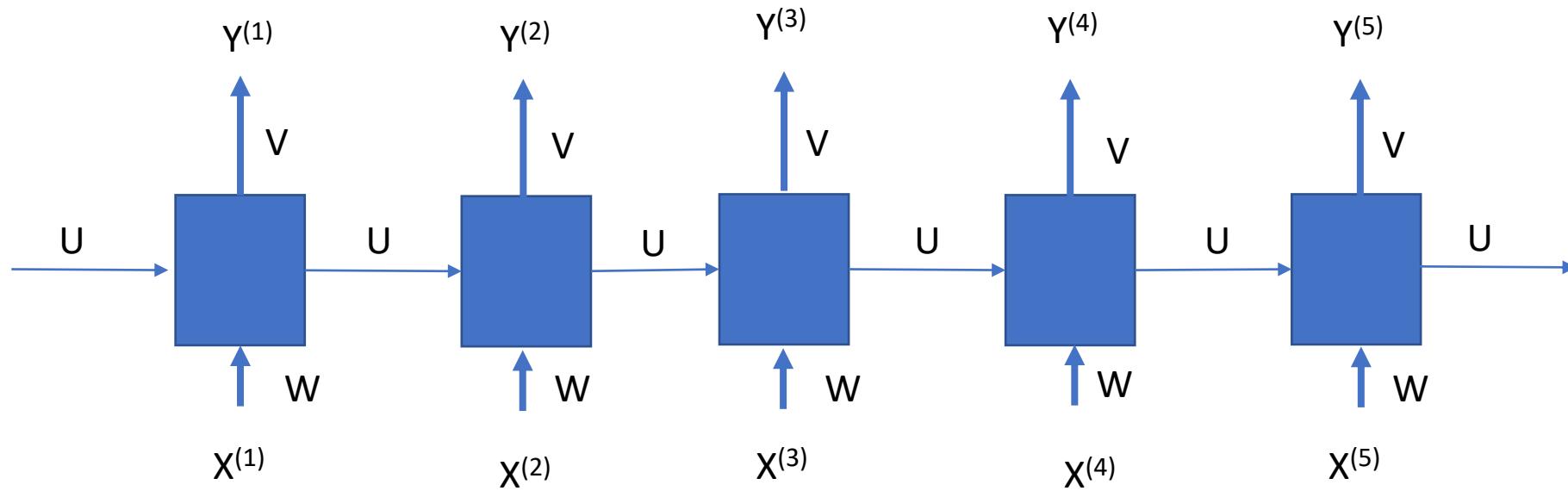
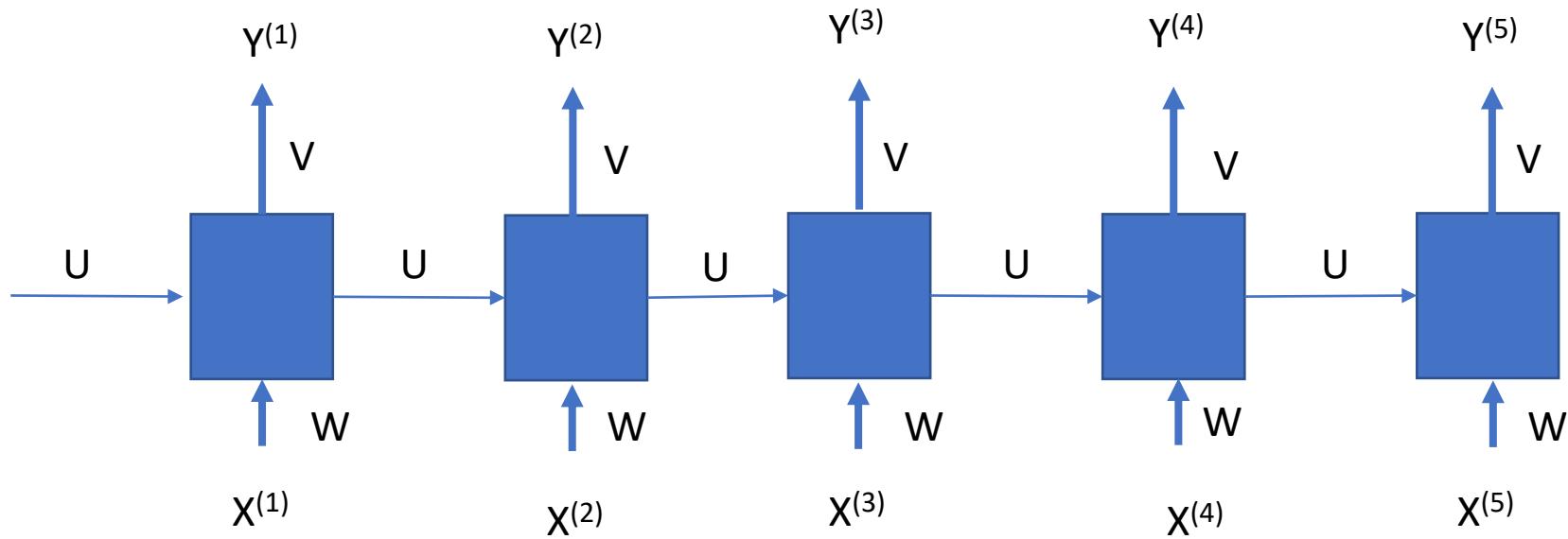


Backpropagation in RNN

Backpropagation through Time

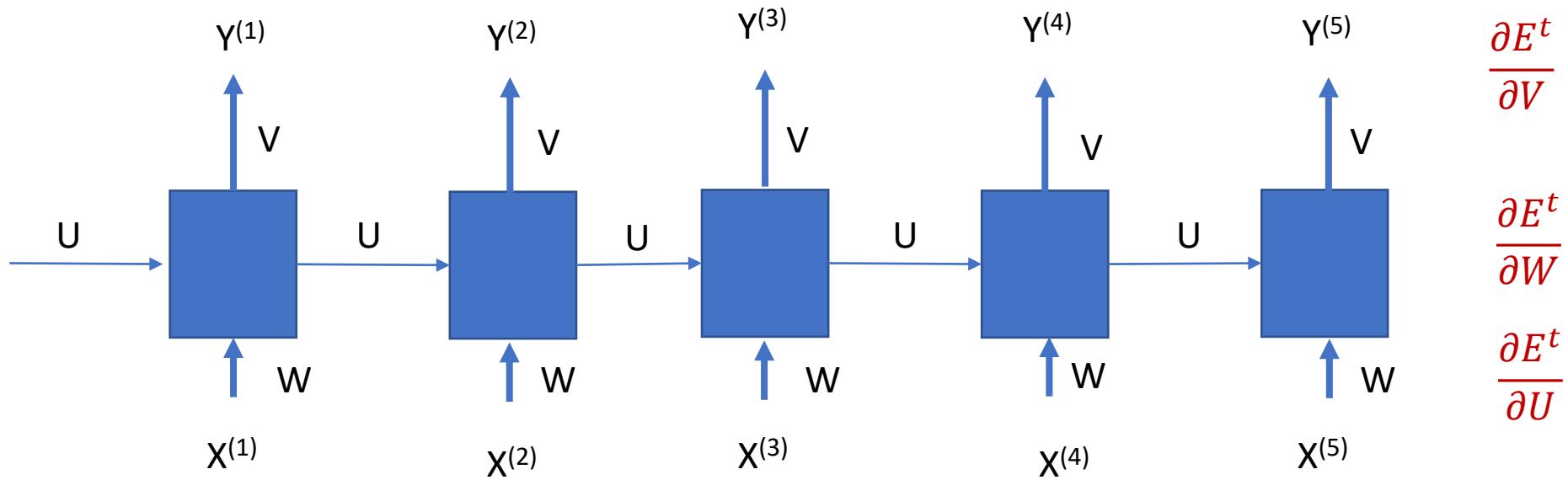


Backpropagation through Time



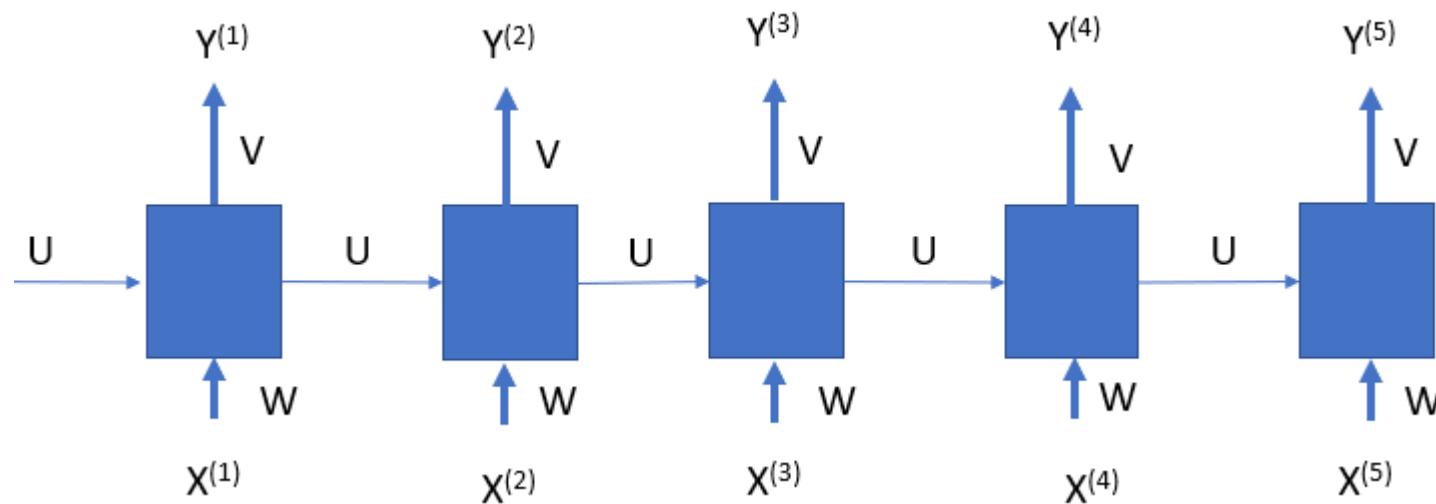
$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U) \quad \overline{y^t}^T = \text{softmax}(\overline{h^t}^T V) \quad E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

Backpropagation through Time



$$\bar{h}^T = \tanh(\bar{x}^T W + \bar{h}^{T-1} U) \quad \bar{y}^T = softmax(\bar{h}^T V) \quad E = \sum_t E^t = \sum_t -\bar{y}^t \log(y^t)$$

Estimating The Parameter V Matrix



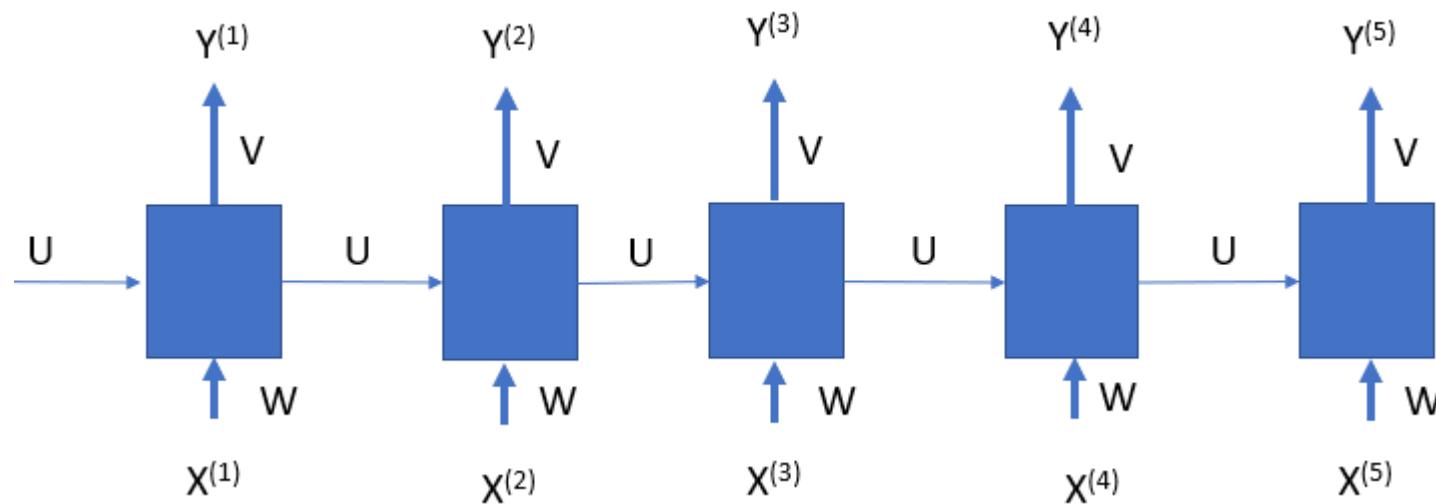
$$\bar{h}^T = \tanh(\bar{x}^T W + \bar{h}^{t-1} T U)$$

$$\bar{y}^T = \text{softmax}(\bar{h}^T V)$$

$$E = \sum_t E^t = \sum_t -\bar{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

Estimating The Parameter V Matrix



$$\bar{h}^T = \tanh(\bar{x}^T W + \bar{h}^{t-1} T U)$$

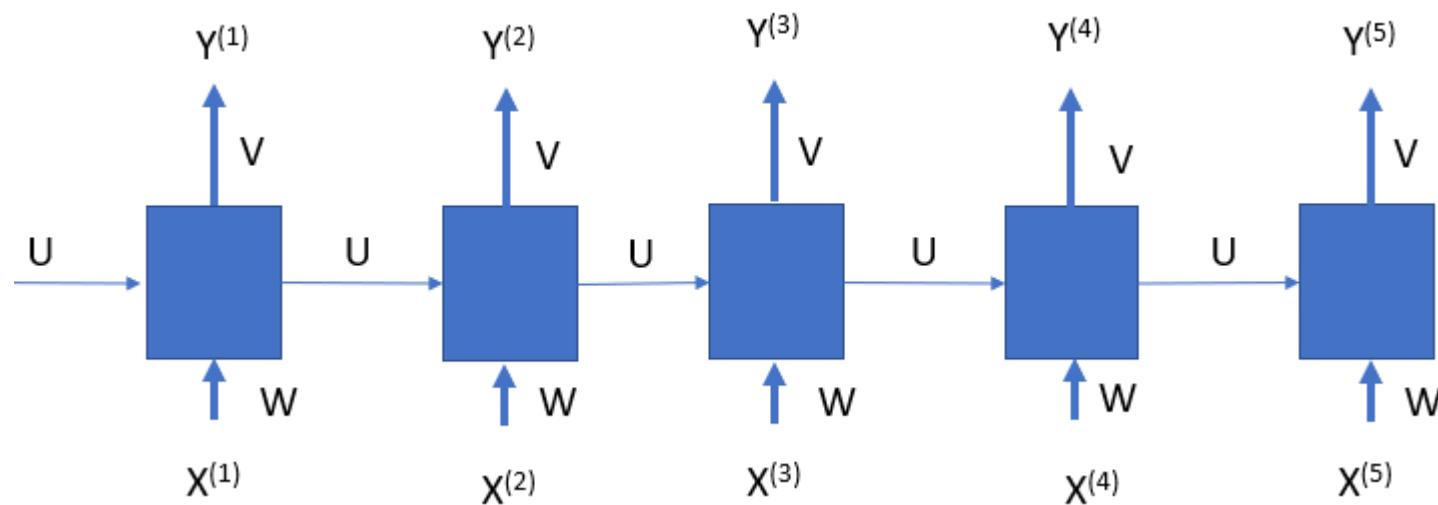
$$\bar{y}^T = \text{softmax}(\bar{h}^T V)$$

$$E = \sum_t E^t = \sum_t -\bar{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\bar{y}^3 = \text{softmax} \left(\sigma \left(\bar{x}^3 W + \sigma \left(\bar{x}^2 W + \sigma \left(\bar{x}^1 W + \bar{h}^0 U \right) U \right) V \right) \right)$$

Estimating The Parameter V Matrix



$$\bar{h}^T = \tanh(\bar{x}^T W + \bar{h}^{t-1} U)$$

$$\bar{y}^T = \text{softmax}(\bar{h}^T V)$$

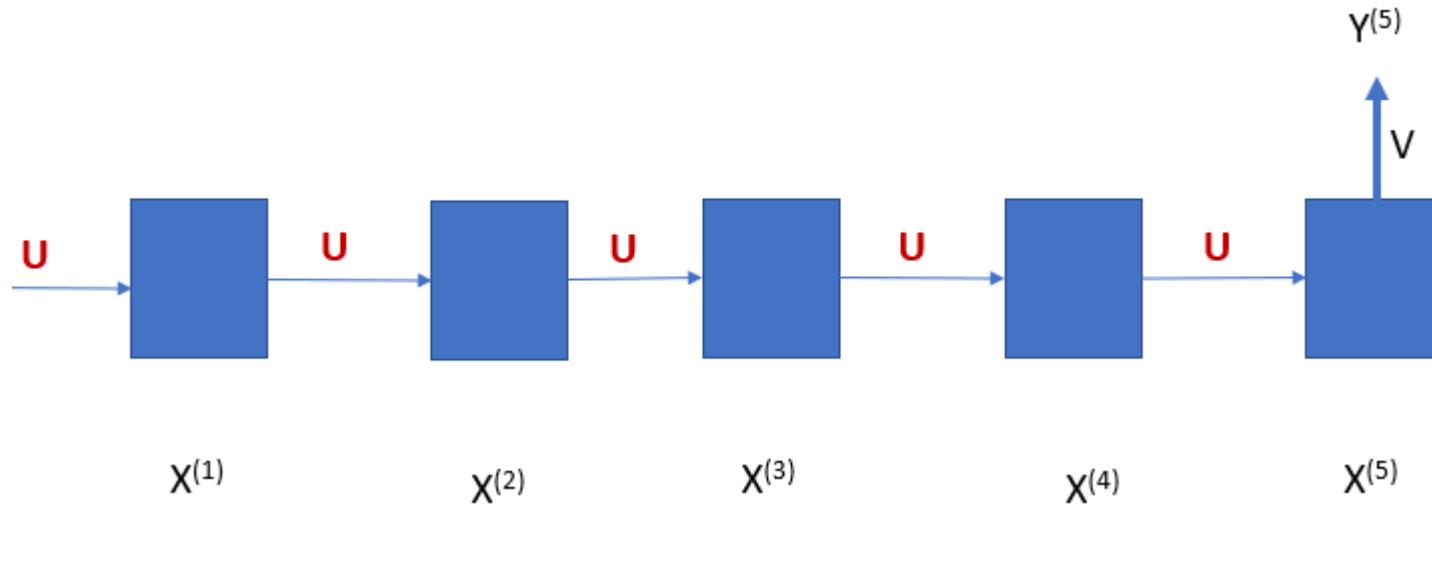
$$E = \sum_t E^t = \sum_t -\bar{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\bar{y}^3 = \text{softmax} \left(\sigma(\bar{x}^3 W + \sigma(\bar{x}^2 W + \sigma(\bar{x}^1 W + \bar{h}^0 U) U) V \right)$$

It depends only on current time step

Estimating The Parameter U Matrix



$$\overline{h}^T = \tanh(\overline{x}^T W + \overline{h}^{t-1} T U)$$

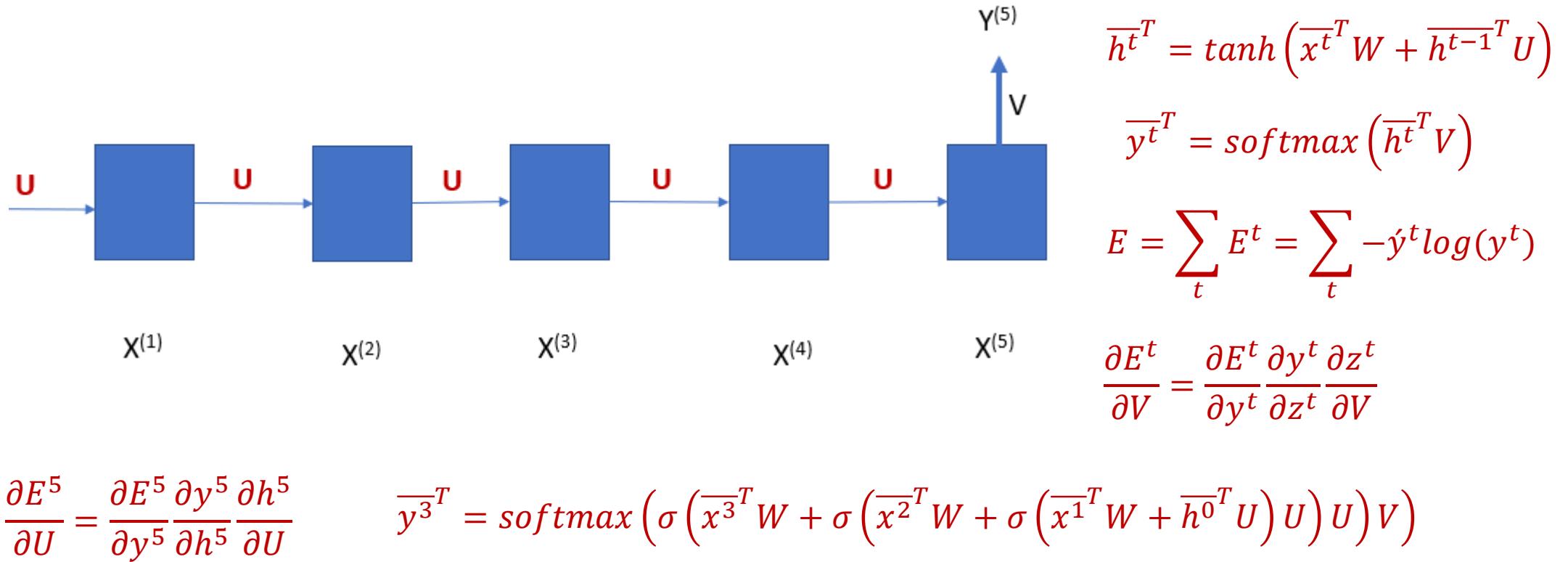
$$\overline{y}^T = \text{softmax}(\overline{h}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

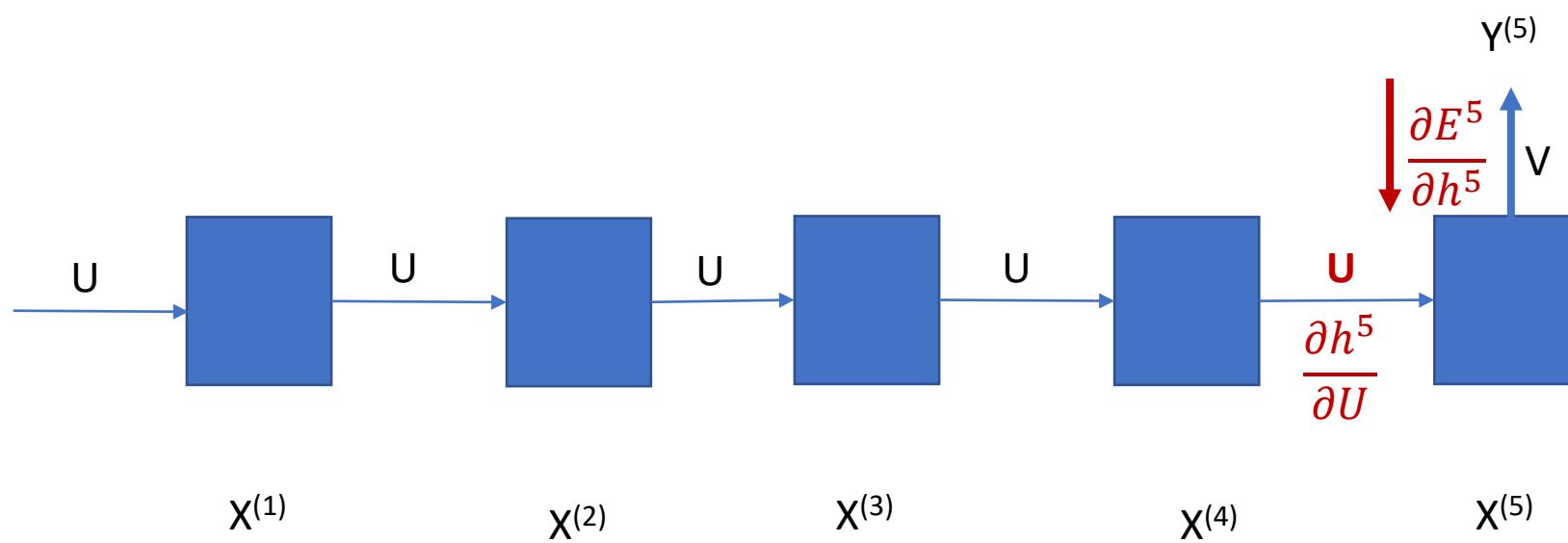
$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^5}{\partial U} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial U}$$

Estimating The Parameter U Matrix



Estimating The Parameter U Matrix



$$\frac{\partial E^5}{\partial U} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial U}$$

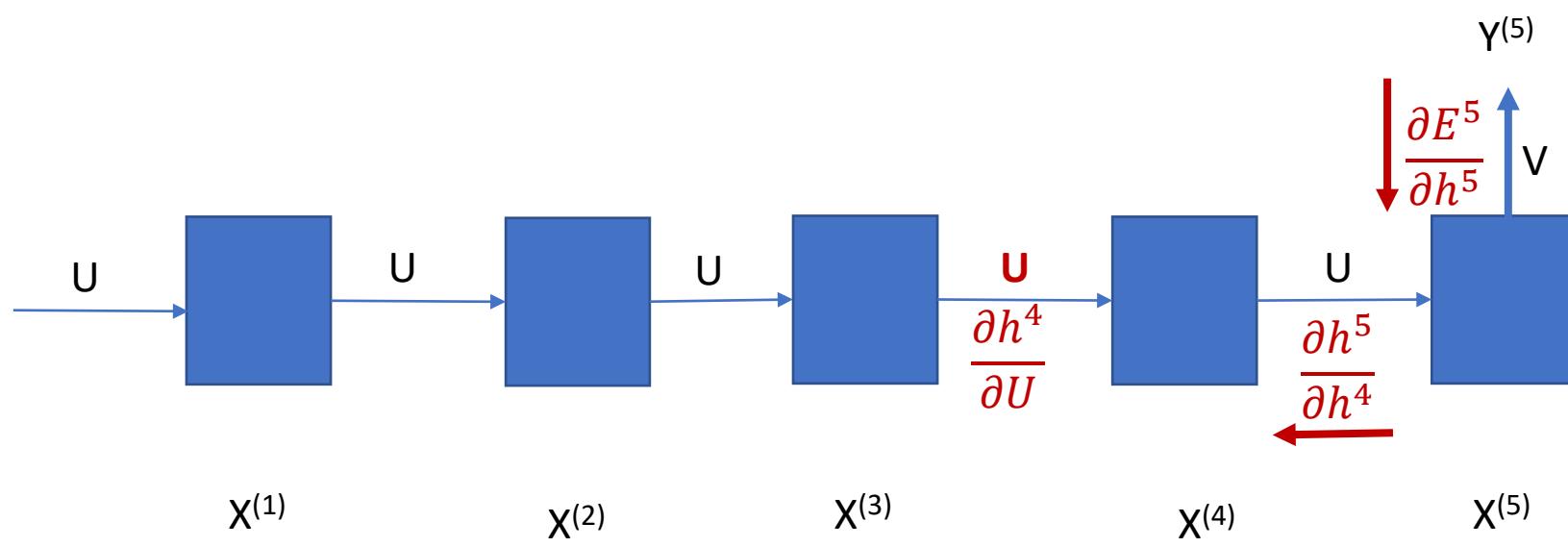
$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

Estimating The Parameter U Matrix



$$\bar{h}^T = \tanh(\bar{x}^T W + \bar{h}^{T-1} U)$$

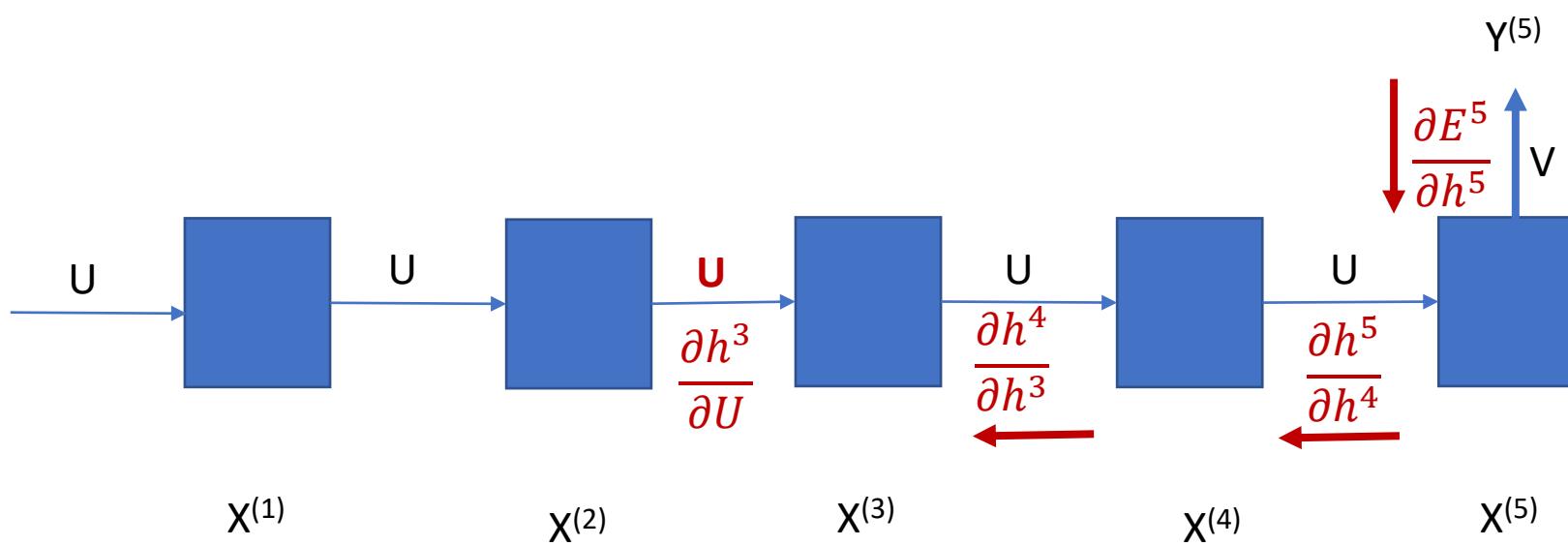
$$\bar{y}^T = \text{softmax}(\bar{h}^T V)$$

$$E = \sum_t E^t = \sum_t -\bar{y}^t \log(\bar{y}^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^5}{\partial U} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial U}$$

Estimating The Parameter U Matrix



$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

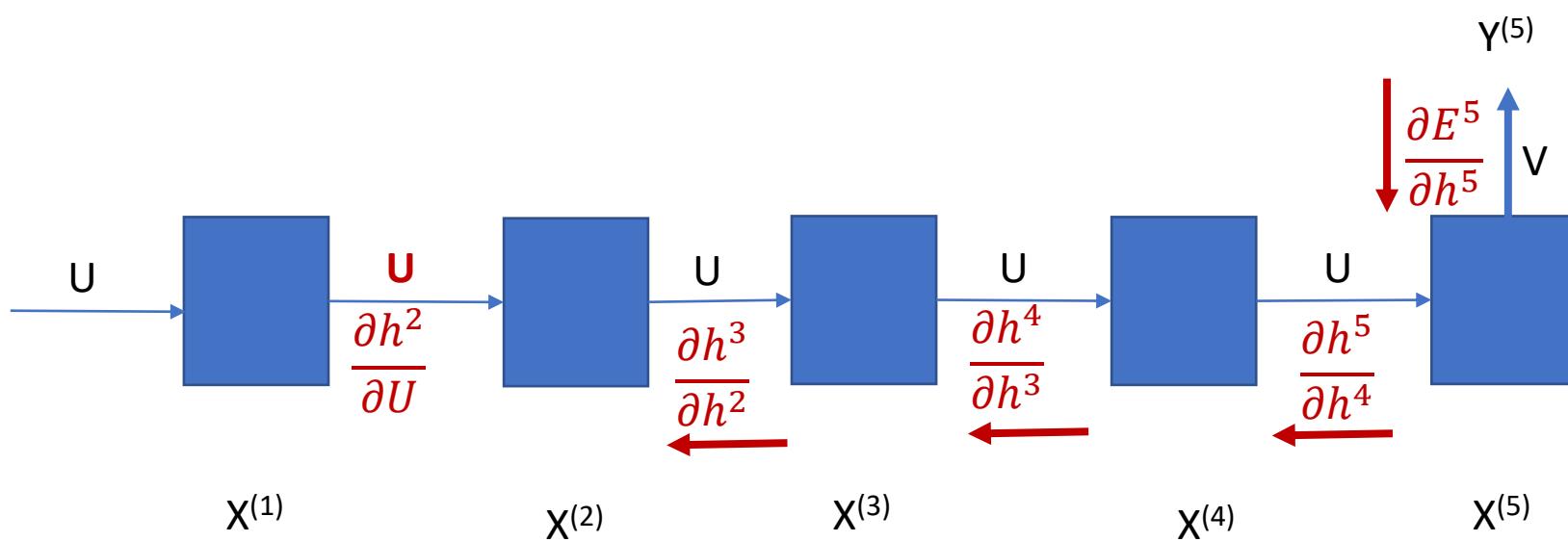
$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^5}{\partial U} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial U}$$

Estimating The Parameter U Matrix



$$\bar{h}^T = \tanh(\bar{x}^T W + \bar{h}^{t-1}^T U)$$

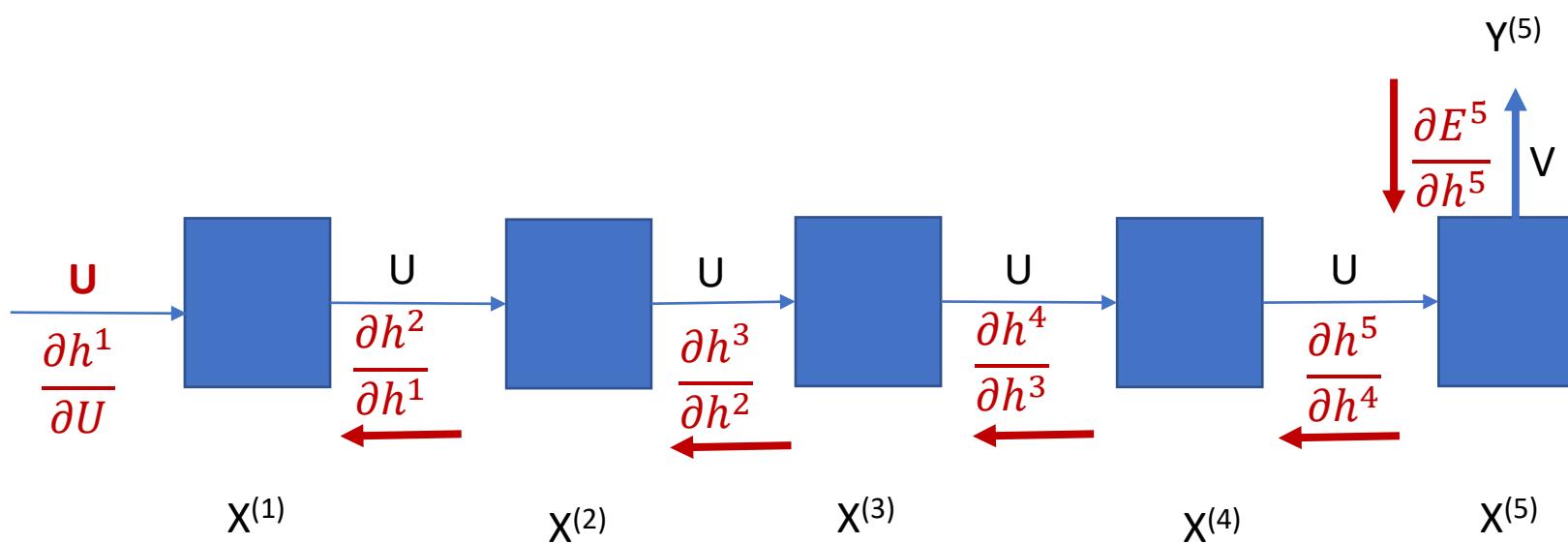
$$\bar{y}^T = \text{softmax}(\bar{h}^T V)$$

$$E = \sum_t E^t = \sum_t -\bar{y}^t \log(\bar{y}^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^5}{\partial U} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial U}$$

Estimating The Parameter U Matrix



$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

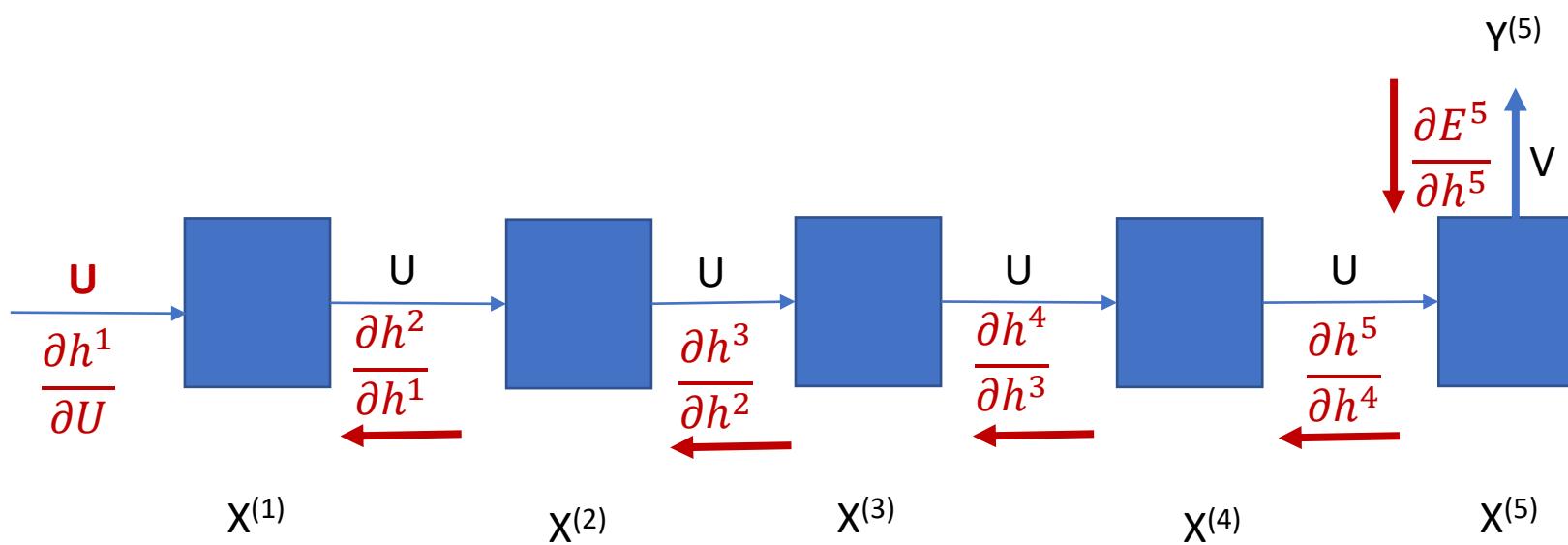
$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^5}{\partial U} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial h^1} \frac{\partial h^1}{\partial U}$$

Estimating The Parameter U Matrix



$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

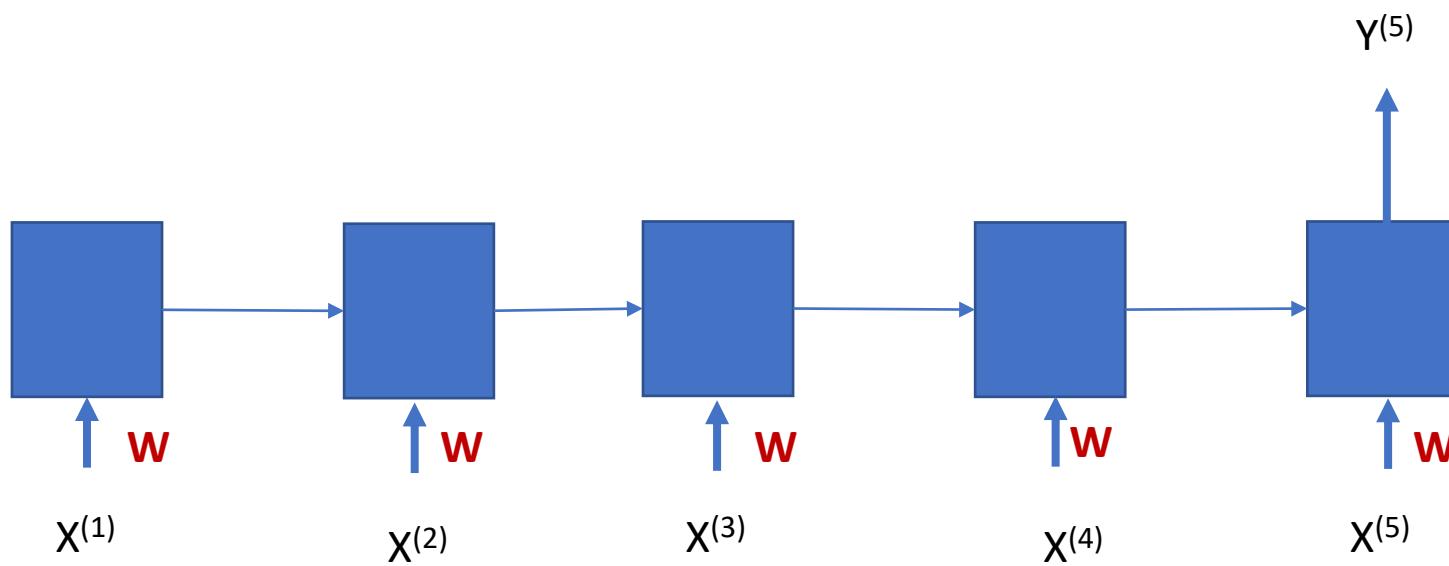
$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^5}{\partial U} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial U} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial h^1} \frac{\partial h^1}{\partial U}$$

$$= \frac{\partial E^t}{\partial y^t} \sum_{i=1}^t \frac{\partial y^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

Estimating The Parameter W Matrix



$$\frac{\partial E^t}{\partial W}$$

$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

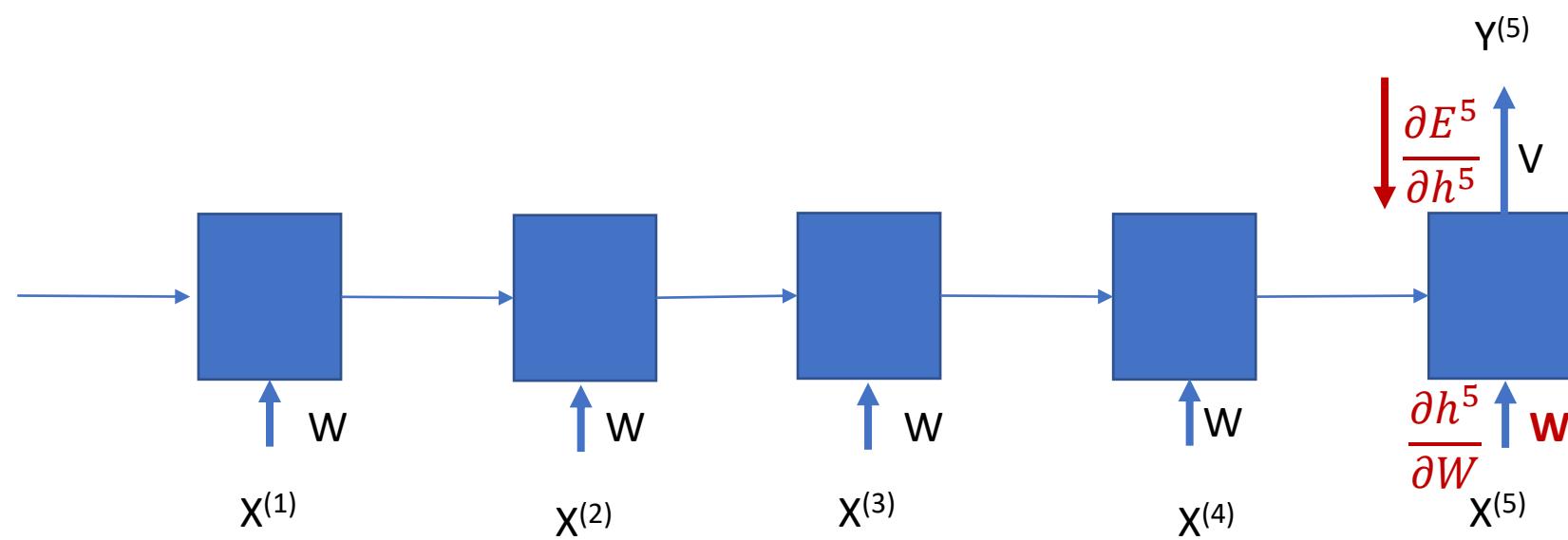
$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\dot{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^t}{\partial U} = \sum_{i=1}^t \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

Estimating The Parameter W Matrix



$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial W}$$

$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

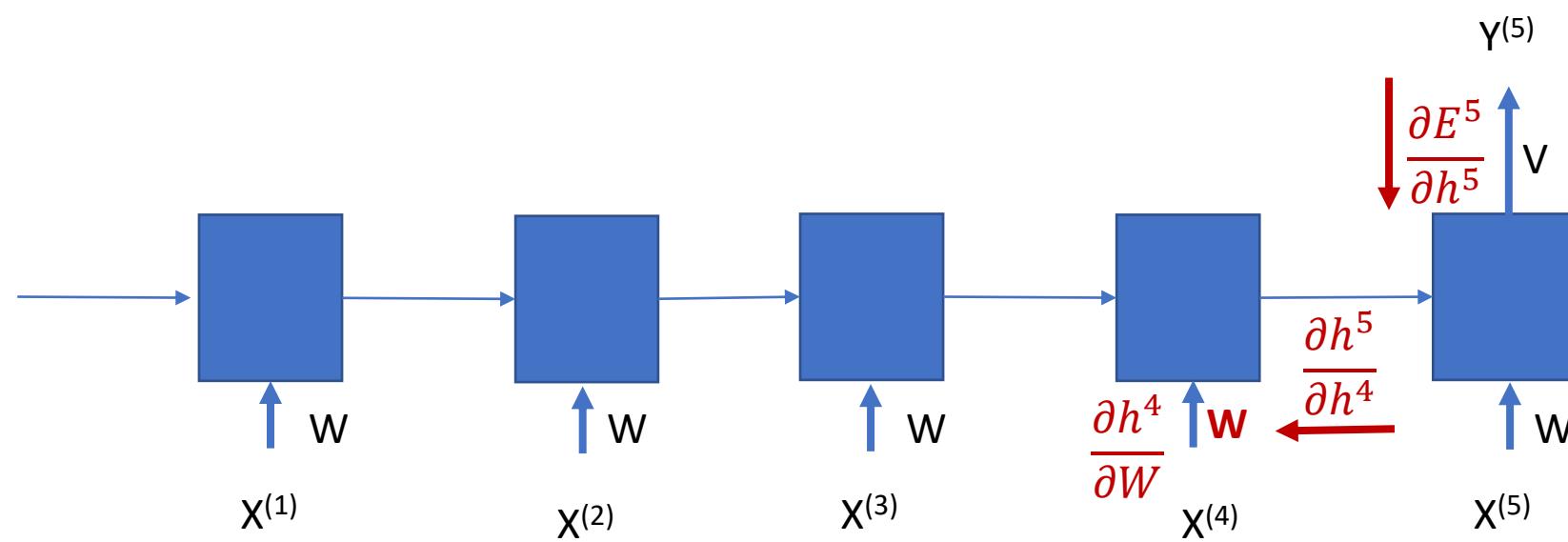
$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^t}{\partial U} = \sum_{i=1}^t \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

Estimating The Parameter W Matrix



$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial W}$$

$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

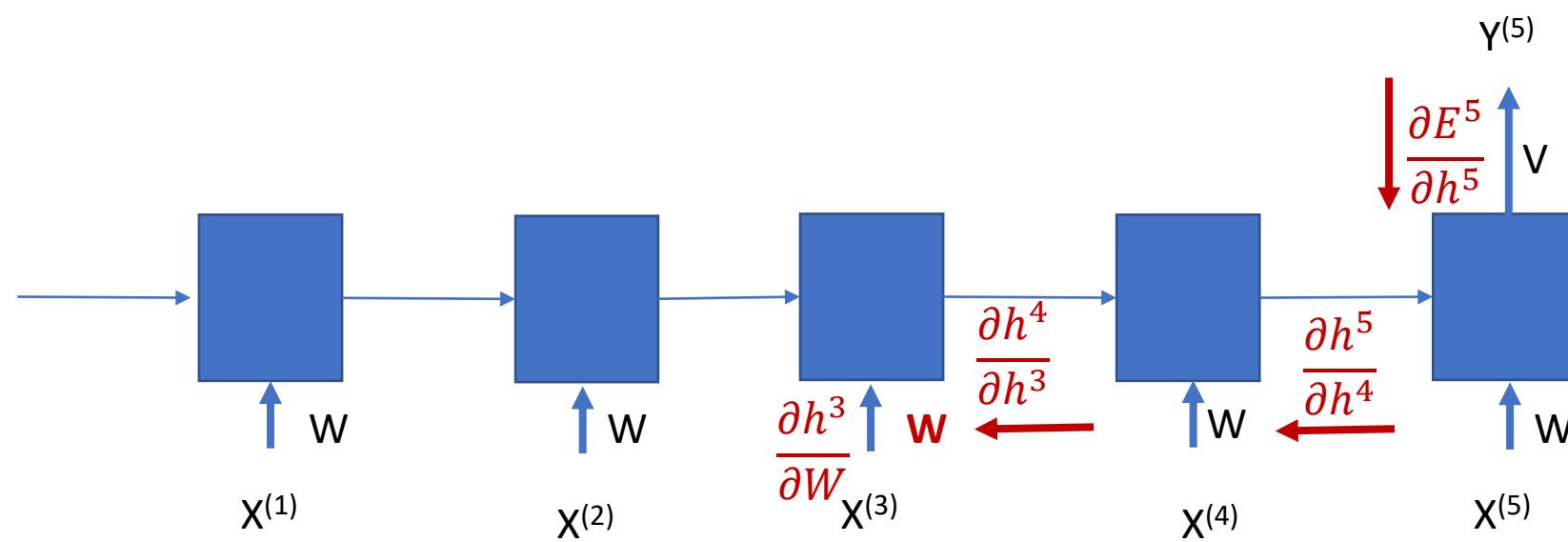
$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^t}{\partial U} = \sum_{i=1}^t \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

Estimating The Parameter W Matrix



$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial W}$$

$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

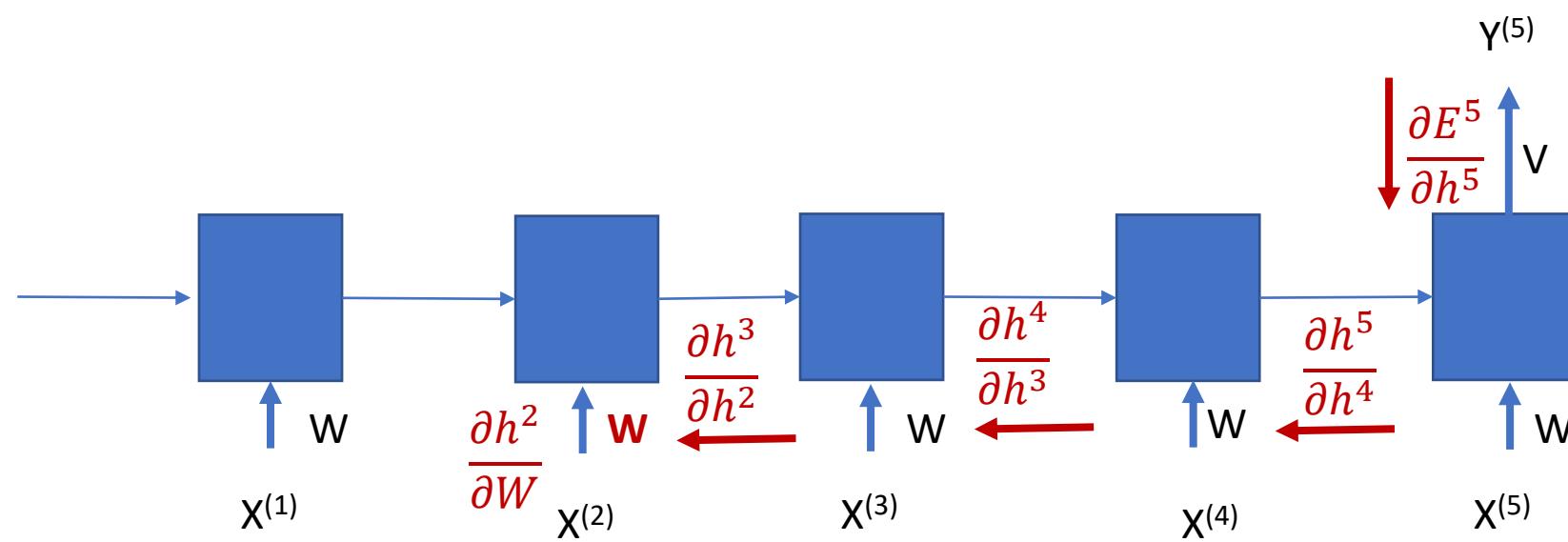
$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^t}{\partial U} = \sum_{i=1}^t \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

Estimating The Parameter W Matrix



$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial W}$$

$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

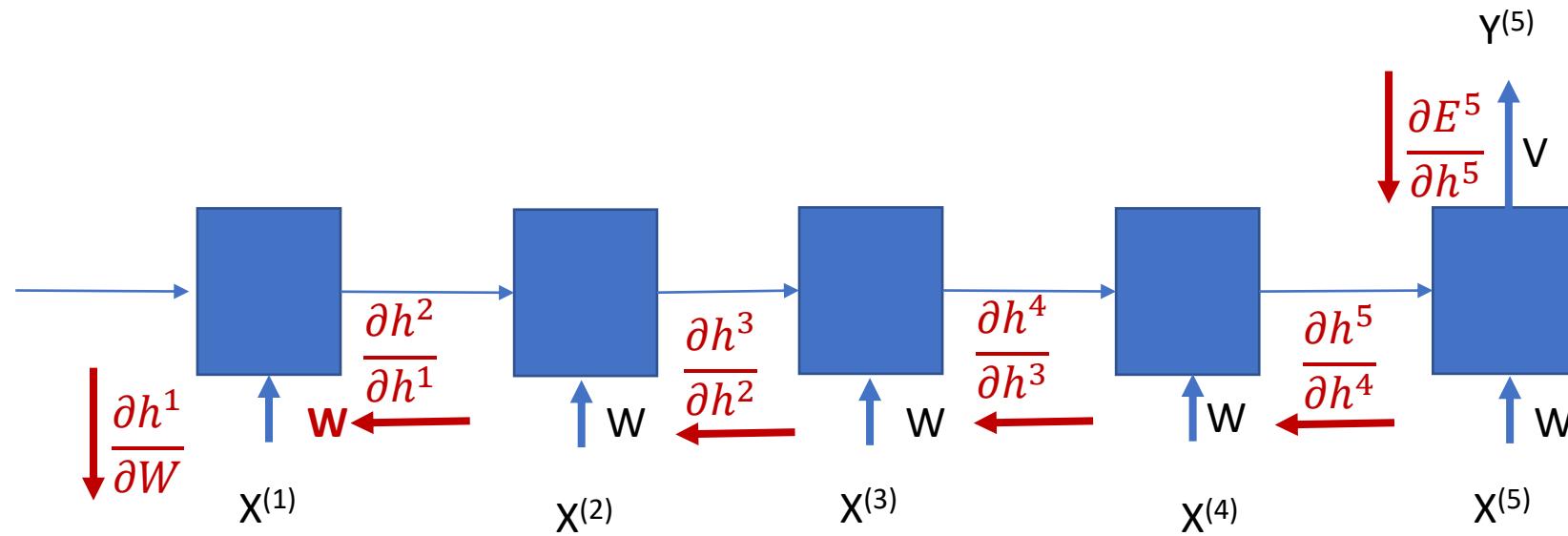
$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^t}{\partial U} = \sum_{i=1}^t \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

Estimating The Parameter W Matrix



$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial h^1} \frac{\partial h^1}{\partial W}$$

$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

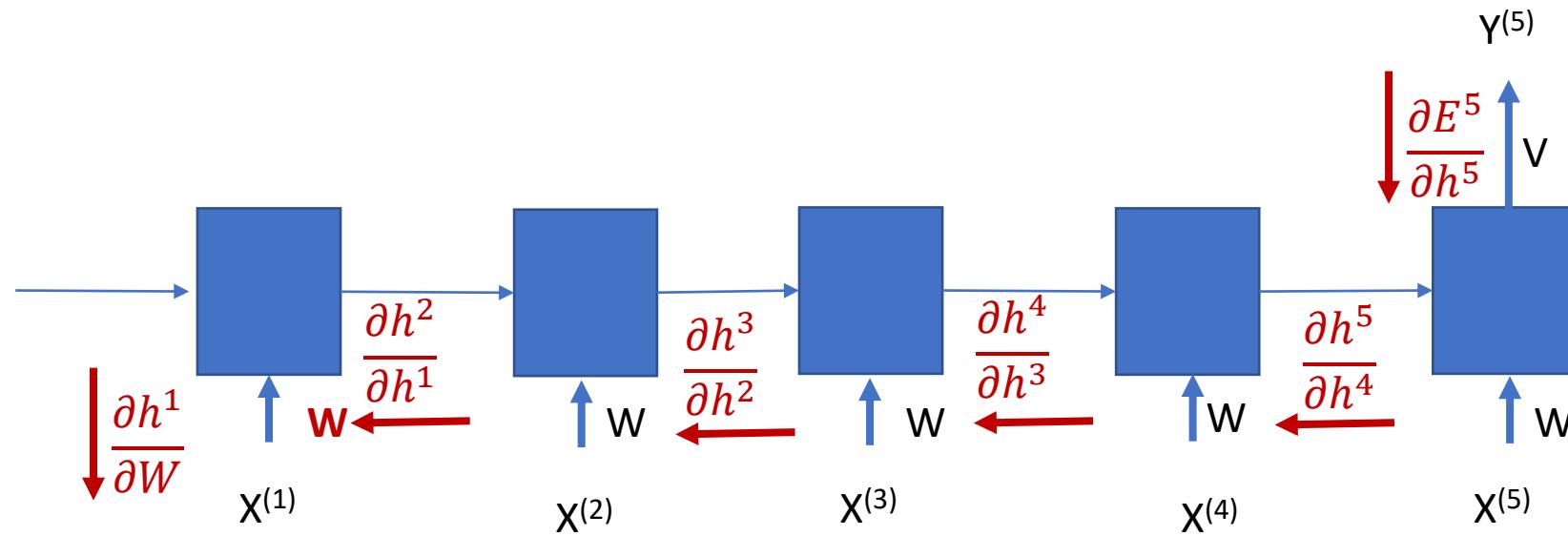
$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^t}{\partial U} = \sum_{i=1}^t \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

Estimating The Parameter W Matrix



$$\overline{h^t}^T = \tanh(\overline{x^t}^T W + \overline{h^{t-1}}^T U)$$

$$\overline{y^t}^T = \text{softmax}(\overline{h^t}^T V)$$

$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

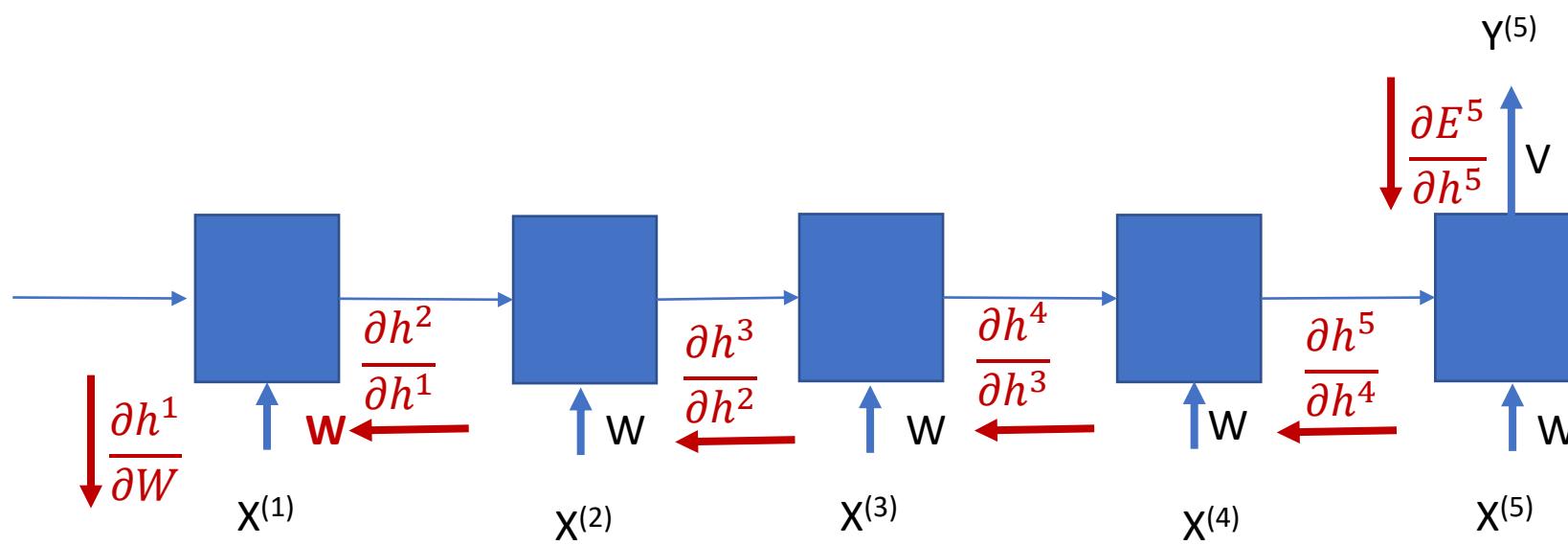
$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^t}{\partial U} = \sum_{i=1}^t \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

$$\frac{\partial E^5}{\partial W} = \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial h^1} \frac{\partial h^1}{\partial W}$$

$$= \frac{\partial E^t}{\partial y^t} \sum_{i=1}^t \frac{\partial y^t}{\partial h^i} \frac{\partial h^i}{\partial W}$$

Vanishing/exploding Gradient



$$\begin{aligned} \frac{\partial E^5}{\partial W} &= \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \frac{\partial h^5}{\partial h^4} \frac{\partial h^4}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \boxed{\frac{\partial h^5}{\partial h^4}} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \boxed{\frac{\partial h^5}{\partial h^4}} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial W} + \frac{\partial E^5}{\partial y^5} \frac{\partial y^5}{\partial h^5} \boxed{\frac{\partial h^5}{\partial h^4}} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial h^1} \frac{\partial h^1}{\partial W} \\ &= \frac{\partial E^t}{\partial y^t} \sum_{i=1}^t \frac{\partial y^t}{\partial h^i} \frac{\partial h^i}{\partial W} \end{aligned}$$

$$\overline{h^t}^T = \tanh \left(\overline{x^t}^T W + \overline{h^{t-1}}^T U \right)$$

$$\overline{y^t}^T = \text{softmax} \left(\overline{h^t}^T V \right)$$

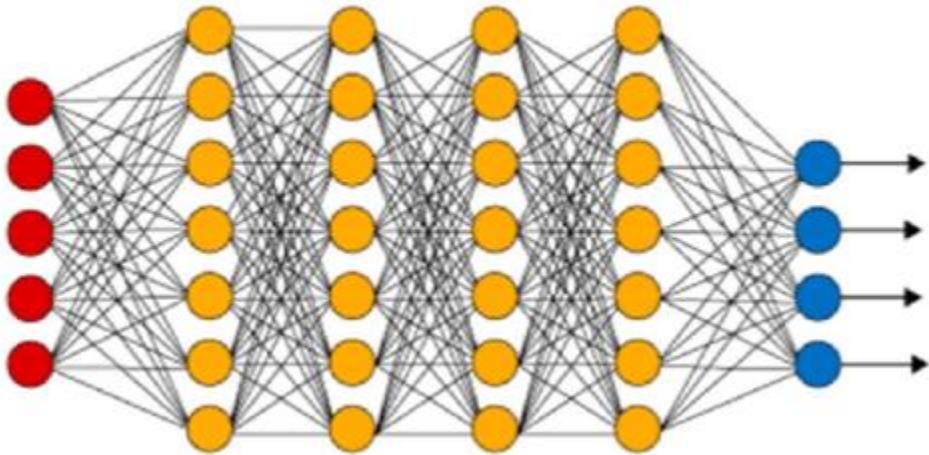
$$E = \sum_t E^t = \sum_t -\hat{y}^t \log(y^t)$$

$$\frac{\partial E^t}{\partial V} = \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial z^t} \frac{\partial z^t}{\partial V}$$

$$\frac{\partial E^t}{\partial U} = \sum_{i=1}^t \frac{\partial E^t}{\partial y^t} \frac{\partial y^t}{\partial h^t} \frac{\partial h^t}{\partial h^i} \frac{\partial h^i}{\partial U}$$

$$\boxed{\frac{\partial h^5}{\partial h^4}} \frac{\partial h^4}{\partial h^3} \frac{\partial h^3}{\partial h^2} \frac{\partial h^2}{\partial h^1} \frac{\partial h^1}{\partial W}$$

Vanishing/exploding gradient problem may happen in all deep networks



Vanishing/exploding gradient problem may happen in all deep networks

