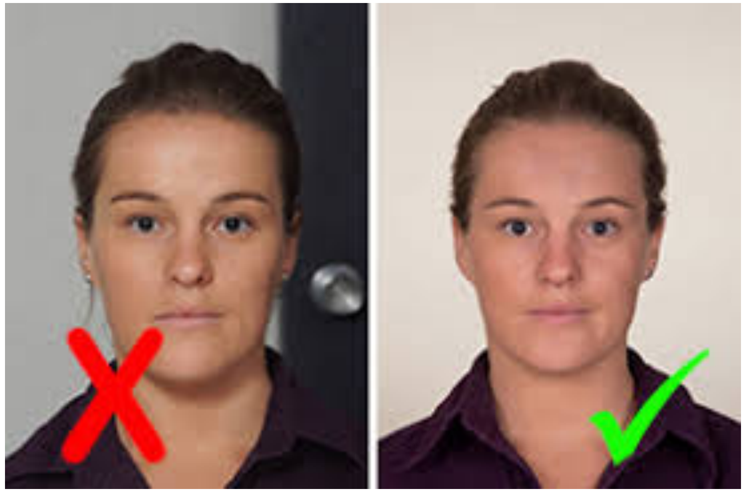
PRINCIPAL COMPONENT ANALYSIS(PCA)

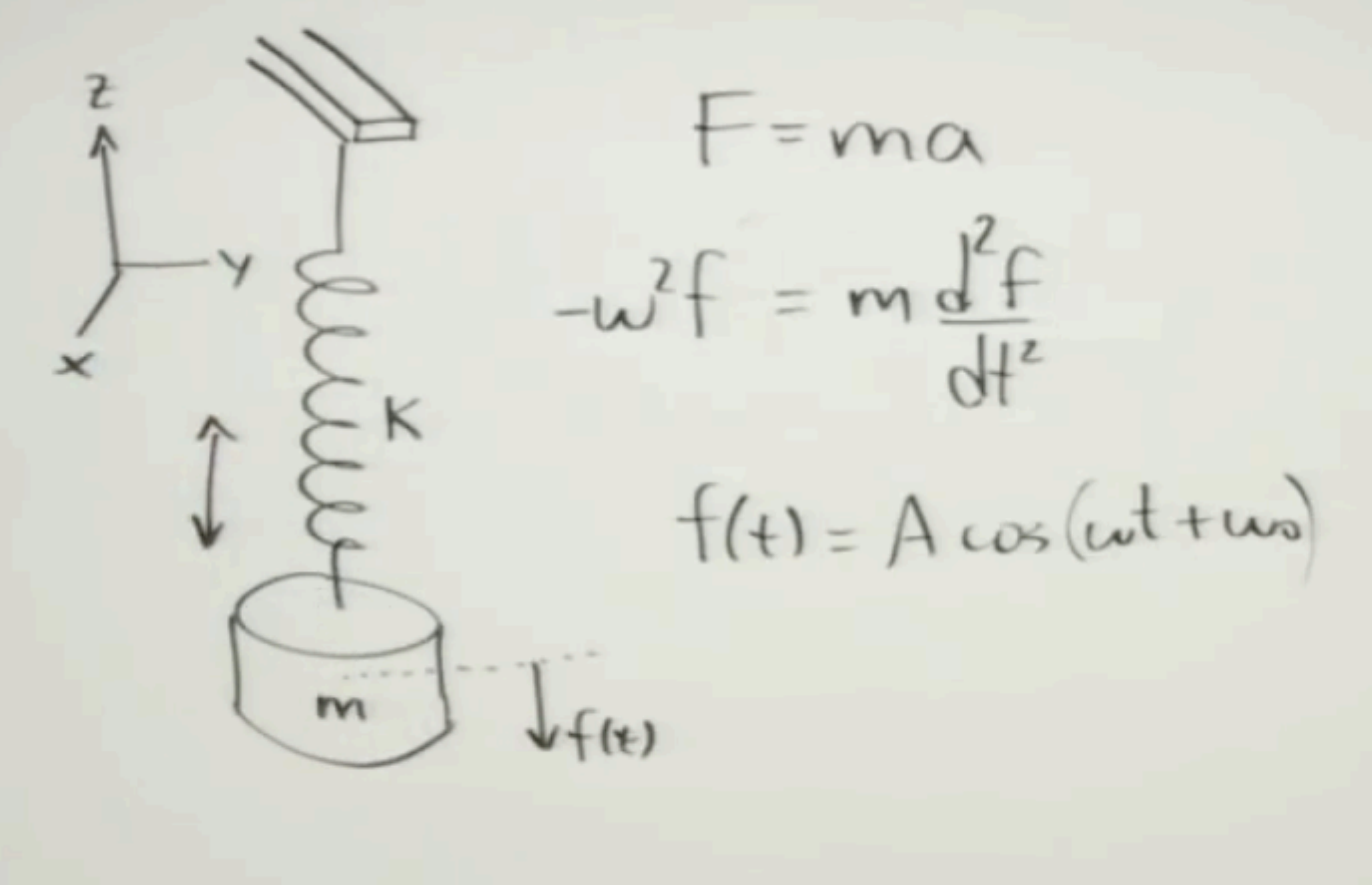
By Deepen naorem

- Latent(hidden) representation Method
- A method or mechanism to see or view data(matrix) in different ways.
- Data matrix $\begin{bmatrix} 2 & 4 \\ 5 & 7 \end{bmatrix}$
- Change of Basis



Let us consider a scenario

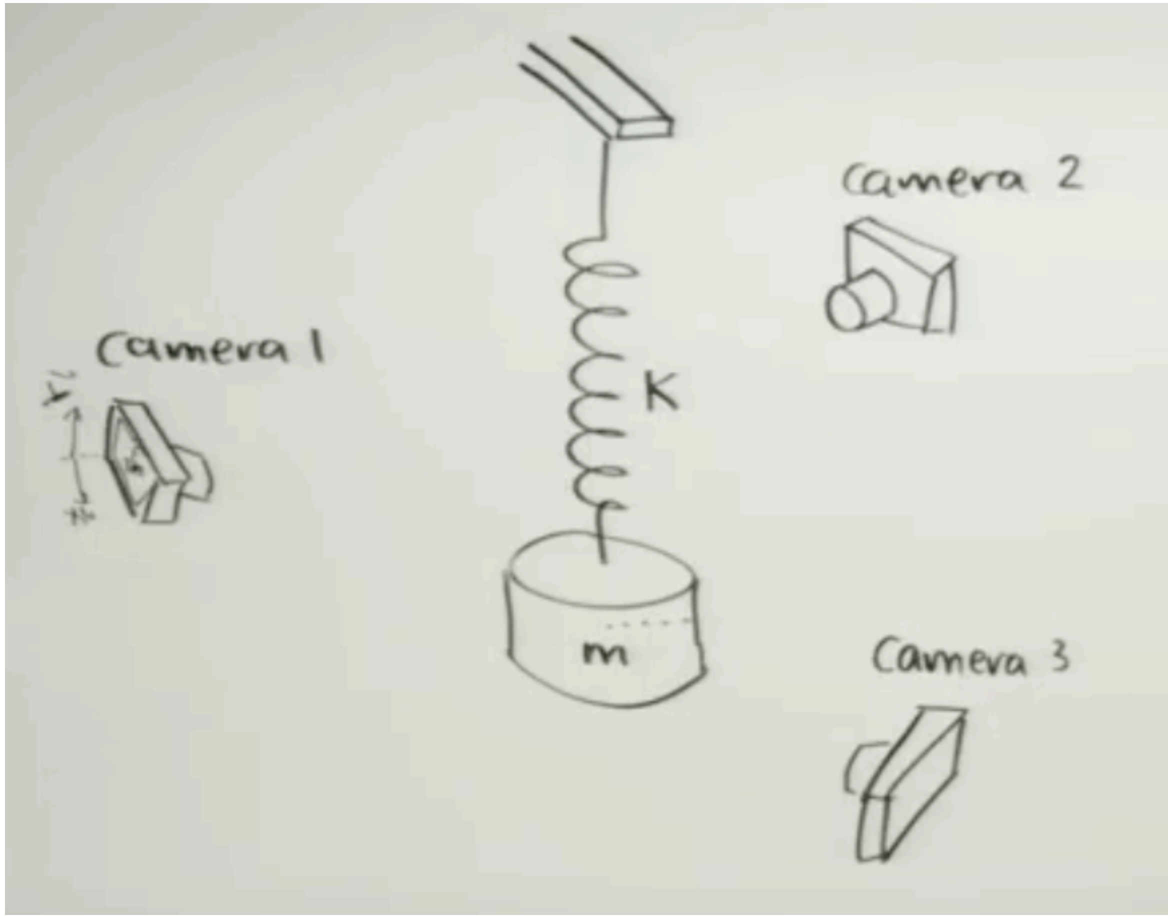
System



Let us also consider a parallel universe

- Newton =DUMB

Can we understand what is happening in the system without F=ma?



- From the camera we have

Camera 1 : $(\vec{x}_a, \vec{y}_a)$

Camera 2 : $(\vec{x}_b, \vec{y}_b)$

Camera 3 : $(\vec{x}_c, \vec{y}_c)$

Data Matrix

$$X = \begin{bmatrix} \vec{x}_a \\ \vec{y}_a \\ \vec{x}_b \\ \vec{y}_b \\ \vec{x}_c \\ \vec{y}_c \end{bmatrix}$$

Fundamental issues

- Noise
- Redundancy
  - Are the measures independent of each other??
  - One degree of freedom, but we have 6 sets of data
  - We just need the Z-direction dynamics
  - We need 1-degree of freedom
- PCA tells us we need only one camera at certain angle which will give the whole things or the entire system.

Before moving on let us understand

- Variance
- Co-variance
- Co-variance matrix

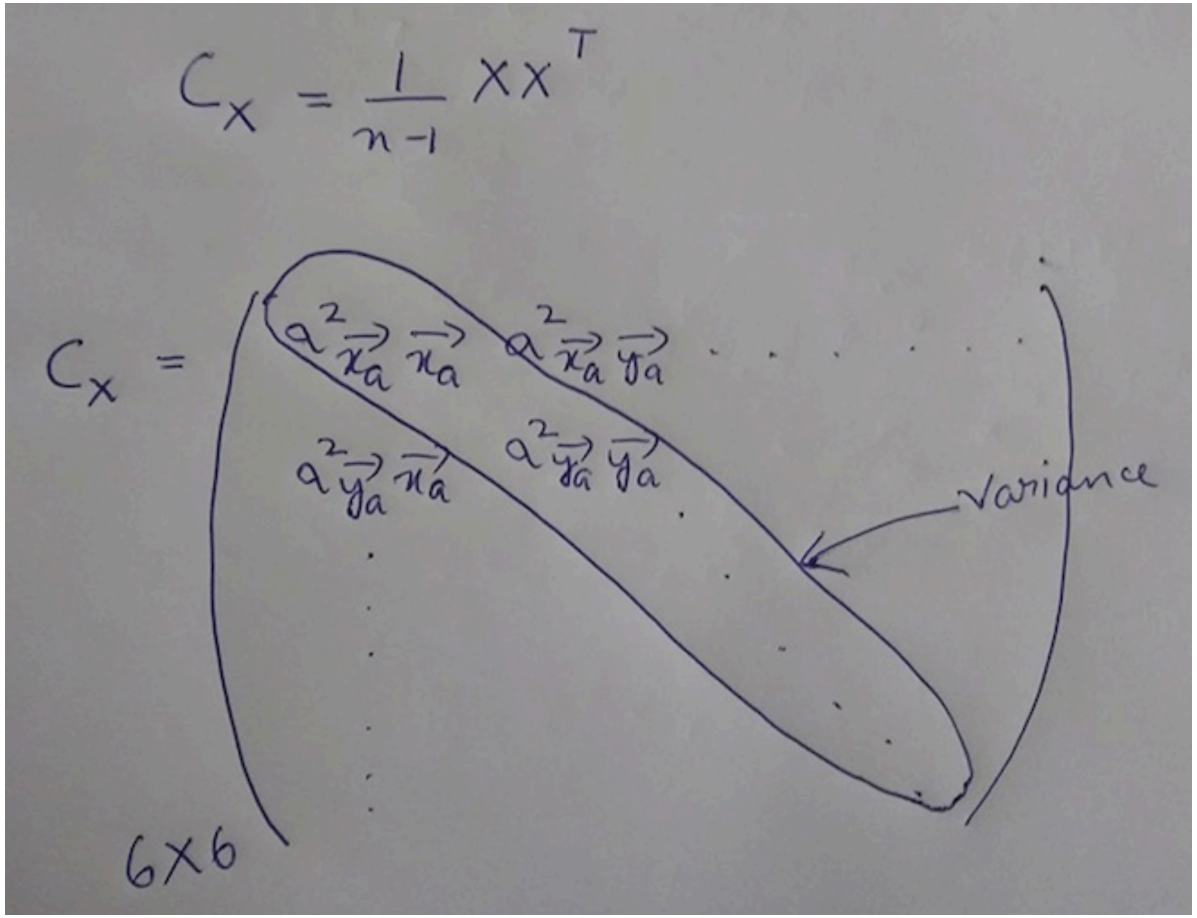
Characteristics of data matrix X

- What we see from the three cameras are not statistically independent.
- lots of data is redundant.
- Need to remove the redundant data.
- Reduce from 6 to 1 degree of freedom.

$$X = \begin{bmatrix} \vec{x}_a \\ \vec{y}_a \\ \vec{x}_b \\ \vec{y}_b \\ \vec{x}_c \\ \vec{y}_c \end{bmatrix}$$

Co-variance matrix

- C_x is a symmetric matrix
- i.e $C_x = C_x^T$
- Self adjoint
- Hermitian



Inspection of covariance matrix

- Small off-diagonal elements implies statistically independent.
- Big off-diagonal elements implies they are sharing a lot of stuffs.
- Lot of redundancy
- Big diagonal elements implies a lot of system stuff is happening there.
- They are the one that matters.

Diagonalized the system i.e the matrix

- Change the basis that I am working.
- What does diagonalize means??
  - Co-variance =0
  - No redundancy.

$$C_x = \begin{pmatrix} \ddots & & 0 \\ & \ddots & \\ 0 & & \ddots \end{pmatrix}$$

This is similar to SVD and EVD

The biggest diagonal gives the strongest contribution in the system.

Using Eigen value decomposition(EVD)

- $X.X^T = S \; \Lambda \; S^{-1}$
- $X.X^T$ is symmetric all the Eigen vectors are orthogonal to each other.
  i.e $S^{-1} = S^*$ or $S^{-1} = S^T$
- Λ is the diagonal matrix with Eigen values of $X.X^T$

What is the new frame of reference or basis needed to remove redundancy?

- It should be related to the original set of measurement.
- We need to rotate data matrix-X
- $Y=S^TX$ (New frame of reference)

Proof of no redundancy

$$Y = S^T X$$

$$C_Y = \frac{1}{n-1} Y Y^T = \frac{1}{n-1} S^T X (S^T X)^T$$

$$= \frac{1}{n-1} S^T X X^T S$$

$$= \frac{1}{n-1} S^T S \Lambda S^{-1} S$$

$$= \frac{1}{n-1} \Lambda$$

$$\Lambda = \begin{pmatrix} \alpha_1 & & & 0 \\ & \alpha_2 & & \\ & & \ddots & \\ 0 & & & \ddots \end{pmatrix}$$

We figure out the right way to look at the data or our problem.

Thank you