\sum_{i=1}^{r} I(A^i \preceq \{z\})$. A^i is the ith approximation set (run) of the optimizer and $I(.)$ is the indicator function, which evaluates to one if its argument is true and zero if its argument is false.

An attainment surface of a given approximate set A is the union of all tightest goals that are known to be attainable as a result of A. Formally, this is the set $\{z \in \Re^n : A \preceq z \wedge \neg A \prec\prec z\}$ [16].

3.3 Parameters

In this paper, 25 runs of ASREA, NSGA-II and SPEA2 were performed using different seeds for the performance assessment of the proposed algorithm. All three MOEAs are run using identical parameters on the ZDT and DTLZ functions:

Population	100
Number of generations	500
Crossover probability of individual	0.9
Crossover probability of variable	0.5
Mutation probability of individual	1.0
Mutation probability of variable	1/(nb of variables)
Distribution crossover index	15
Distribution mutation index	20

The performance of evolved non-dominated fronts of the three MOEAs is evaluated at 10^3, 10^4 and 5×10^4 function evaluations (EVAL).

3.4 Results and Discussion

As recommended in [16], minimum and maximum limits on each objective (cf. Table 1) are used to normalize and further scale the approximate sets of MOEAs and the reference set of the given ZDT and DTLZ functions between 1 and 2. On this basis, the reference set (z^*) of (1, 1) and

Table 1: Objective function bounds for normalization

Functions→ Bounds↓	ZDT3		ZDT4		ZDT6	
	f_1	f_2	f_1	f_2	f_1	f_2
Lower	0.0	-0.773	0.0	0.0	0.281	0.0
Upper	0.873	5.915	1.0	149.672	1.0	9.141

DTLZ1			DTLZ2		
f_1	f_2	f_3	f_1	f_2	f_3
0.0	0.0	0.0	0.0	0.0	0.0
425.975	408.277	481.420	2.425	2.416	2.202

DTLZ3		
f_1	f_2	f_3
0.0	0.0	0.0
1701.967	1534.385	1436.245

(2, 2) are used to evaluate I_{R2} values whereas, (2.1, 2.1) is assigned as the reference point (z^+) for calculating the $I_{\bar{H}}$ values. The suggested range of I_{R2} and $I_{\bar{H}}$ lies between -1 and $+1$, where -1 is the best and $+1$ is considered as worst. Interested readers may refer to [16, 23] for more details on performance assessment of MOEAs.

Table 2 shows the statistical I_{R2} values (which show the proximity w.r.t. the reference set) of each MOEA over 25 runs (the best values are in bold face).

At 5×10^4 EVAL, ASREA shows the best results in the mean, median, best and worst values of I_{R2} (proximity with the Pareto-optimal (P-O) front).

ASREA also gets the best statistical I_{R2} values for ZDT3, ZDT4, ZDT6 and DTLZ2 at 10^4 EVAL, and for ZDT3, ZDT6, DTLZ1, DTLZ 2 and DTLZ3 at 10^3 EVAL.

This suggests that ASREA obtains better results than NSGA-II and SPEA2 on more problems at three stages of EVAL and also, ASREA approaches the P-O front faster, before showing the best convergence on the P-O front all in all.

One interesting thing to ponder here is that negative values are observed for I_{R2} for the ZDT6 function. This suggests that the evolved non-dominated solutions of the three MOEAs explore regions of P-O front which are not represented by the limited points of the reference set.

Table 3 shows the statistical $I_{\bar{H}}$ values (which represents the spread of solutions in the objective space) of each MOEA over 25 runs. ASREA displays the best mean, median, best and worst $I_{\bar{H}}$ values for DTLZ1 and DTLZ2 problems at 5×10^4 EVAL, for ZDT4 and DTLZ1 at 10^4 and for ZDT3 and ZDT6 functions at 10^3 EVAL.

For the rest of the EVAL stages, ASREA's statistical $I_{\bar{H}}$ values are close to NSGA-II but better than SPEA2 for all ZDT and DTLZ functions except for ZDT4 at 10^3 EVAL, ZDT6 at 10^4 EVAL and DTLZ2 at 5×10^4 EVAL where SPEA2 shows the best spread of all.

For all ZDT functions, figure 2 shows 0% attainment surface plots in which:

1. The theoretical Pareto Front is in black,
2. Obtained P-O front for NSGA-II is in dark red (dark grey).
3. Obtained P-O front for SPEA2 is in blue (grey).
4. Obtained P-O front for ASREA is in green (light grey).

Plots are overlapped from black to light grey (ASREA) so that results for ASREA are always visible over SPEA2, NSGA-II and also, over the theoretical front (in black), meaning that ASREA always appears, and the other curves appear where they differ from ASREA[3].

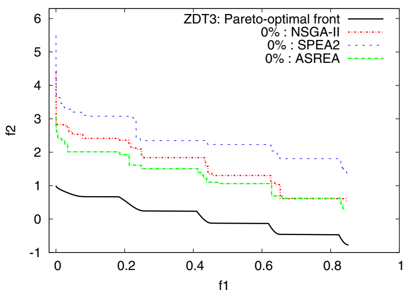
In the initial stage (10^3 EVAL), ASREA shows the closest proximity with the P-O front, and maintains a good spread for all functions, except ZDT4 where SPEA2 has an edge.

In the intermediate stage (10^4 EVAL), ASREA shows the closest proximity for all functions over NSGA-II and SPEA2, and equivalent spread for all functions.

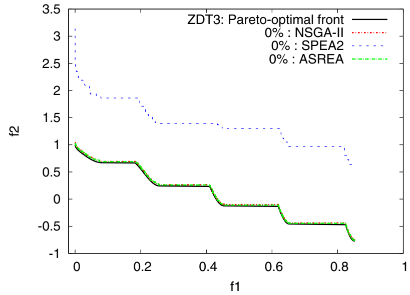
On final stage (5×10^4 EVAL), all three algorithms approximate the P-O front but for SPEA2 on ZDT4 function (where it was best at initial stage).

Finally, fig. 3 shows a comparison between the number of comparisons necessary for the ranking and the number of function evaluations (EVAL) for NSGA-II, SPEA2 and ASREA. The figure clearly shows the significant computation savings thanks to the stochastic ranking procedure of ASREA against the deterministic ranking operators of NSGA-

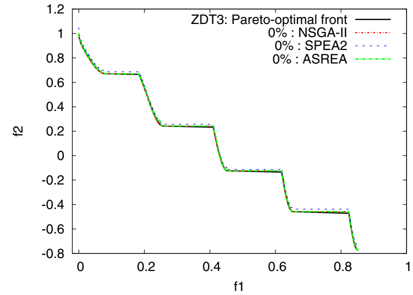
[3]In electronic version, the plots are in color and can be enlarged. Electronic plots can be obtained by sending a mail to the authors.



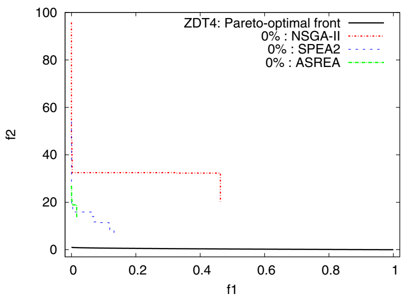
(a) ZDT3 at 1000 EVAL



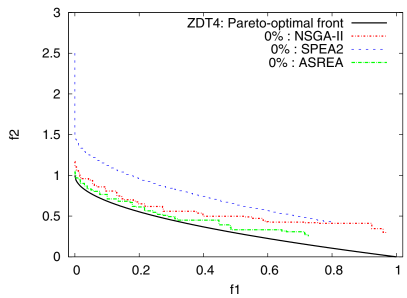
(b) ZDT3 at 10,000 EVAL



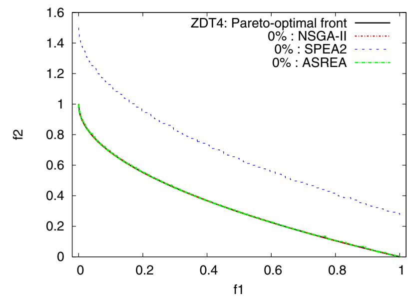
(c) ZDT3 at 50,000 EVAL



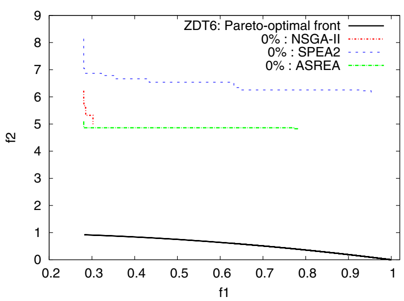
(d) ZDT4 at 1000 EVAL



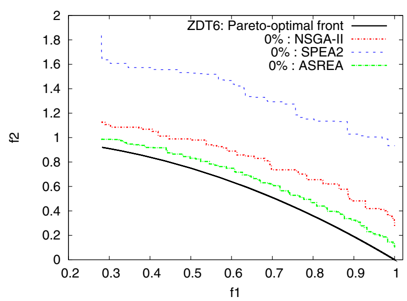
(e) ZDT4 at 10,000 EVAL



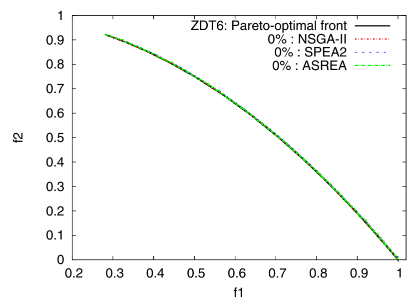
(f) ZDT4 at 50,000 EVAL



(g) ZDT6 at 1000 EVAL



(h) ZDT6 at 10,000 EVAL



(i) ZDT6 at 50,000 EVAL

Figure 2: Attainment surface plots of ASREA, NSGA-II and SPEA2 for ZDT functions.

Table 2: R-indicator values of ASREA, NSGA-II and SPEA2 on ZDT AND DTLZ functions.

Functions→		ZDT3			ZDT4			ZDT6		
MOEAs→		ASREA	NSGA-II	SPEA2	ASREA	NSGA-II	SPEA2	ASREA	NSGA-II	SPEA2
10^3 EVAL	Mean	**0.0719**	0.0886	0.1381	0.0621	0.0763	**0.0394**	**0.1963**	0.2014	0.2248
	Median	**0.0705**	0.0879	0.1397	0.0601	0.0759	**0.0390**	**0.1982**	0.2026	0.2264
	S.D.	0.0114	**0.0081**	0.0112	0.0162	**0.0095**	0.0152	0.0104	**0.0067**	0.0070
	Best	**0.0566**	0.0718	0.1128	0.0326	0.0450	**0.0150**	**0.1745**	0.1802	0.2098
	Worst	**0.0969**	0.1060	0.1587	0.1084	0.0981	**0.0771**	0.2155	**0.2116**	0.2368
10^4 EVAL	Mean	**0.0017**	0.0030	0.0944	**0.0011**	0.0018	0.0052	**0.0052**	0.0173	0.0534
	Median	**0.0008**	0.0016	0.0938	**0.0011**	0.0017	0.0052	**0.0049**	0.0174	0.0525
	S.D.	**0.0035**	0.0048	0.0107	**0.0004**	0.0009	0.0022	**0.0012**	0.0030	0.0121
	Best	**0.0007**	0.0010	0.0747	**0.0003**	0.0005	0.0010	**0.0033**	0.0105	0.0370
	Worst	**0.0187**	0.0192	0.1174	**0.0021**	0.0042	0.0119	**0.0077**	0.0230	0.0835
$5*10^4$ EVAL	Mean	**0.0006**	**0.0006**	0.0074	**0.0000**	**0.0000**	0.0040	**-0.0008**	**-0.0008**	-0.0006
	Median	**0.0000**	**0.0000**	0.0053	**0.0000**	**0.0000**	0.0036	**-0.0008**	**-0.0008**	-0.0006
	S.D.	**0.0034**	**0.0034**	0.0066	**0.0000**	**0.0000**	0.0017	**0.0000**	**0.0000**	**0.0000**
	Best	**0.0000**	**0.0000**	0.0010	**0.0000**	**0.0000**	0.0008	**-0.0008**	**-0.0008**	-0.0007
	Worst	**0.0173**	**0.0173**	0.0214	**0.0000**	**0.0000**	0.0099	**-0.0008**	-0.0007	-0.0005
Functions→		DTLZ1			DTLZ2			DTLZ3		
MOEAs→		ASREA	NSGA-II	SPEA2	ASREA	NSGA-II	SPEA2	ASREA	NSGA-II	SPEA2
10^3 EVAL	Mean	**0.0003**	0.0004	0.0005	**0.0009**	0.0010	0.0022	**0.0009**	0.0010	0.0014
	Median	**0.0003**	0.0004	0.0005	**0.0009**	**0.0009**	0.0020	**0.0008**	0.0010	0.0013
	S.D.	**0.0001**	**0.0001**	**0.0001**	**0.0002**	**0.0002**	0.0007	0.0002	**0.0001**	0.0004
	Best	**0.0002**	**0.0002**	0.0003	**0.0005**	0.0006	0.0011	**0.0007**	0.0008	**0.0007**
	Worst	**0.0005**	0.0007	0.0008	0.0015	**0.0014**	0.0048	**0.0012**	**0.0012**	0.0026
10^4 EVAL	Mean	0.0001	**0.0000**	0.0001	**0.0002**	0.0004	0.0005	0.0002	**0.0001**	0.0002
	Median	0.0001	**0.0000**	0.0001	**0.0002**	0.0004	0.0005	0.0002	**0.0001**	0.0002
	S.D.	**0.0000**	**0.0000**	**0.0000**	**0.0001**	0.0002	**0.0001**	0.0001	**0.0000**	**0.0000**
	Best	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.0003	0.0001	**0.0000**	0.0001
	Worst	0.0002	**0.0000**	0.0001	**0.0006**	0.0008	0.0009	0.0003	**0.0001**	0.0002
$5*10^4$ EVAL	Mean	**0.0000**	**0.0000**	**0.0000**	**0.0003**	**0.0003**	0.0004	**0.0000**	**0.0000**	0.0001
	Median	**0.0000**	**0.0000**	**0.0000**	**0.0003**	**0.0003**	0.0004	**0.0000**	**0.0000**	0.0001
	S.D.	**0.0000**	**0.0000**	**0.0000**	**0.0002**	**0.0002**	**0.0002**	**0.0000**	**0.0000**	**0.0000**
	Best	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.0001	**0.0000**	**0.0000**	**0.0000**
	Worst	**0.0000**	**0.0000**	0.0001	**0.0006**	0.0007	0.0008	0.0001	**0.0000**	0.0001

Table 3: $I_{\bar{H}}$ indicator values of ASREA, NSGA-II and SPEA2 on ZDT and DTLZ functions.

Functions→		ZDT3			ZDT4			ZDT6		
MOEAs→		ASREA	NSGA-II	SPEA2	ASREA	NSGA-II	SPEA2	ASREA	NSGA-II	SPEA2
10^3 EVAL	Mean	**0.2561**	0.3149	0.4758	0.1960	0.2498	**0.1484**	**0.6192**	0.6499	0.8009
	Median	**0.2519**	0.3180	0.4758	0.1929	0.2446	**0.1484**	**0.6205**	0.6513	0.7956
	S.D.	0.0234	0.0238	**0.0206**	0.0504	**0.0333**	0.0455	0.0432	**0.0342**	0.0397
	Best	**0.2178**	0.2597	0.4333	0.0972	0.2077	**0.0602**	**0.5200**	0.5384	0.7217
	Worst	**0.3114**	0.3731	0.5307	0.3344	0.3363	**0.2488**	**0.7082**	0.7094	0.8531
10^4 EVAL	Mean	**0.0070**	0.0072	0.2945	**0.0036**	0.0059	0.0162	**0.0137**	0.0416	0.1471
	Median	**0.0062**	0.0071	0.2906	**0.0036**	0.0056	0.0161	**0.0138**	0.0409	0.1408
	S.D.	**0.0020**	**0.0020**	0.0254	**0.0015**	0.0030	0.0064	**0.0023**	0.0063	0.0385
	Best	0.0052	**0.0045**	0.2412	**0.0011**	0.0017	0.0030	**0.0094**	0.0282	0.0959
	Worst	0.0135	**0.0133**	0.3564	**0.0070**	0.0130	0.0355	**0.0197**	0.0547	0.2117
$5*10^4$ EVAL	Mean	0.0026	**0.0006**	0.0089	0.0002	**0.0000**	0.0132	-0.0012	**-0.0032**	-0.0029
	Median	0.0023	**0.0003**	0.0084	0.0001	**0.0000**	0.0119	-0.0013	**-0.0032**	-0.0029
	S.D.	**0.0012**	0.0013	0.003	**0.0000**	**0.0000**	0.0054	0.0004	**0.0000**	**0.0000**
	Best	0.0016	**0.0003**	0.0045	0.0001	**0.0000**	0.0027	-0.0017	**-0.0032**	-0.0030
	Worst	0.0083	**0.0071**	0.0161	0.0004	**0.00004**	0.0320	0.0000	**-0.0031**	-0.0027
Functions→		DTLZ1			DTLZ2			DTLZ3		
MOEAs→		ASREA	NSGA-II	SPEA2	ASREA	NSGA-II	SPEA2	ASREA	NSGA-II	SPEA2
10^3 EVAL	Mean	0.0013	**0.0008**	0.0014	0.0596	**0.0463**	0.1245	0.0123	**0.0110**	0.0170
	Median	0.0010	**0.0007**	0.0012	0.0562	**0.0465**	0.1251	0.0105	**0.0100**	0.0157
	S.D.	0.0010	**0.0004**	0.0008	0.0125	**0.0076**	0.0183	0.0076	**0.0036**	0.0062
	Best	**0.0002**	0.0003	0.0005	0.0424	**0.0294**	0.0878	**0.0046**	0.0070	0.0070
	Worst	0.0043	**0.0016**	0.0041	0.1031	**0.0614**	0.1546	0.0432	**0.0220**	0.0303
10^4 EVAL	Mean	**0.0000**	**0.0000**	**0.0000**	0.0129	**0.0109**	0.0206	0.0003	**0.0000**	0.0002
	Median	**0.0000**	**0.0000**	**0.0000**	0.0123	**0.0095**	0.0198	0.00019	**0.0000**	0.0002
	S.D.	**0.0000**	**0.0000**	**0.0000**	**0.0023**	0.0029	0.0034	0.0003	**0.0000**	0.0001
	Best	**0.0000**	**0.0000**	**0.0000**	0.0097	**0.0069**	0.0153	**0.0000**	**0.0000**	**0.0000**
	Worst	0.0002	**0.0000**	**0.0000**	0.0176	**0.0172**	0.0298	0.0013	**0.0001**	0.0004
$5*10^4$ EVAL	Mean	**0.0000**	**0.0000**	**0.0000**	0.0118	0.0094	**0.0088**	**0.0000**	**0.0000**	0.0001
	Median	**0.0000**	**0.0000**	**0.0000**	0.0116	0.0093	**0.0087**	**0.0000**	**0.0000**	0.0001
	S.D.	**0.0000**	**0.0000**	**0.0000**	0.0023	0.0023	**0.0011**	**0.0000**	**0.0000**	**0.0000**
	Best	**0.0000**	**0.0000**	**0.0000**	0.0083	**0.0065**	0.0067	**0.0000**	**0.0000**	**0.0000**
	Worst	**0.0000**	**0.0000**	**0.0000**	0.0168	0.0141	**0.0108**	**0.0000**	**0.0000**	0.0001

II and SPEA2. The figure is only shown for ZDT3 because similar curves are obtained for the other test functions.

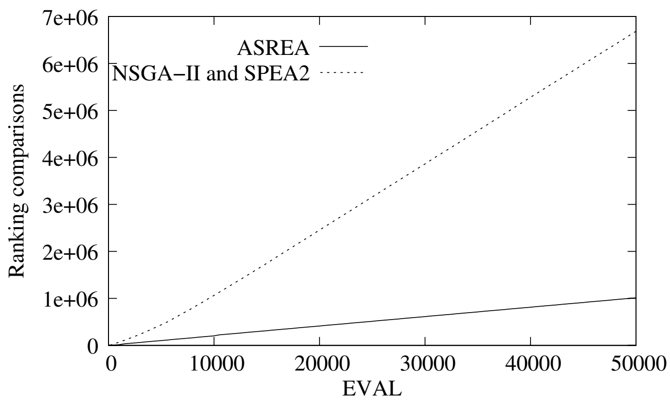


Figure 3: Rank comparison for ZDT3.

4. CONCLUSIONS AND FUTURE WORK

The aim of this work was to create a more "consistent" MOEA than the existing ones, that would couple a stochastic ranking to a stochastic evolutionary algorithm, so as to cut down the usual $O(mn^2)$ complexity of the ranking stage.

Not only was this achieved in ASREA (making it a really cheap algorithm with a ranking complexity in $O(man)$, cf. section 2.2), but the resulting algorithm with drastic but subtle selection strategy obtained astonishingly good results (even with small populations) on all different benchmark tests that were performed and six of which were presented in this paper:

In final stage: on all tests, ASREA always converged to the known P-O front as least as well as NSGA-II and SPEA2 with a comparable spread.

In intermediate stage: ASREA has always been the fastest algorithm towards the P-O front for all ZDT functions and DTLZ1 while maintaining a good spread.

In early stage: ASREA also managed to be the fastest on ZDT3, ZDT6, DTLZ2 and DTLZ3 functions.

This behavior makes ASREA particularly well suited for large real-world problems for three reasons:

1. Typically, the optimal front is not known, and rarely attained in a real-world problem, so being the fastest with good spread in early and intermediate stages will give an edge to ASREA over NSGA-II and SPEA2.

2. If the evaluation function is fast enough, really large populations can be used thanks to the complexity that is linear in objective space.

3. As for NSGA-II and SPEA2, ASREA is an "out of the box" algorithm that does not need any particular parameters to work well (several tests on dimensions larger than 2, seem to show that a good way to choose the archive size could be 10× the number of objectives).

Future work now consists in testing ASREA on many-objectives, to evaluate its performance and determine an optimal archive size. Selection strategy and clustering techniques may require attention.

5. REFERENCES

[1] D. W. Corne, N. R. Jerram, J. D. Knowles, M. J. Oates, and M. J. Pesa-ii: Region-based selection in evolutionary multiobjective optimization. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCOÕ2001*, pages 283–290. Morgan Kaufmann Publishers, 2001.

[2] D. W. Corne, J. D. Knowles, and M. J. Oates. The pareto envelope-based selection algorithm for multiobjective optimization. In *Proceedings of the Parallel Problem Solving from Nature VI Conference*, pages 839–848. Springer, 2000.

[3] K. Deb and R. B. Agrawal. Simulated binary crossover for continuous search space. *Complex Systems*, 9(2):115–148, 1995.

[4] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan. A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, 2002.

[5] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. Scalable test problems for evolutionary multiobjective optimization. In A. Abraham, L. Jain, and R. Goldberg, editors, *Evolutionary multiobjective optimization: theoritical advances and applications*, pages 105–145. Springer-Verlag, London, 2005.

[6] L. J. Fogel, A. J. Owens, and M. J. Walsh. *Artificial Intelligence through Simulated Evolution*. New York: John Wiley, 1966.

[7] C. M. Fonseca and P. J. Fleming. On the performance assessment and comparison of stochastic multiobjective optimizers. In *Proceedings of Parallel Problem Solving from Nature IV (PPSN-IV)*, pages 584–593, 1996.

[8] C. M. Fonseca, V. G. Fonseca, and L. Paquete. Exploring the performance of stochastic multiobjective optimizers with the second-order attainment functions. In *Proceedings of the Third Evolutionary Multi-Criterion Optimization (EMO-05) Conference*, pages 250–264, 2005.

[9] V. G. Fonseca, C. M. Fonseca, and A. O. Hall. Inferential performance assessment of stochastic optimizers and the attainment function. In *Proceedings of the First Evolutionary Multi-Criterion Optimization (EMO-01) Conference*, pages 213–225, 2001.

[10] M. P. Fourman. Compaction of symbolic layout using genetic algorithms. In *Proceedings of the 1st International Conference on Genetic Algorithms*, pages 141–153, Hillsdale, NJ, USA, 1985. L. Erlbaum Associates Inc.

[11] D. E. Goldberg. *Genetic algorithms in search, optimization and machine learning*. Addison Wesley, 1989.

[12] M. P. Hansen and A. Jaszkiewicz. Evaluating the Quality of Approximations to the Non-Dominated Set. Imm-rep-1998-7, Technical University of Denmark, 1998.

[13] J. Horn, N. Nafploitis, and D. E. Goldberg. A niched Pareto genetic algorithm for multi-objective optimization. In *Proceedings of the First IEEE Conference on Evolutionary Computation*, pages 82–87, 1994.

[14] KanGAL. Nsga-ii in c with gnuplot (real + binary + constraint handling): Revision 1.1. http://www.iitk.ac.in/kangal/codes.shtml, June 2005.

[15] J. Knowles and D. Corne. Properties of an adaptive archiving algorithm for storing nondominated vectors. *IEEE Transactions on Evolutionary Computation*, 7(2):100–116, 2003.

[16] J. Knowles, L. Thiele, and E. Zitzler. A Tutorial on the Performance Assessment of Stochastic Multiobjective Optimizers. TIK Report 214, Computer Engineering and Networks Laboratory (TIK), ETH Zurich, Feb. 2006.

[17] M. Laumanns, L. Thiele, K. Deb, and E. Zitzler. Archiving with Guaranteed Convergence And Diversity in Multi-objective Optimization. In *Genetic and Evolutionary Computation Conference (GECCO 2002)*, pages 439–447, New York, NY, USA, July 2002. Morgan Kaufmann Publishers.

[18] G. T. Parks and I. Miller. Selective breeding in a multiobjective genetic algorithm. In A. E. Eiben, M. Schoenauer, and H. P. Schwefel, editors, *Proceedings of the Parallel Problem Solving from Nature V (PPSN-V)*, pages 250–259, Amsterdam, Holland, 1998. Springer-Verlag.

[19] TIK. A platform and programming language independent interface for search algorithms. http://www.tik.ethz.ch/˜ sop/pisa/.

[20] E. Zitzler, K. Deb, and L. Thiele. Comparison of multiobjective evolutionary algorithms : Empirical results. *Evolutionary Computation Journal*, 8(2):125–148, 2000.

[21] E. Zitzler, M. Laumanns, and L. Thiele. SPEA2: Improving the Strength Pareto Evolutionary Algorithm for Multiobjective Optimization. In K. Giannakoglou et al., editors, *Evolutionary Methods for Design, Optimisation and Control with Application to Industrial Problems (EUROGEN 2001)*, pages 95–100. International Center for Numerical Methods in Engineering (CIMNE), 2002.

[22] E. Zitzler and L. Thiele. Multiobjective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach. *IEEE Transactions on Evolutionary Computation*, 3(4):257–271, 1999.

[23] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. Grunert da Fonseca. Performance Assessment of Multiobjective Optimizers: An Analysis and Review. *IEEE Transactions on Evolutionary Computation*, 7(2):117–132, 2003.

APPENDIX

A. TEST FUNCTIONS

Problem definition for ZDT functions [20]:
Minimize $\Gamma(\mathbf{x}) = (f_1(x_1),\ f_2(\mathbf{x}))$
subjected to: $f_2(\mathbf{x}) = g(x_2, ..., x_m)h(f_1(x_1), g(x_2, ..., x_m))$
where $\mathbf{x} = (x_1, ..., x_m)$

Problem definitions for DTLZ functions [5]
Minimize $\Gamma(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ ...,\ f_M(\mathbf{x}))$
where M = 3 in this paper.

Table 4: ZDT and DTLZ functions

Functions	Properties
ZDT3: $f_1(x_1) = x_1$, $g(x_2, ..., x_m) = 1 + 9.\sum_{i=2}^{m} x_i/(m-1)$, $h(f_1, g) = 1 - \sqrt{f_1/g} - (f_1/g)\sin(10\pi f_1)$, where $m = 30$, and $x_i \in [0, 1]$	Discontinuous convex P-O front
ZDT4: $f_1(x_1) = x_1$, $g(x_2, ..., x_m) = 1 + 10(m-1) + \sum_{i=2}^{m}(x_i^2 - 10\cos(4\pi x_i))$, $h(f_1, g) = 1 - \sqrt{f_1/g}$, where $m = 10$, $x_1 \in [0, 1]$, and $x_2, ..., x_m \in [-5, 5]$	21^9 local optimal P-O fronts
ZDT6: $f_1(x_1) = 1 - \exp(-4x_1)\sin^6(6\pi x_1)$, $g(x_2, ..., x_m) = 1 + 9.(\sum_{i=2}^{m} x_i/(m-1))^{0.25}$, $h(f_1, g) = 1 - (f_1/g)^2$, where $m = 10$, and $x_i \in [0, 1]$	1. Non-uniform distribution of P-O solutions. 2. Density of solutions is lower near the P-O front and highest away from the front
DTLZ1: $f_1(\mathbf{x}) = 0.5x_1x_2\cdots x_{M-1}(1 + g(\mathbf{x}_M))$, $f_2(\mathbf{x}) = 0.5x_1x_2\cdots(1 - x_{M-1})(1 + g(\mathbf{x}_M))$, ... , $f_{M-1}(\mathbf{x}) = 0.5x_1(1 - x_2)(1 + g(\mathbf{x}_M))$, $f_M(\mathbf{x}) = 0.5(1 - x_1)(1 + g(\mathbf{x}_M))$, subjected to $0 \le x_i \le 1$, for $i = 1, 2, ..., n$, $g(\mathbf{x}_M) = 100[\lvert\mathbf{x}_M\rvert + \Sigma_{x_i \in x_M}(x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5))]$	$(11^k - 1)$ local Pareto-optimal fronts, where k = 5. Hyper-plane P-O front
DTLZ2: $f_1(\mathbf{x}) = (1 + g(\mathbf{x})_M)\cos(x_1\pi/2)\cos(x_2\pi/2)\cdots\cos(x_M - 2\pi/2)\cos(x_{M-1}\pi/2)$, $f_2(\mathbf{x}) = (1 + g(\mathbf{x})_M)\cos(x_1\pi/2)\cos(x_2\pi/2)\cdots\cos(x_M - 2\pi/2)\sin(x_{M-1}\pi/2), \cdots, f_{M-1}(\mathbf{x}) = (1 + g(\mathbf{x}_M))\cos(x_1\pi/2)\sin(x_2\pi/2)$, $f_M(\mathbf{x}) = (1 + g(\mathbf{x}_M))\sin(x_2\pi/2)$, subjected to $0 \le x_i \le 1$, for $i = 1, 2, ..., n, g(\mathbf{x_M}) = \Sigma_{x_i \in \mathbf{x}_M}(x_i - 0.5)^2$	Unit spherical P-O surface
DTLZ3: $f_1(\mathbf{x}) = (1 + g(\mathbf{x})_M)\cos(x_1\pi/2)\cos(x_2\pi/2)\cdots\cos(x_M - 2\pi/2)\cos(x_{M-1}\pi/2)$, $f_2(\mathbf{x}) = (1 + g(\mathbf{x})_M)\cos(x_1\pi/2)\cos(x_2\pi/2)\cdots\cos(x_M - 2\pi/2)\sin(x_{M-1}\pi/2), \cdots, f_{M-1}(\mathbf{x}) = (1 + g(\mathbf{x}_M))\cos(x_1\pi/2)\sin(x_2\pi/2)$, $f_M(\mathbf{x}) = (1 + g(\mathbf{x}_M))\sin(x_2\pi/2)$, subjected to $0 \le x_i \le 1$, for $i = 1, 2, ..., n$, $g(\mathbf{x}_M) = 100[\lvert\mathbf{x}_M\rvert + \Sigma_{x_i \in x_M}(x_i - 0.5)^2 - \cos(20\pi(x_i - 0.5))]$	$3^k - 1$ local P-O fronts and one global P-O front, where k = 10.