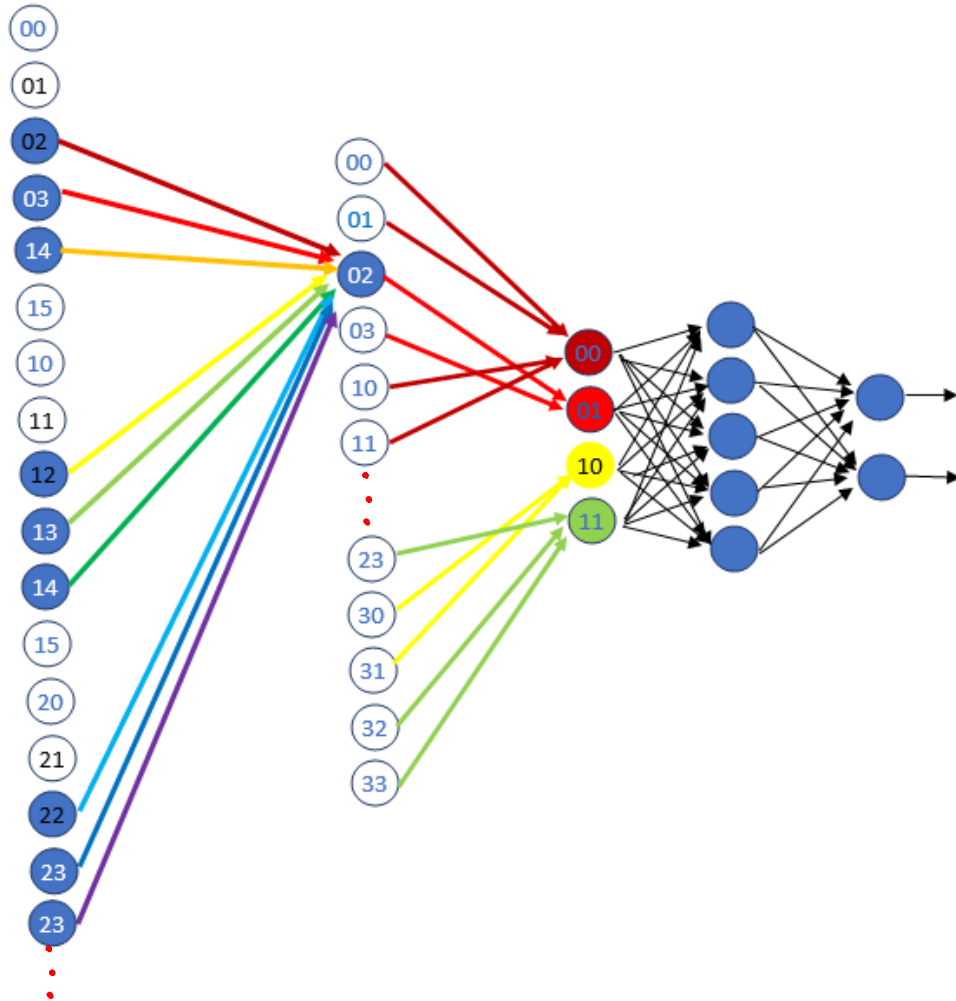


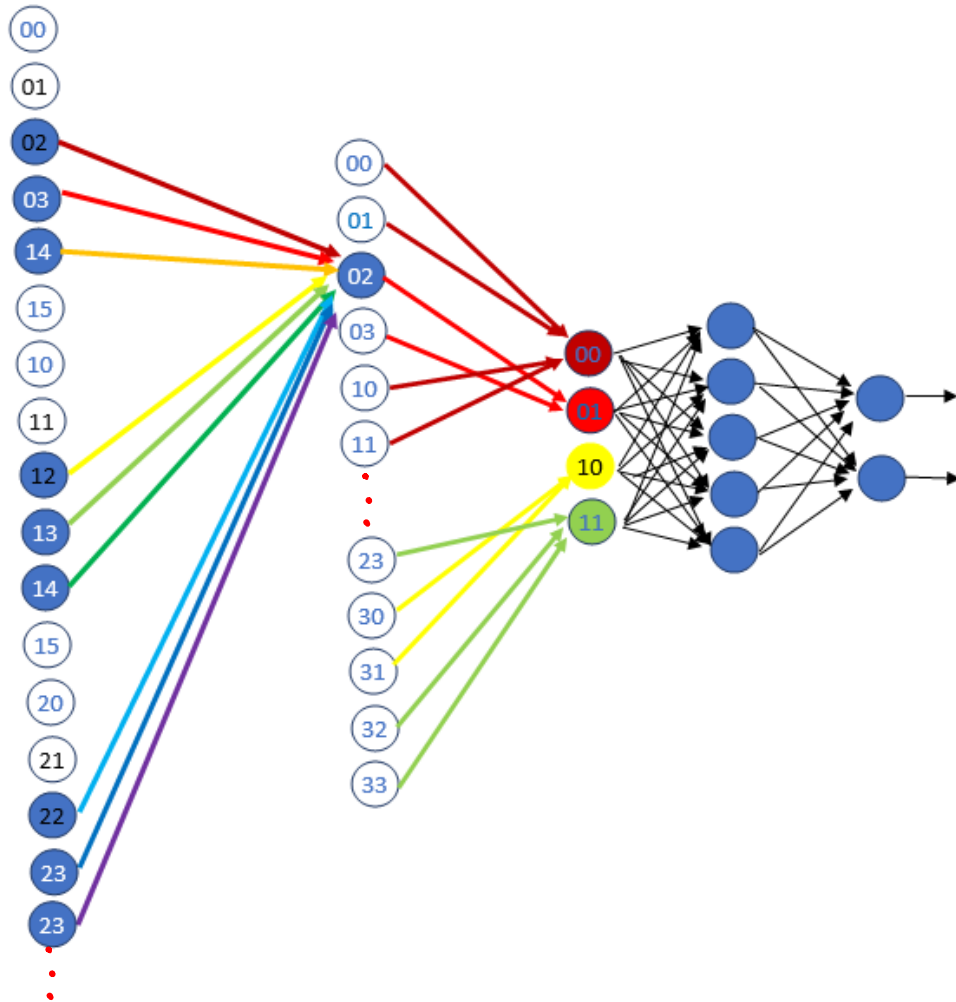
Backpropagation in CNN



Parameters are

- Filter Matrices
- Weight Matrix of the Dense layer

Backpropagation in CNN



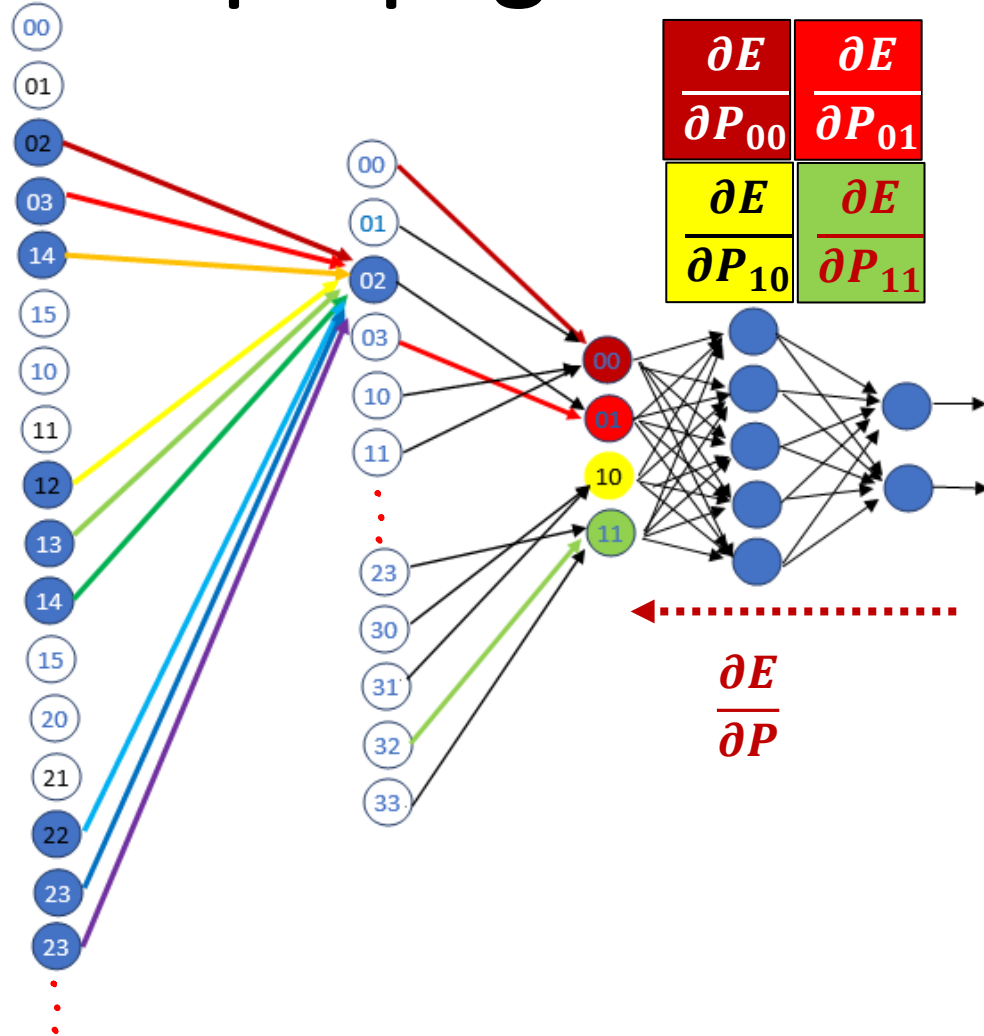
5	6	7	5	6	7
6	7	8	6	7	8
7	8	9	7	8	9
5	6	7	5	6	7
6	7	8	6	7	8
7	8	9	7	8	9



41	44		

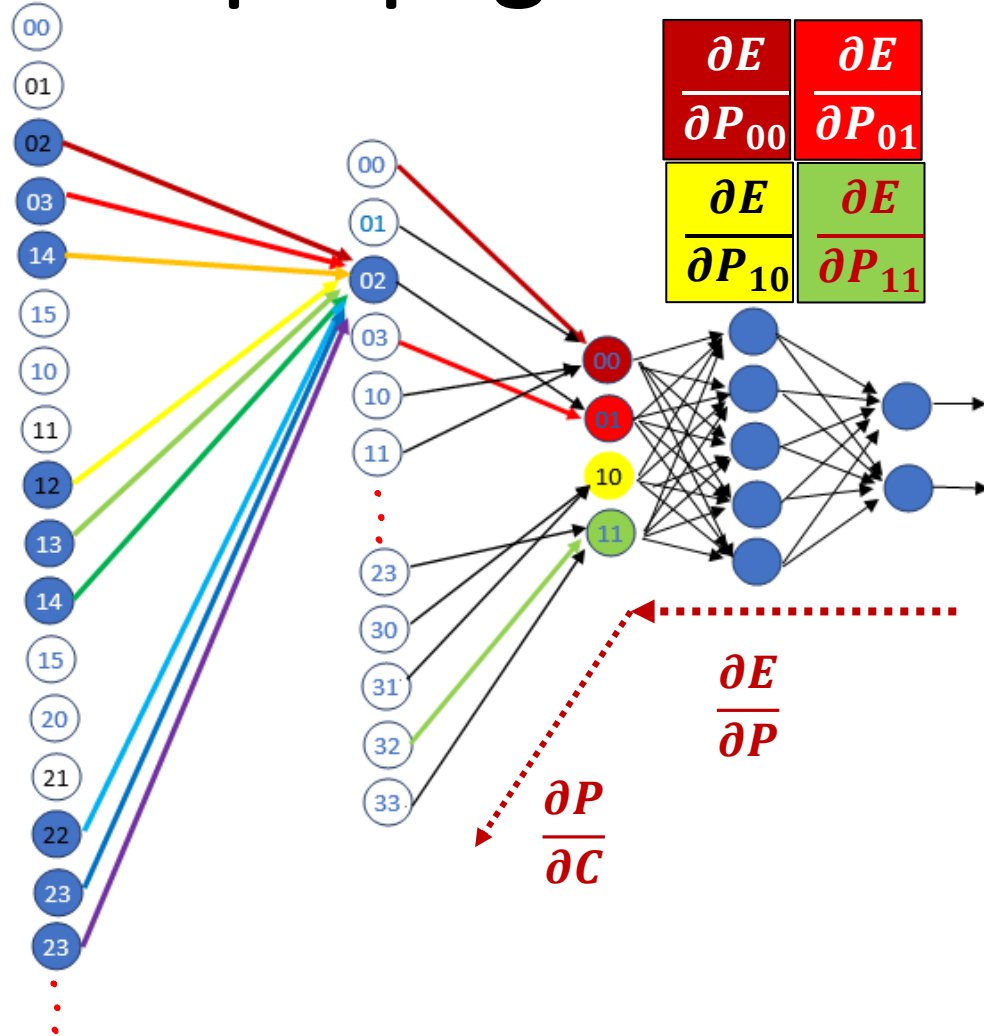
$$\frac{\partial E}{\partial F} = \frac{\partial C}{\partial F} \times \frac{\partial P}{\partial C} \times \frac{\partial E}{\partial P}$$

Backpropagation in CNN



$$\frac{\partial E}{\partial F} = \frac{\partial C}{\partial F} \times \frac{\partial P}{\partial C} \times \frac{\partial E}{\partial P}$$

Backpropagation in CNN

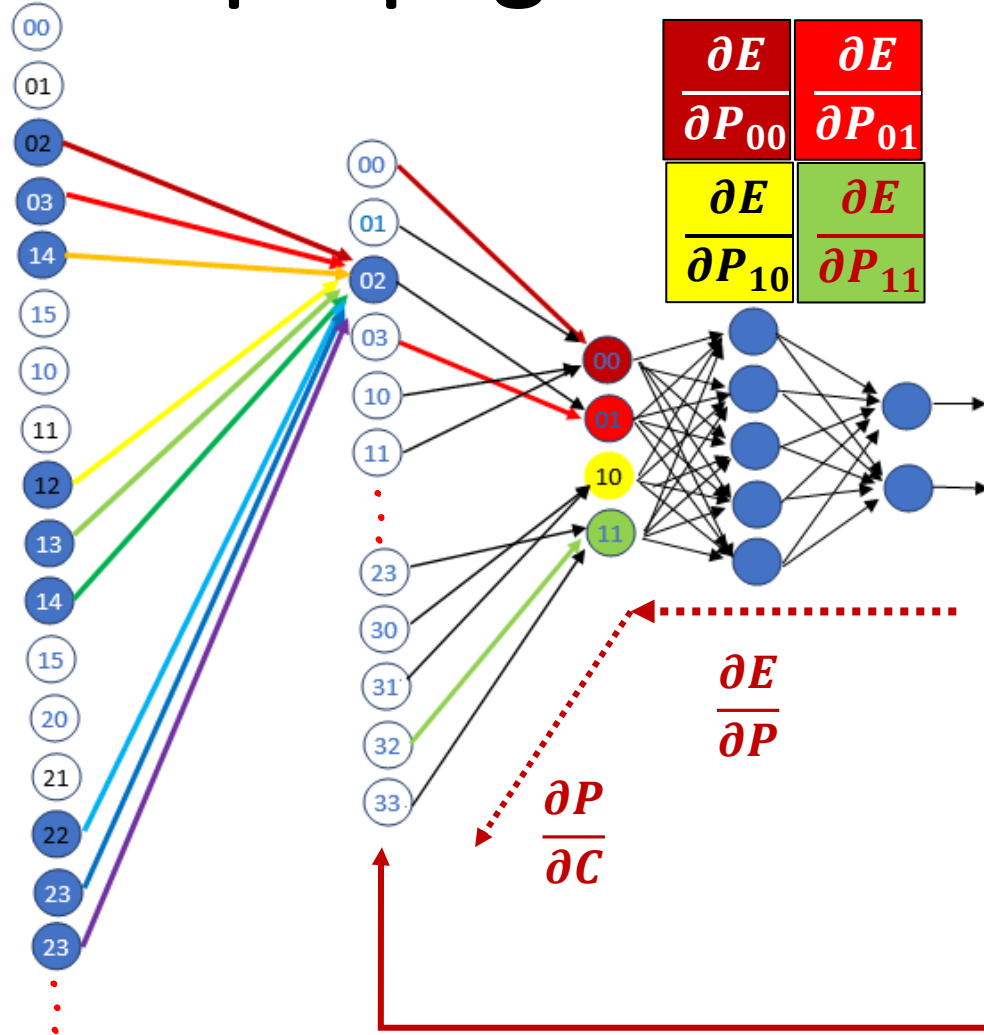


$$\frac{\partial E}{\partial F} = \frac{\partial C}{\partial F} \times \frac{\partial P}{\partial C} \times \frac{\partial E}{\partial P}$$

$$\frac{\partial E}{\partial C_{00}} = \frac{\partial P_{00}}{\partial C_{00}} \times \frac{\partial E}{\partial P_{00}}$$

$$\frac{\partial E}{\partial C_{00}} = \frac{\partial}{\partial C_{00}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0$$

Backpropagation in CNN



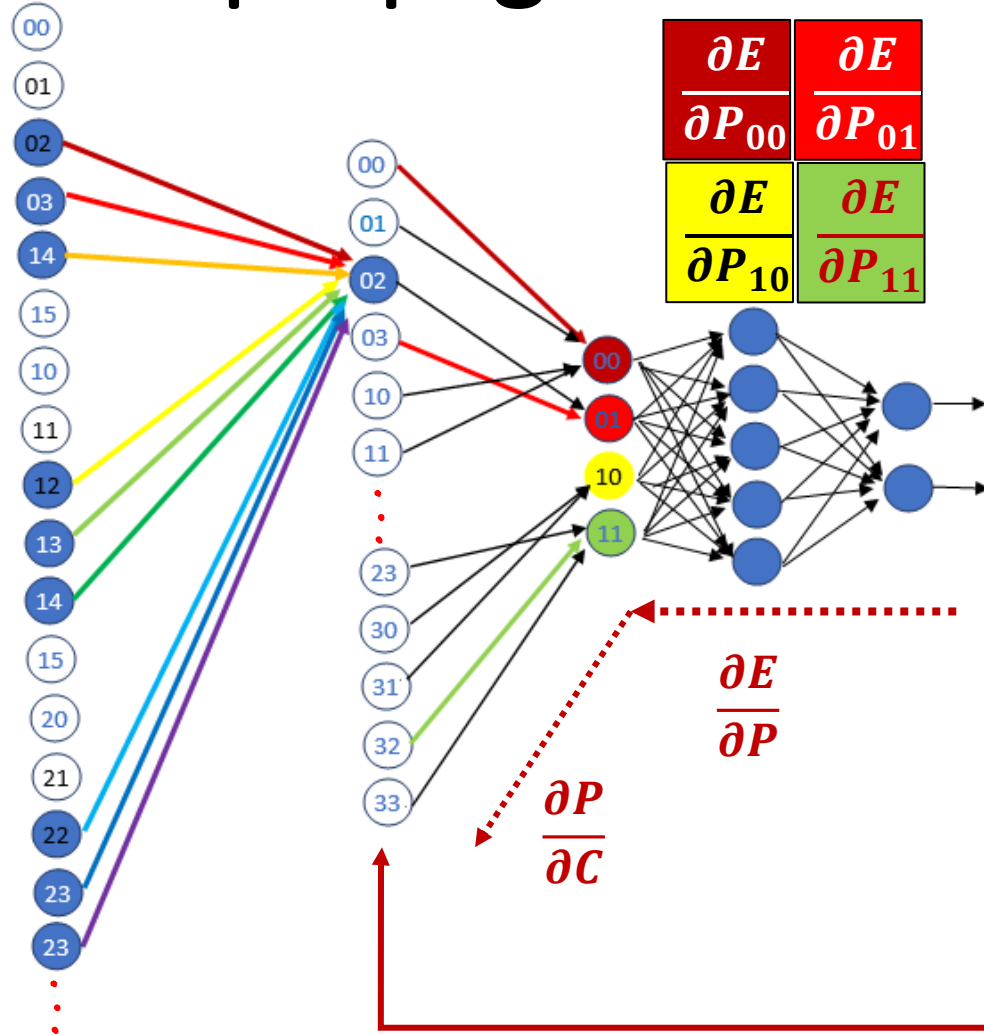
$$\frac{\partial E}{\partial F} = \frac{\partial C}{\partial F} \times \frac{\partial P}{\partial C} \times \frac{\partial F}{\partial P}$$

$$\frac{\partial E}{\partial C_{00}} = \frac{\partial P_{00}}{\partial C_{00}} \times \frac{\partial E}{\partial P_{00}}$$

$$\frac{\partial E}{\partial C_{00}} = \frac{\partial}{\partial C_{00}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0$$

0			

Backpropagation in CNN



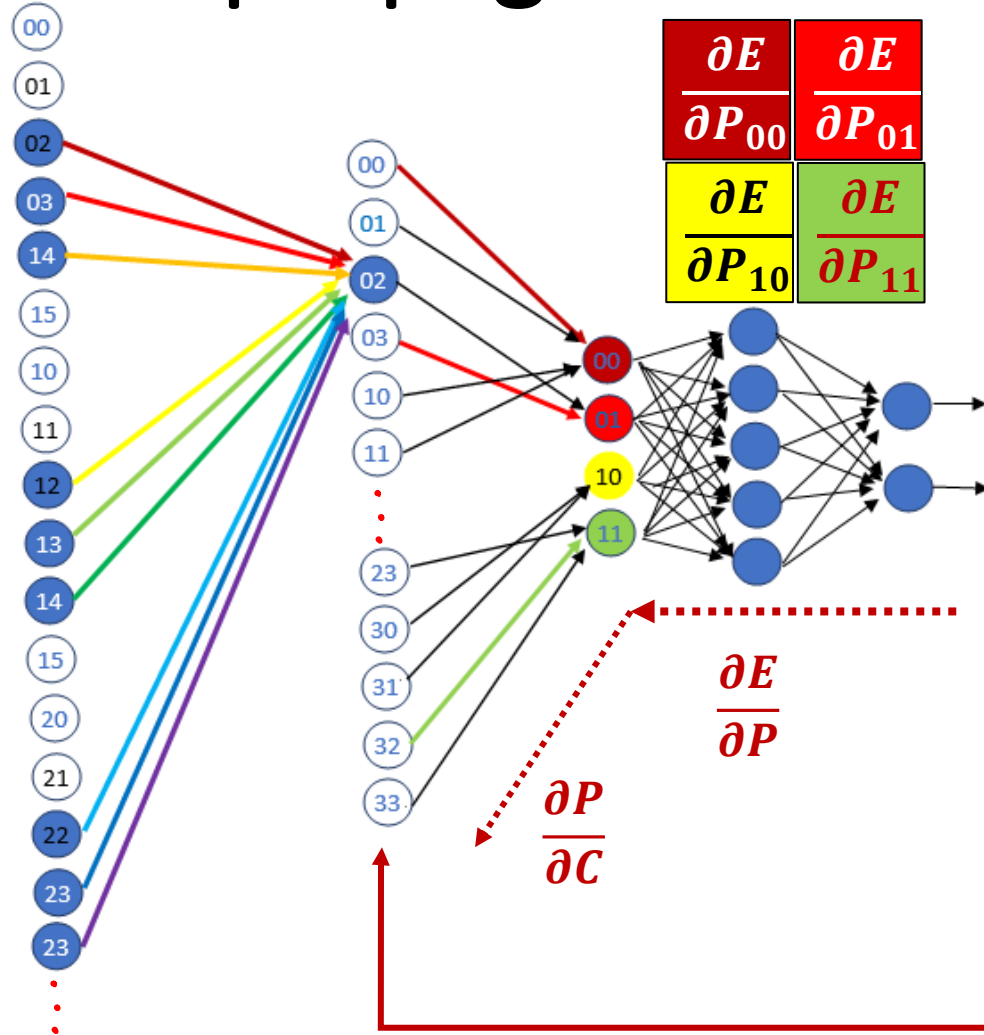
$$\frac{\partial E}{\partial C_{00}} = \frac{\partial P_{00}}{\partial C_{00}} \times \frac{\partial E}{\partial P_{00}}$$

$$\frac{\partial E}{\partial C_{00}} = \frac{\partial}{\partial C_{00}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0$$

$$\frac{\partial E}{\partial C_{01}} = \frac{\partial}{\partial C_{01}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = \mathbf{C_{01}} \frac{\partial E}{\partial P_{00}}$$

0	$\mathbf{C_{01}}$		

Backpropagation in CNN

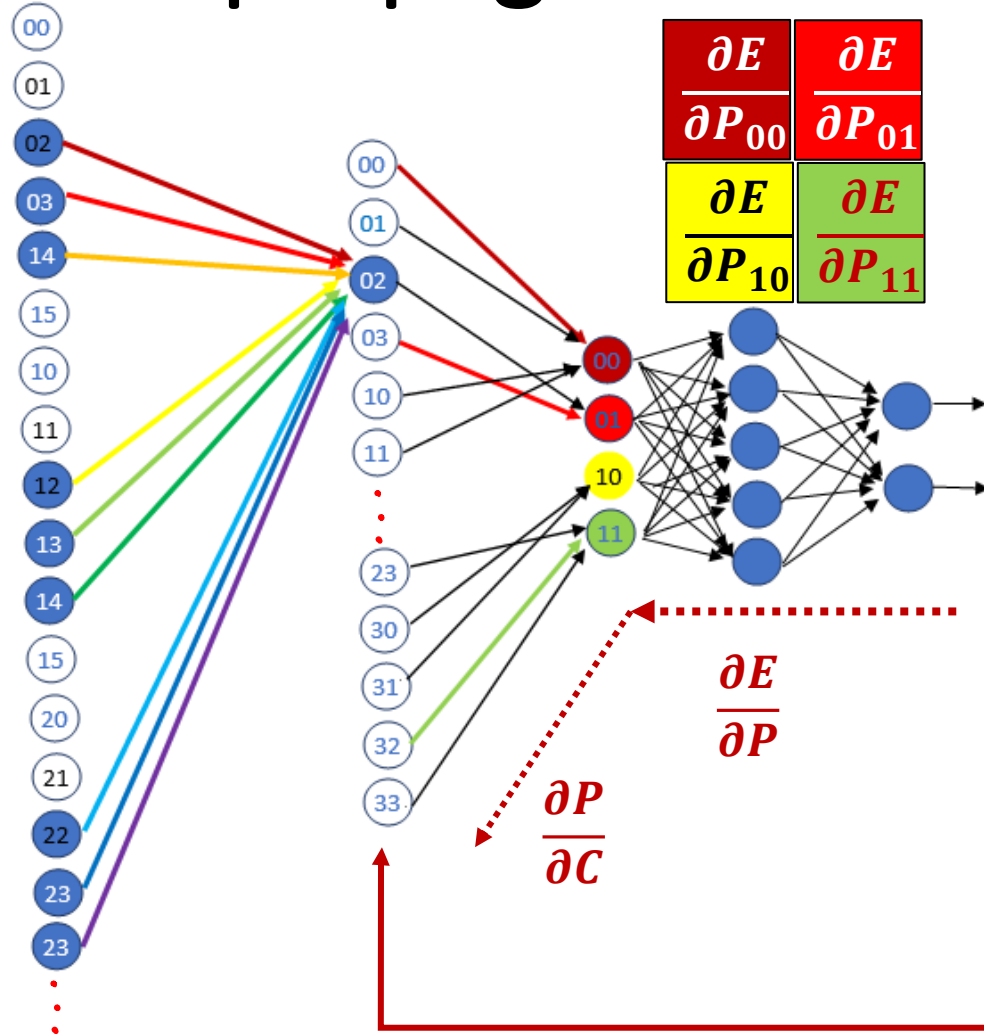


$$\frac{\partial E}{\partial C_{00}} = \frac{\partial P_{00}}{\partial C_{00}} \times \frac{\partial E}{\partial P_{00}}$$

$$\begin{aligned} \frac{\partial E}{\partial C_{00}} &= \frac{\partial}{\partial C_{00}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0 \\ \frac{\partial E}{\partial C_{01}} &= \frac{\partial}{\partial C_{01}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = \mathbf{C_{01}} \frac{\partial E}{\partial P_{00}} \\ \frac{\partial E}{\partial C_{10}} &= \frac{\partial}{\partial C_{10}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0 \end{aligned}$$

0	C_{01}		
0			

Backpropagation in CNN

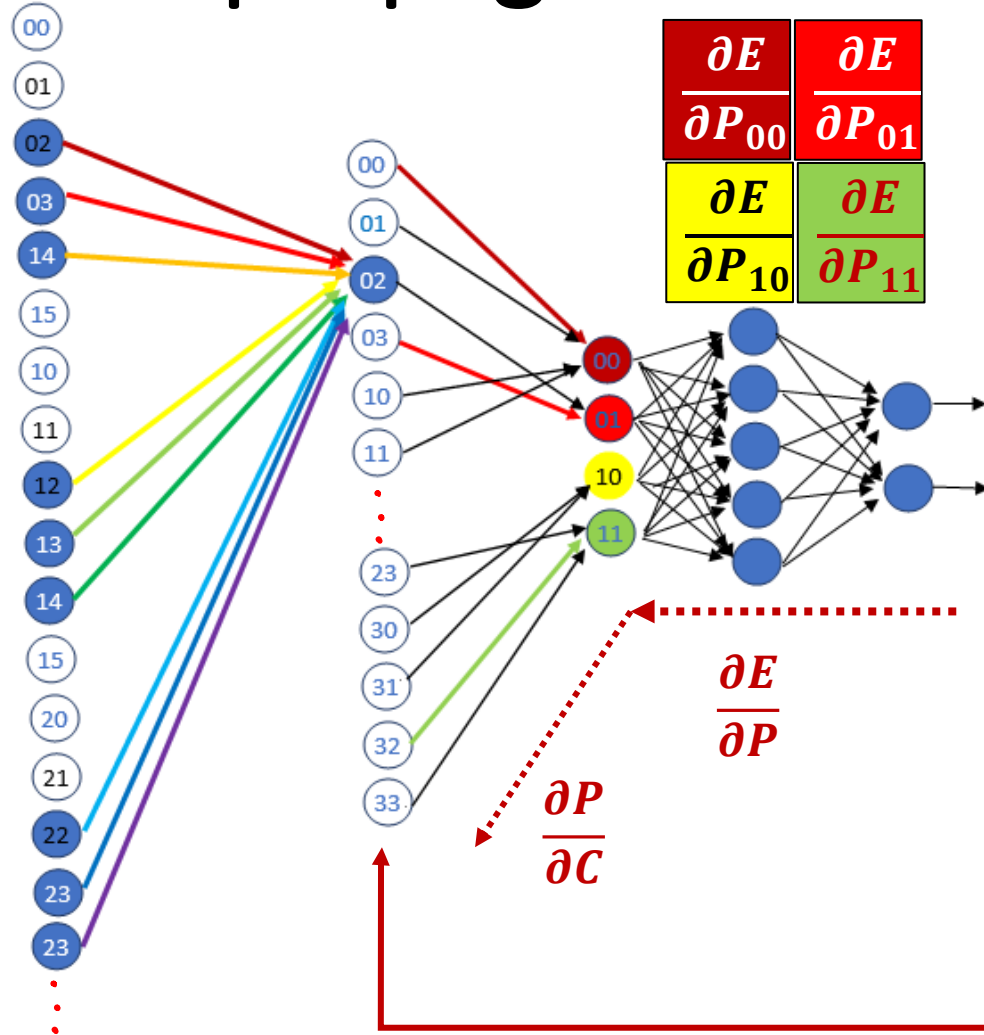


$$\frac{\partial E}{\partial C_{00}} = \frac{\partial P_{00}}{\partial C_{00}} \times \frac{\partial E}{\partial P_{00}}$$

$$\begin{aligned} \frac{\partial E}{\partial C_{00}} &= \frac{\partial}{\partial C_{00}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0 \\ \frac{\partial E}{\partial C_{01}} &= \frac{\partial}{\partial C_{01}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = \mathbf{C_{01}} \frac{\partial E}{\partial P_{00}} \\ \frac{\partial E}{\partial C_{10}} &= \frac{\partial}{\partial C_{10}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0 \\ \frac{\partial E}{\partial C_{11}} &= \frac{\partial}{\partial C_{11}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0 \end{aligned}$$

0	C_{01}		
0	0		

Backpropagation in CNN



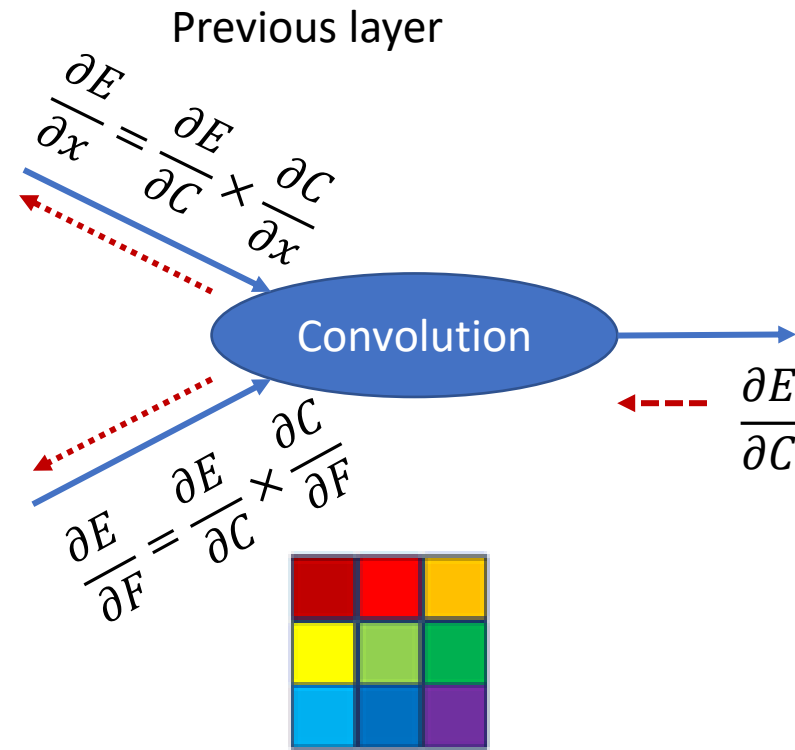
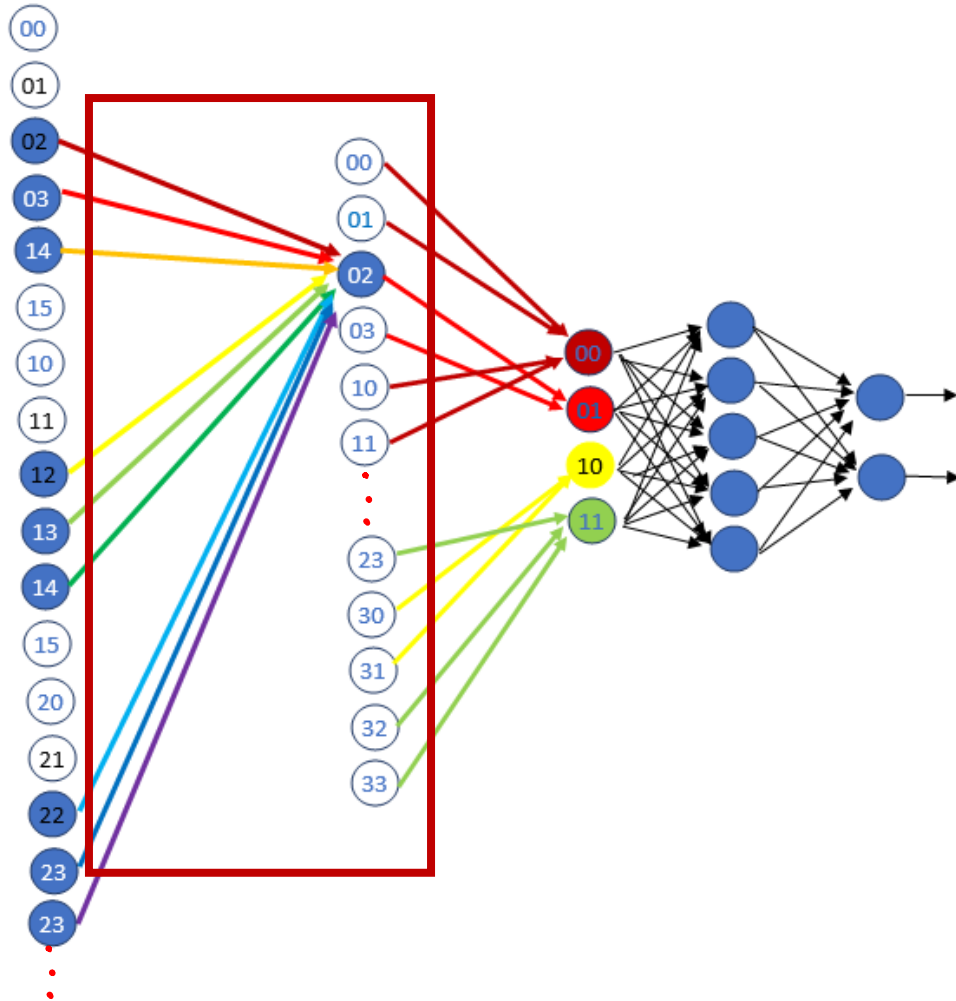
$$\frac{\partial E}{\partial C_{00}} = \frac{\partial P_{00}}{\partial C_{00}} \times \frac{\partial E}{\partial P_{00}}$$

$$\begin{aligned} \frac{\partial E}{\partial C_{00}} &= \frac{\partial}{\partial C_{00}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0 \\ \frac{\partial E}{\partial C_{01}} &= \frac{\partial}{\partial C_{01}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = \mathbf{C_{01}} \frac{\partial E}{\partial P_{00}} \\ \frac{\partial E}{\partial C_{10}} &= \frac{\partial}{\partial C_{10}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0 \\ \frac{\partial E}{\partial C_{11}} &= \frac{\partial}{\partial C_{11}} \max(C_{00}, \mathbf{C_{01}}, C_{10}, C_{11}) \frac{\partial E}{\partial P_{00}} = 0 \end{aligned}$$

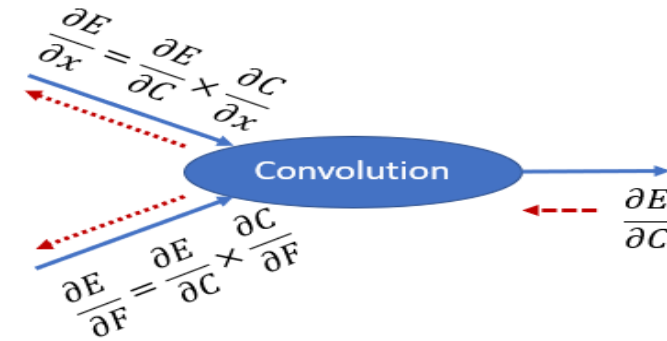
0	$\mathbf{C_{01}}$	0	$\mathbf{C_{03}}$
0	0	0	0
$\mathbf{C_{20}}$	0	0	0
0	0	$\mathbf{C_{32}}$	0

Backpropagation in CNN

$$\frac{\partial E}{\partial F} = \frac{\partial C}{\partial F} \times \frac{\partial P}{\partial C} \times \frac{\partial E}{\partial P}$$



Backpropagation in CNN



$$C_{00} = A_{00}F_{00} + A_{01}F_{01} + A_{02}F_{02} + A_{10}F_{10} + A_{11}F_{11} + A_{12}F_{12} + A_{20}F_{20} + A_{21}F_{21} + A_{22}F_{22}$$

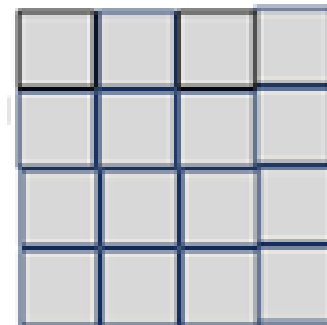
$$C_{01} = A_{01}F_{00} + A_{02}F_{01} + A_{03}F_{02} + A_{11}F_{10} + A_{12}F_{11} + A_{13}F_{12} + A_{21}F_{20} + A_{22}F_{21} + A_{23}F_{22}$$

$$C_{02} = A_{02}F_{00} + A_{03}F_{01} + A_{04}F_{02} + A_{12}F_{10} + A_{13}F_{11} + A_{14}F_{12} + A_{22}F_{20} + A_{23}F_{21} + A_{24}F_{22}$$

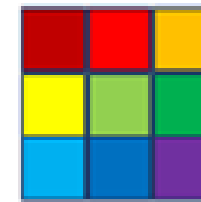
$$C_{03} = A_{03}F_{00} + A_{04}F_{01} + A_{05}F_{02} + A_{13}F_{10} + A_{14}F_{11} + A_{15}F_{12} + A_{23}F_{20} + A_{24}F_{21} + A_{25}F_{22}$$

⋮

$$\frac{\partial E}{\partial F} \quad \frac{\partial E}{\partial F_{ij}} = \sum_{k=0, l=0} \frac{\partial E}{\partial C_{kl}} \times \frac{\partial C_{kl}}{\partial F_{ij}}$$



Convolution matrix



Filter

Backpropagation in CNN

$$C_{00} = A_{00}F_{00} + A_{01}F_{01} + A_{02}F_{02} + A_{10}F_{10} + A_{11}F_{11} + A_{12}F_{12} + A_{20}F_{20} + A_{21}F_{21} + A_{22}F_{22}$$

$$C_{01} = A_{01}F_{00} + A_{02}F_{01} + A_{03}F_{02} + A_{11}F_{10} + A_{12}F_{11} + A_{13}F_{12} + A_{21}F_{20} + A_{22}F_{21} + A_{23}F_{22}$$

$$C_{02} = A_{02}F_{00} + A_{03}F_{01} + A_{04}F_{02} + A_{12}F_{10} + A_{13}F_{11} + A_{14}F_{12} + A_{22}F_{20} + A_{23}F_{21} + A_{24}F_{22}$$

$$C_{03} = A_{03}F_{00} + A_{04}F_{01} + A_{05}F_{02} + A_{13}F_{10} + A_{14}F_{11} + A_{15}F_{12} + A_{23}F_{20} + A_{24}F_{21} + A_{25}F_{22}$$

⋮

$$\frac{\partial E}{\partial F} \quad \frac{\partial E}{\partial F_{ij}} = \sum_{k=0, l=0} \frac{\partial E}{\partial C_{kl}} \times \frac{\partial C_{kl}}{\partial F_{ij}}$$

$$\frac{\partial E}{\partial F_{00}} = \frac{\partial E}{\partial C_{00}} \times A_{00} + \frac{\partial E}{\partial C_{01}} \times A_{01} + \frac{\partial E}{\partial C_{02}} \times A_{02} + \frac{\partial E}{\partial C_{03}} \times A_{03} + \frac{\partial E}{\partial C_{10}} \times A_{10} + \frac{\partial E}{\partial C_{11}} \times A_{11} + \dots$$

$$\frac{\partial E}{\partial F_{01}} = \frac{\partial E}{\partial C_{00}} \times A_{01} + \frac{\partial E}{\partial C_{01}} \times A_{02} + \frac{\partial E}{\partial C_{02}} \times A_{03} + \frac{\partial E}{\partial C_{03}} \times A_{04} + \frac{\partial E}{\partial C_{10}} \times A_{11} + \frac{\partial E}{\partial C_{11}} \times A_{12} + \dots$$

Backpropagation in CNN

$\frac{\partial E}{\partial F_{00}}$	$\frac{\partial E}{\partial F_{01}}$	$\frac{\partial E}{\partial F_{02}}$
$\frac{\partial E}{\partial F_{10}}$	$\frac{\partial E}{\partial F_{11}}$	$\frac{\partial E}{\partial F_{12}}$
$\frac{\partial E}{\partial F_{20}}$	$\frac{\partial E}{\partial F_{21}}$	$\frac{\partial E}{\partial F_{22}}$

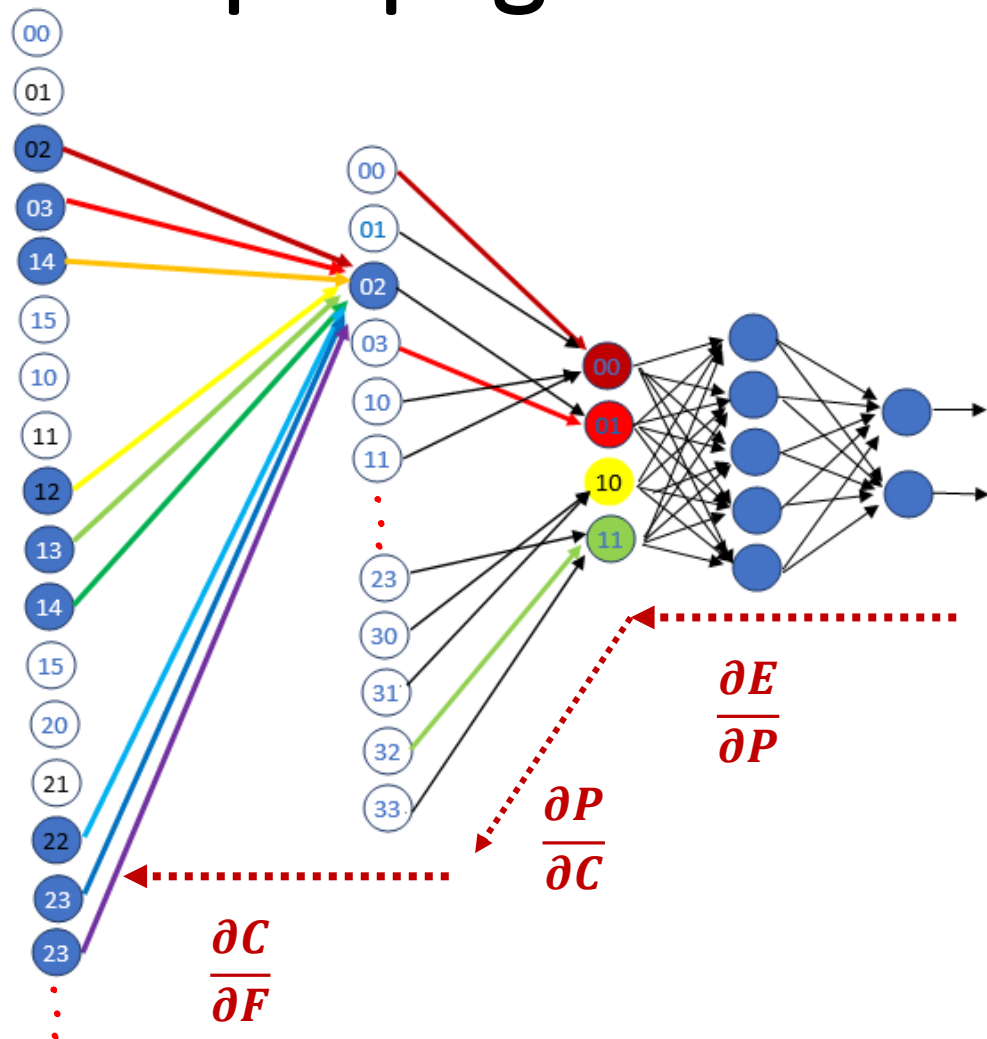
= Convolution

5	6	7	5	6	7
6	7	8	6	7	8
7	8	9	7	8	9
5	6	7	5	6	7
6	7	8	6	7	8
7	8	9	7	8	9

$\frac{\partial E}{\partial C_{00}}$	$\frac{\partial E}{\partial C_{01}}$	$\frac{\partial E}{\partial C_{02}}$	$\frac{\partial E}{\partial C_{03}}$
$\frac{\partial E}{\partial C_{10}}$	$\frac{\partial E}{\partial C_{11}}$	$\frac{\partial E}{\partial C_{12}}$	$\frac{\partial E}{\partial C_{13}}$
$\frac{\partial E}{\partial C_{20}}$	$\frac{\partial E}{\partial C_{21}}$	$\frac{\partial E}{\partial C_{22}}$	$\frac{\partial E}{\partial C_{23}}$
$\frac{\partial E}{\partial C_{30}}$	$\frac{\partial E}{\partial C_{31}}$	$\frac{\partial E}{\partial C_{32}}$	$\frac{\partial E}{\partial C_{33}}$

$$\frac{\partial E}{\partial F_{00}} = \frac{\partial E}{\partial C_{00}} \times A_{00} + \frac{\partial E}{\partial C_{01}} \times A_{01} + \frac{\partial E}{\partial C_{02}} \times A_{02} + \frac{\partial E}{\partial C_{03}} \times A_{03} + \frac{\partial E}{\partial C_{10}} \times A_{10} + \frac{\partial E}{\partial C_{11}} \times A_{11} + \dots$$

Backpropagation in CNN



Convolution

5	6	7	5	6	7
6	7	8	6	7	8
7	8	9	7	8	9
5	6	7	5	6	7
6	7	8	6	7	8
7	8	9	7	8	9

$\frac{\partial E}{\partial C_{00}}$	$\frac{\partial E}{\partial C_{01}}$	$\frac{\partial E}{\partial C_{02}}$	$\frac{\partial E}{\partial C_{03}}$
$\frac{\partial E}{\partial C_{10}}$	$\frac{\partial E}{\partial C_{11}}$	$\frac{\partial E}{\partial C_{12}}$	$\frac{\partial E}{\partial C_{13}}$
$\frac{\partial E}{\partial C_{20}}$	$\frac{\partial E}{\partial C_{21}}$	$\frac{\partial E}{\partial C_{22}}$	$\frac{\partial E}{\partial C_{23}}$
$\frac{\partial E}{\partial C_{30}}$	$\frac{\partial E}{\partial C_{31}}$	$\frac{\partial E}{\partial C_{32}}$	$\frac{\partial E}{\partial C_{33}}$

Backpropagation in CNN

$$\frac{\partial E}{\partial F} = \text{Convolution} \left(A, \frac{\partial E}{\partial C} \right) \quad \frac{\partial E}{\partial C} = \frac{\partial P}{\partial C} \times \frac{\partial E}{\partial P}$$

$\frac{\partial E}{\partial F_{00}}$	$\frac{\partial E}{\partial F_{01}}$	$\frac{\partial E}{\partial F_{02}}$
$\frac{\partial E}{\partial F_{10}}$	$\frac{\partial E}{\partial F_{11}}$	$\frac{\partial E}{\partial F_{12}}$
$\frac{\partial E}{\partial F_{20}}$	$\frac{\partial E}{\partial F_{21}}$	$\frac{\partial E}{\partial F_{22}}$

Filter Parameters

=

5	6	7	5	6	7
6	7	8	6	7	8
7	8	9	7	8	9
5	6	7	5	6	7
6	7	8	6	7	8
7	8	9	7	8	9

Gradient Convolution

0	C_{01}	0	C_{03}
0	0	0	0
C_{20}	0	0	0
0	0	C_{32}	0

Gradient Pooling

×

$\frac{\partial E}{\partial P_{00}}$	$\frac{\partial E}{\partial P_{01}}$
$\frac{\partial E}{\partial P_{10}}$	$\frac{\partial E}{\partial P_{11}}$

Gradient from next layer