Neutrino Mass Models and Their Experimental Signatures

K.S. Babu

Oklahoma State University

GIAN Course on Electrowak Symmetry Breaking, Flavor Physics and BSM

IIT Guwahati, Guwahati, Assam, India

December 18 -- 22, 2017

Neutrino Flavor Oscillations

Neutrino flavor eigenstates are admixtures of mass eigenstates

$$\begin{aligned} \nu_e &= \cos\theta\ \nu_1 + \sin\theta\ \nu_2 \\ \nu_\mu &= -\sin\theta\ \nu_1 + \cos\theta\ \nu_2 \end{aligned}$$

$$P(\nu_e \to \nu_\mu;\ L) = \sin^2 2\theta\ \ \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$= \sin^2 2\theta\ \ \sin^2 \left\{1.27 \left(\frac{\Delta m^2}{\mathrm{eV}^2}\right) \left(\frac{L}{\mathrm{km}}\right) / \left(\frac{E}{\mathrm{GeV}}\right)\right\}$$

$$\Delta m^2 = m_2^2 - m_1^2$$

For $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin\theta \sim 1$, $L \sim 10^3$ km, $E \sim$ GeV,

oscillation probability is of order one

A global fit to neutrino oscillation data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 0.83$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.271 \to 0.345$
$\theta_{12}/^\circ$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$31.38 \to 35.99$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.385 \to 0.635$	$0.587^{+0.020}_{-0.024}$	$0.393 \to 0.640$	$0.385 \to 0.638$
$\theta_{23}/^\circ$	$41.6^{+1.5}_{-1.2}$	$38.4 \to 52.8$	$50.0^{+1.1}_{-1.4}$	$38.8 \to 53.1$	$38.4 \to 53.0$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.01934 \to 0.02392$	$0.02179^{+0.00076}_{-0.00076}$	$0.01953 \to 0.02408$	$0.01934 \to 0.02397$
$\theta_{13}/^\circ$	$8.46^{+0.15}_{-0.15}$	$7.99 \to 8.90$	$8.49^{+0.15}_{-0.15}$	$8.03 \to 8.93$	$7.99 \to 8.91$
$\delta_{\rm CP}/^\circ$	261^{+51}_{-59}	$0 \to 360$	277^{+40}_{-46}	$145 \to 391$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.03 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \rm eV^2}$	$+2.524^{+0.039}_{-0.040}$	$+2.407 \to +2.643$	$-2.514^{+0.038}_{-0.041}$	$-2.635 \to -2.399$	$\begin{bmatrix} +2.407 \to +2.643 \\ -2.629 \to -2.405 \end{bmatrix}$

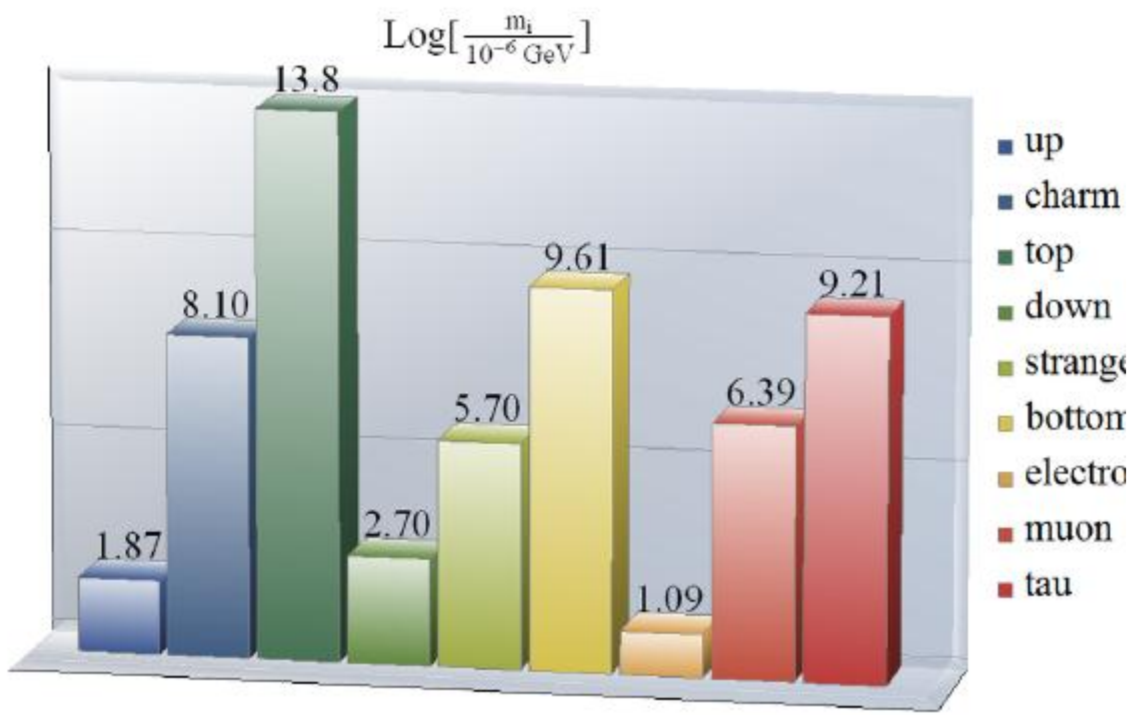
Estaban, Gonzalez-Garcia, Maltoni, Martinze-Soler, Schwetz (2017)

Global data has a slight preference for $\delta_{CP} \sim 3\pi/2$, with $\delta_{CP} = 0$ excluded at 90% CL by recent T2K results

Neutrinos and the Fermion Mass Puzzle

Charged Fermion Mass Hierarchy

- **up-type quarks**
- $m_u \sim 6.5 \times 10^{-6}$
- $m_c \sim 3.3 \times 10^{-3}$
- $m_t \sim 1$
- **down-type quarks**
- $m_d \sim 1.5 \times 10^{-5}$
- $m_s \sim 3 \times 10^{-4}$
- $m_b \sim 1.5 \times 10^{-2}$
- **charged leptons**
- $m_e \sim 3 \times 10^{-6}$
- $m_\mu \sim 6 \times 10^{-4}$
- $m_\tau \sim 1 \times 10^{-2}$



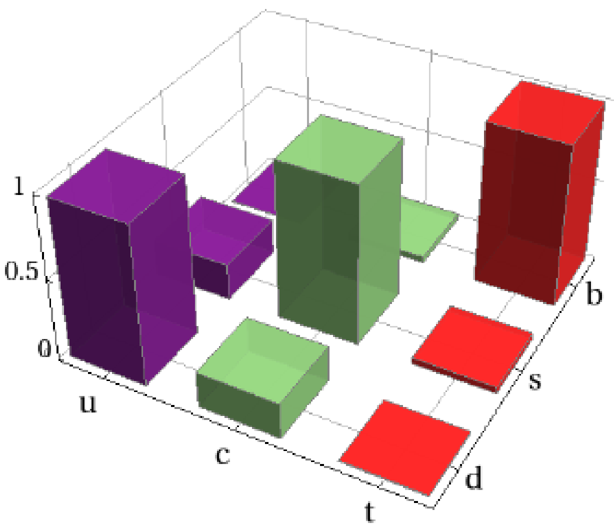
Neutrino masses not strongly hierarchical

3 masses within an order of magnitude consistent!

Quark and Lepton Mixing Parameters

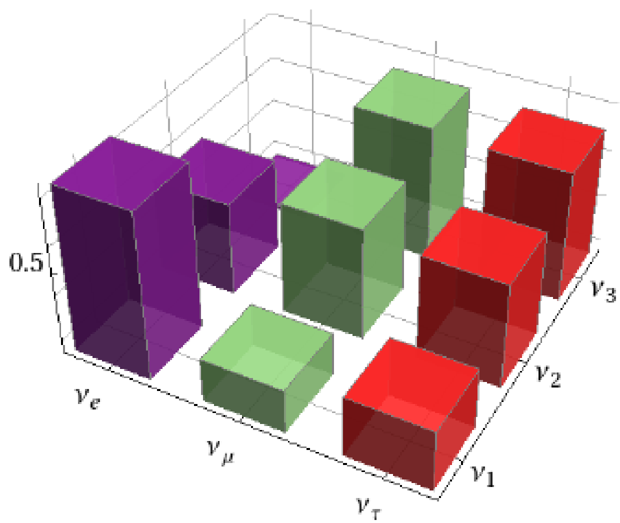
- **Quark Mixings**

$$V_{CKM} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix}$$



- **Leptonic Mixings**

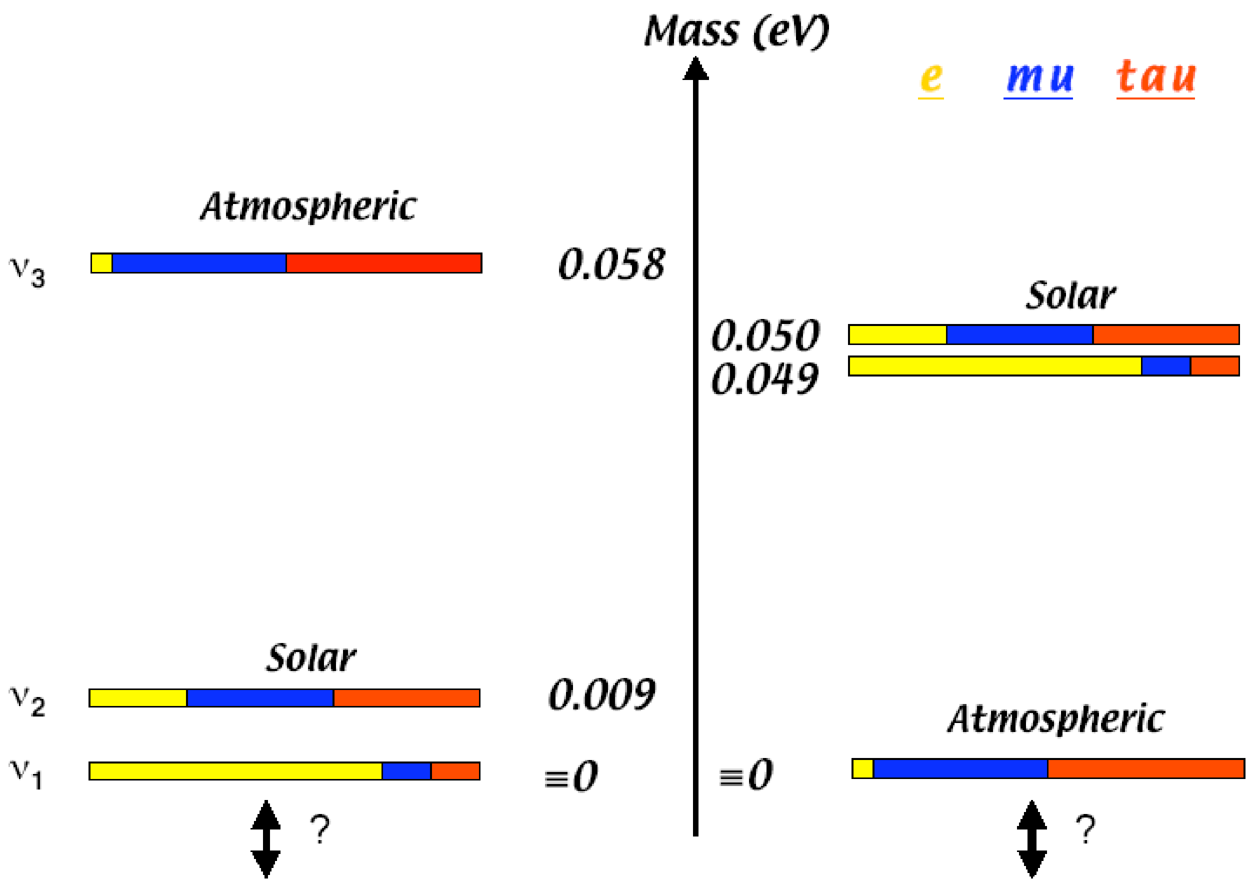
$$U_{PMNS} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix}$$



Pressing Questions for Neutrinos

- Are neutrinos their own antiparticles?
- Is there CP violation in neutrino oscillations?
- Is the mass hierarchy normal or inverted?
- Are there light sterile neutrinos?
- What is the scale of neutrino mass generation?
- What explains the pattern of neutrino mixings?
- Can neutrinos be unified with quarks?
- Is neutrino CP violation related to baryon asymmetry?

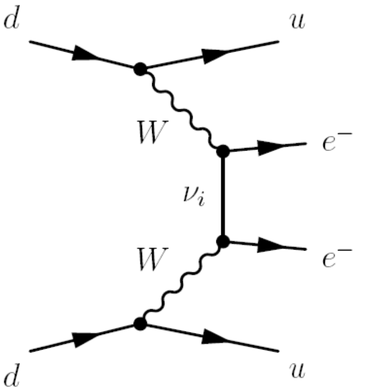
Neutrino Mass Ordering



Future long-baseline experiments can test the ordering

INO, JUNO, DUNE, HyperK,...

Neutrinoless Double Beta Decay

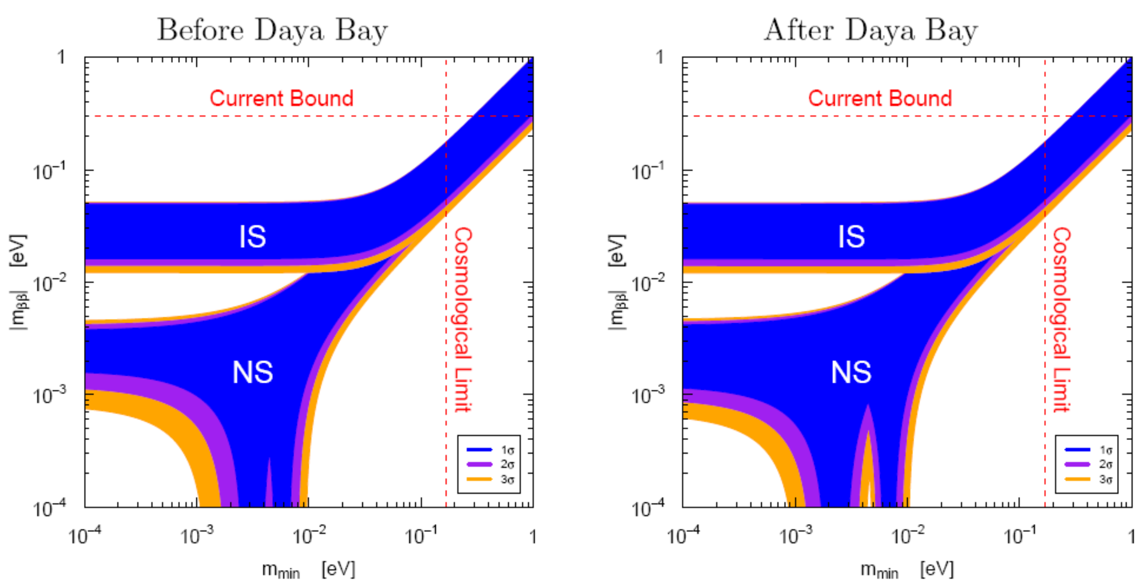


$$N(A,Z) \to N(A,Z+2) + e^- + e^-$$

(Assumes neutrino mass is the only contribution)

$$m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$$

$$|m_{\beta\beta}| = \left|\cos^2\theta_{12}\cos^2\theta_{13} m_1 + e^{2i\alpha_{12}}\sin^2\theta_{12}\cos^2\theta_{13} m_2 + e^{2i\alpha_{13}}\sin^2\theta_{13} m_3\right|$$



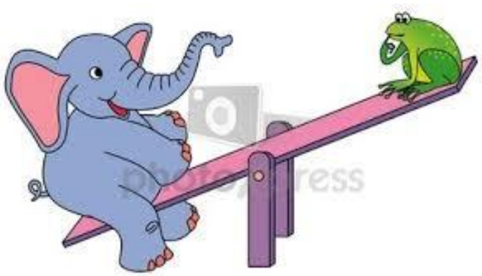
Bilenky, Giunti, 2012

Seesaw Mechanism for Neutrino Masses

(ν, ν^c) mass matrix: $\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$

With $M_R \gg m_D$, the mass eigenvalues are:

$$m_\nu \simeq -\frac{m_D^2}{M_R}, \qquad M_{\text{heavy}} \simeq M_R$$



Minkowski (1977)
Gell-Mann, Ramond, Slansky (1979)
Yanagida (1979)
Mohapatra, Senjanovic (1979)

$$m_D \approx 10^2 \text{ GeV}, M_R \approx 10^{14} \text{ GeV} \Rightarrow m_\nu \approx 0.05 \text{ eV}$$

$$m_D \approx \text{MeV}, M_R \approx \text{TeV} \Rightarrow m_\nu \approx \text{eV}$$

ν^c with this mass can also produce baryon asymmetry of universe

Variety of Seesaw Mechanisms

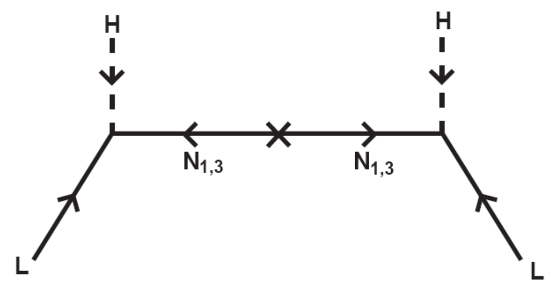
Effective neutrino mass operator:

$$\mathcal{L}_{\text{eff}} = \frac{LLHH}{M} \;\Rightarrow\; m_\nu \sim \frac{v^2}{M}$$

Neutrino oscillations can probe $M \sim 10^{14}$ GeV

Type (I,III) seesaw

Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, & Slansky (1980)
Mohapatra & Senjanovic (1980)

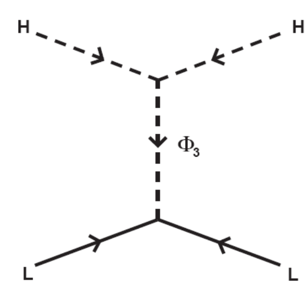


$N_1 : (1,1,0)$, $N_3 : (1,3,0)$

Foot, Lew, He, & Joshi (1989)

Type II seesaw

Mohapatra & Senjanovic (1980)
Schechter & Valle (1980)
Lazarides, Shafi, & Wetterich (1981)



$\Phi_3 : (1,3,+1)$

Variety of Seesaw Mechanisms (cont.)

Inverse seesaw:

In addition to the right-handed neutrino ν^c, a second singlet Weyl fermion N is added, one per generation

Neutrino mass matrix spanning $(\nu,\, \nu^c,\, N)$ fields:

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_R \\ 0 & M_R^T & \mu \end{pmatrix}$$

Mohapatra (1986)
Mohapatra, Valle (1986)

$$M_\nu = m_D\, M_R^{-1}\, \mu\, (M_R^T)^{-1}\, m_D^T$$

If $\mu \sim M_R \gg m_D$, then this reduces to ususal (double) seesaw

$\mu \ll m_D, M_R$ protected by lepton number

In this case low scale seesaw natural, and testable at colliders

Dev, Mohapatra (2010); Khalil (2010); A. Das, Okada (2013); Dev, Pilaftsis (2013); Arganda, Herrero, Marcano, Weiland (2015); Soumya C, R. Mohanta (2017); A. Mukherjee, D. Borah, M.K. Das (2017); B. Karmakar, A. Sil (2017); Sinha, Samanta, Ghosal (2016); Humbert, Lindner, J. Smirnov (2016),….

Testing Type-I Seesaw Experimentally

The new particle in type-I seesaw is the SM singlet neutrino N

N has no direct interaction with the gauge bosons

Production of N must go through mixing or via Higgs coupling

Mixing of N with ν constrained by:

- Neutrino mass
- Lepton number violation
- Charged lepton flavor violation
- Lepton universality
- Direct searches for N

Neutrino mass and neutrinoless double beta decay most severe

Simple way to avoid neutrino mass and $\beta\beta_{0\nu}$ constraints exists

Type-I Seesaw Tests

Consider the following forms for m_D and M_N in seesaw:

$$m_D = m_D^0\, V \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}; \; M_R = \begin{pmatrix} M_{11} & 0 & 0 \\ 0 & 0 & M_{23} \\ 0 & M_{23} & 0 \end{pmatrix}$$

Here V is an arbitrary unitary matrix

With this structure, contribution to light neutrino masses is zero!

$$M_\nu^\ell = -m_D M_R^{-1} m_D^T = 0$$

There is an unbroken lepton number symmetry, under which (N_1, N_2, N_3) have charges $(0, 1, -1)$

$\nu_{e,\mu,\tau}$ have charge $+1$

$\nu_{e,\mu,\tau}$ mix with N_2; this mixing can be relatively large

Babu, T. Ghosh (2017)

Type-I Seesaw Tests (cont.)

Define three mixing angles for $\nu_i - N_2$ mixing – with $N_2 \to \nu_4$:

$$U_{e4} = \left(\frac{m_D^0}{M_{23}}\right) V_{13}, \quad U_{\mu 4} = \left(\frac{m_D^0}{M_{23}}\right) V_{23}, \quad U_{\tau 4} = \left(\frac{m_D^0}{M_{23}}\right) V_{33}$$

Charged current interactions become:

$$\begin{aligned}
\mathcal{L}_{c.c.} &= \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}}_L \gamma_\mu \begin{pmatrix} 1-\frac{1}{2}|U_{e4}|^2 & -\frac{1}{2}(U_{e4}^* U_{\mu 4}) & -\frac{1}{2}(U_{e4}^* U_{\tau 4}) \\ -\frac{1}{2}(U_{e4} U_{\mu 4}^*) & 1-\frac{1}{2}|U_{\mu 4}|^2 & -\frac{1}{2}(U_{\mu 4}^* U_{\tau 4}) \\ -\frac{1}{2}(U_{e4} U_{\tau 4}^*) & -\frac{1}{2}(U_{\mu 4} U_{\tau 4}^*) & 1-\frac{1}{2}|U_{\tau 4}|^2 \end{pmatrix} \nu_\ell \, W^{\mu -} \\
&+ \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}}_L \gamma_\mu \begin{pmatrix} U_{e4} \\ U_{\mu 4} \\ U_{\tau 4} \end{pmatrix} \nu_4 \, W^{\mu -} + h.c.
\end{aligned}$$

Neutral current interactions:

$$\begin{aligned}
\mathcal{L}_{n.c.} &= \frac{g}{2c_W} \overline{\nu_\ell} \, \gamma_\mu \begin{pmatrix} 1-|U_{e4}|^2 & U_{e4}^* U_{\mu 4} & U_{e4}^* U_{\tau 4} \\ U_{e4} U_{\mu 4}^* & 1-|U_{\mu 4}|^2 & U_{\mu 4}^* U_{\tau 4} \\ U_{e4} U_{\tau 4}^* & U_{\mu 4} U_{\tau 4}^* & 1-|U_{\tau 4}|^2 \end{pmatrix} \nu_\ell \, Z^\mu + \frac{g}{2c_W} \overline{\nu_\ell} \, \gamma_\mu \begin{pmatrix} U_{e4}^* \\ U_{\mu 4}^* \\ U_{\tau 4}^* \end{pmatrix} \nu_4 \, Z^\mu \\
&+ \frac{g}{2c_W} \overline{\nu_4} \, \gamma_\mu \begin{pmatrix} U_{e4} & U_{\mu 4} & U_{\tau 4} \end{pmatrix} \nu_\ell \, Z^\mu + \frac{g}{2c_W} (|U_{e4}|^2 + |U_{\mu 4}|^2 + |U_{\tau 4}|^2) \, \overline{\nu_4} \, \gamma_\mu \, \nu_4 \, Z^\mu \; .
\end{aligned}$$

Type-I Seesaw Tests (cont.)

If only one among $(U_{e4}, U_{\mu 4}, U_{\tau 4})$ is nozero, lepton universailty violation gives the best limit

$$|U_{e4}|^2 < 0.8 \times 10^{-2},\ |U_{\mu 4}|^2 \leq 2 \times 10^{-3} \text{ and } |U_{\tau 4}|^2 \leq 0.9 \times 10^{-2}$$

If two of the mixing parameters are nozero, lepton flavor violation constraints are significant

De Gouvea, Kobach (2016)
Atre, Han, Pascoli (2009)

$$(U_{\tau 4}, U_{e4}):\ |U_{\tau 4}|^2 \leq 4 \times 10^{-3} \text{ and } |U_{e4}|^2 \leq 4 \times 10^{-3}.$$

$$(U_{\tau 4}, U_{\mu 4}):\ |U_{\tau 4}|^2 \leq 7 \times 10^{-3} \text{ and } |U_{\mu 4}|^2 \leq 0.8 \times 10^{-3}.$$

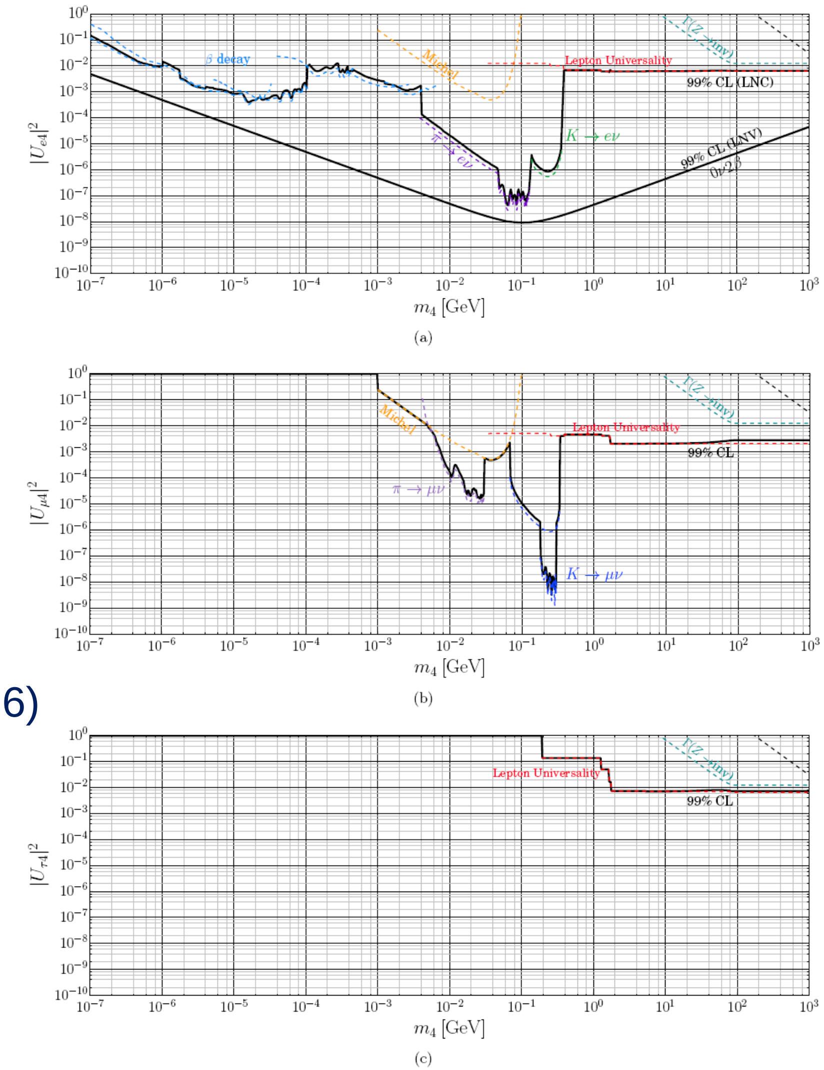
$$(U_{e4}, U_{\mu 4}):\ |U_{\mu 4}|^2 \leq 2 \times 10^{-3} \text{ and } |U_{e4}|^2 \leq 4 \times 10^{-3}.$$

Production of N via its mixing challenging, but LFV tests possible

Gauged $B - L$ or left-right symmetry makes production testable

See talks in WG III: B. Dev, A. Patra, K. Ghosh,…

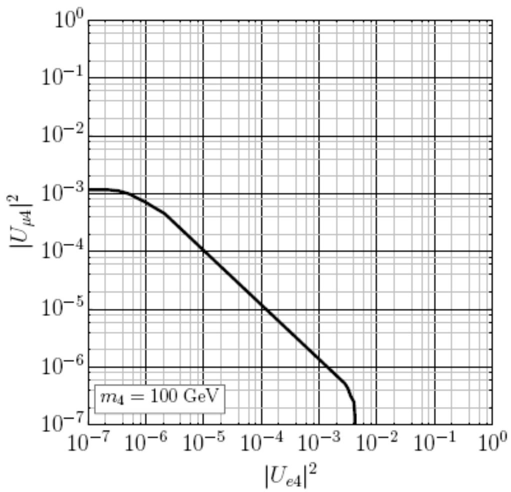
Global constraints on neutrino mixing



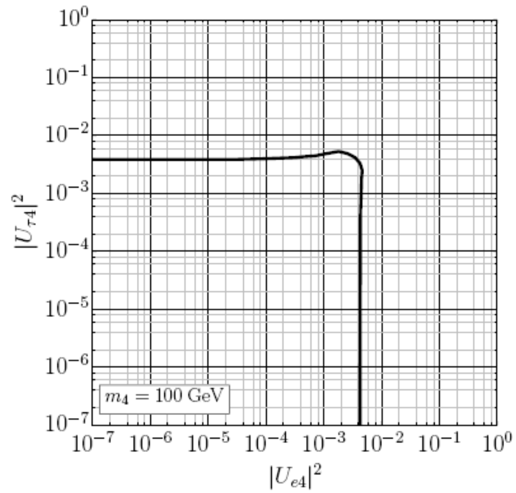
De Gouvea, Kobach (2016)
Atre, Han, Pascoli (2009)

Lepton number violating constraints do not apply!

Constraints with 2 nonzero mixings

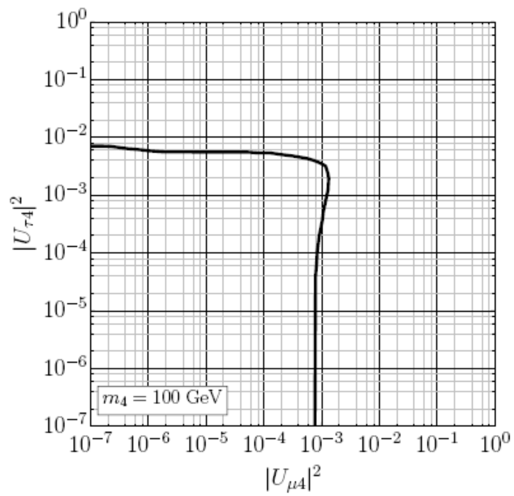


(a)



(b)

De Gouvea, Kobach (2016)



(c)

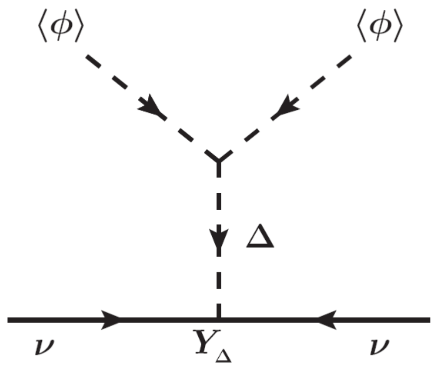
Testing Type-II Seesaw

Introduces a weak triplet $\Delta(1,3,+2)$

$$\Delta = \frac{\sigma_i}{\sqrt{2}}\,\Delta_i = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & \frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

Neutrino Yukawa couplings:

$$\mathcal{L}_Y \supset -\frac{(\mathbf{Y}_\Delta)_{ij}}{\sqrt{2}}\,\ell_i^T C\, i\sigma_2\,\Delta\,\ell_j + h.c.$$



Δ^0 acquries an induced VEV

$$\langle\Delta\rangle = \frac{\mu v^2}{\sqrt{2}\mu_\Delta^2} \ll v \qquad \mathbf{m}_\nu \simeq \mathbf{Y}_\Delta\,\frac{\mu\, v^2}{2\mu_\Delta^2}$$

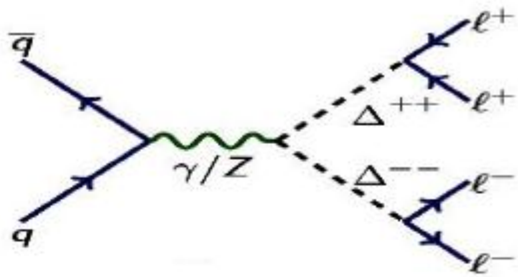
Same sign dilepton decay of Δ^{++} best test

Photon initiated production can be significant Babu, Jana (2017)

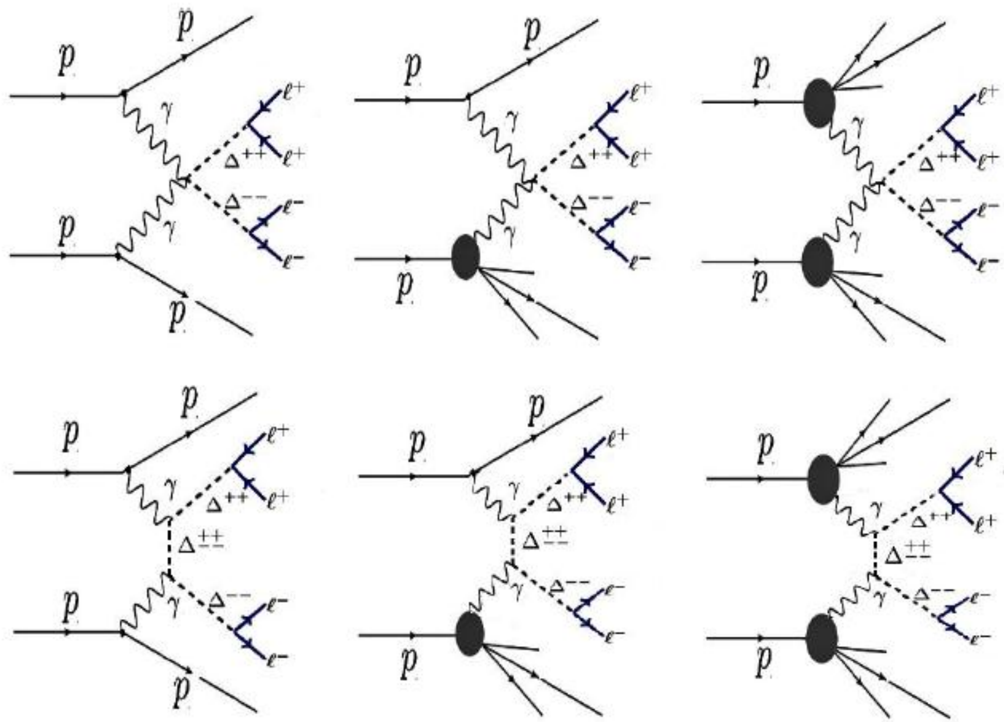
Rizzo (1982); Huitu, Maalampi, Pietila, Raidal (1997); Gunion, Loomis, Pitts (1996); Akeryod, Aoki (2005); Han, Mukhopadhyaya, Ci, Wang (2005), N. Sahu, Uma Sankar (2005); Sarma, Devi, Singh (2007); Chao, Luo, Xing, Zhao (2007); Perez, Han, Huang, Li, Wang (2008); McDonald, Sahu, Sarkar (2008); Chiang, Nomura, Tsumura (2012); Dev, D. Ghosh, Okada, Saha (2013); Nayak, Parida (2015); Cai, Han, Ruiz (2017),……

Production of doubly charged scalar

Drell-Yan:



Photon initiated processes



Babu, Jana (2017)

Improved limits on doubly charged mass

Including photon initiated processes

Benchmark Point	ATLAS limit(GeV)	Limits from our analysis (GeV) (DY)	(DY+PF)
$\Delta_L^{\pm\pm} \to e^\pm e^\pm = 100\%$	551	551	**∼630**
$\Delta_L^{\pm\pm} \to e^\pm \mu^\pm = 100\%$	468	470	**607**
$\Delta_L^{\pm\pm} \to \mu^\pm \mu^\pm = 100\%$	516	515	**∼620**
$\Delta_R^{\pm\pm} \to e^\pm e^\pm = 100\%$	374	372	**572**
$\Delta_R^{\pm\pm} \to e^\pm \mu^\pm = 100\%$	402	402	**488**
$\Delta_R^{\pm\pm} \to \mu^\pm \mu^\pm = 100\%$	438	439	**591**

NNPDF23_lo_as_0130_qed

Significant uncertainty due to PDF (of order 30%)

Babu, Jana (2017)

Minimal SO(10) Model

$$\mathcal{L}_{\text{Yukawa}} = Y_{10}\, \mathbf{16}\,\mathbf{16}\,\mathbf{10}_H + Y_{126}\,\mathbf{16}\,\mathbf{16}\,\overline{\mathbf{126}}_H$$

Two Yukawa matrices determine all fermion masses and mixings, including the neutrinos

$$\begin{aligned} M_u &= \kappa_u Y_{10} + \kappa'_u Y_{126} \\ M_d &= \kappa_d Y_{10} + \kappa'_d Y_{126} \\ M_\nu^D &= \kappa_u Y_{10} - 3\kappa'_u Y_{126} \\ M_l &= \kappa_d Y_{10} - 3\kappa'_d Y_{126} \end{aligned}$$

$$\begin{aligned} M_{\nu R} &= \langle \Delta_R \rangle Y_{126} \\ M_{\nu L} &= \langle \Delta_L \rangle Y_{126} \end{aligned}$$

Model has only 11 real parameters plus 7 phases

Babu, Mohapatra (1993)
Bajc, Melfo, Senjanovic, Vissani (2002)
Fukuyama, Okada (2002)
Bajc, Melfo, Senjanovic, Vissani (2004)
Fukuyama, Ilakovac, Kikuchi, Meljanac, Okada (2004)
Aulakh et al (2004)
Bertolini, Frigerio, Malinsky (2004)

Babu, Macesanu (2005)
Bertolini, Malinsky, Schwetz (2006)
Dutta, Mimura, Mohapatra (2007)
Bajc, Dorsner, Nemevsek (2009)
Joshipura, Patel (2011)
Dueck, Rodejohann (2013)

Specific Example: Type I Seesaw

Input at the GUT scale:

$$\begin{aligned} m_u &= 0.0006745 & m_c &= 0.3308 & m_t &= 97.335 \\ m_d &= 0.0009726 & m_s &= 0.02167 & m_b &= 1.1475 \\ m_e &= 0.000344 & m_\mu &= 0.0726 & m_\tau &= 1.350 \text{ GeV} \\ s_{12} &= 0.2248 & s_{23} &= 0.03278 & s_{13} &= 0.00216 \\ & & \delta_{CKM} &= 1.193\ . \end{aligned}$$

Output for neutrinos:

$$\sin^2\theta_{12} \simeq 0.27, \quad \sin^2 2\theta_{23} \simeq 0.90, \quad \sin^2 2\theta_{13} \simeq 0.08$$

$$m_i = \{0.0021 e^{0.11i},\ 0.0098 e^{-3.08i},\ 0.048\} \text{ eV}$$

$$\Delta m^2_{23}/\Delta m^2_{12} \simeq 24$$

K.S. Babu and C. Macesanu (2005)

Minimal SO(10) Global Fit

Observables	Fitted values (type-I)	pulls (type-I)	Fitted values (type-II)	pulls (type-II)
m_d	0.000810163	-0.687161	0.00101285	-0.264898
m_s	0.0208099	-0.198354	0.0225915	0.0844982
m_b	0.999667	-0.00831657	1.08201	2.05031
m_u	0.000495023	0.0751133	0.000507336	0.13668
m_c	0.237348	0.0670883	0.237096	0.0598882
m_t	73.9427	-0.0154941	74.3006	0.075144
m_e	0.000469652	-	0.000469652	-
m_μ	0.0991466	-	0.0991466	-
m_τ	1.68558	-	1.68558	-
$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$	0.030526	0.127968	0.0297114	-0.235285
$\sin\theta^q_{12}$	0.224651	0.0464044	0.224499	-0.0916848
$\sin\theta^q_{23}$	0.0420499	0.0392946	0.0421308	0.103004
$\sin\theta^q_{13}$	0.00349369	-0.0974312	0.00353053	0.0389979
$\sin^2\theta^l_{12}$	0.323245	0.148134	0.3108	-0.610792
$\sin^2\theta^l_{23}$	0.435096	-0.369178	0.113306	-7.02461
$\sin^2\theta^l_{13}$	0.0244287	-	0.0176863	-
$\delta_{CKM}[^\circ]$	69.5262	-0.0314447	69.2051	-0.128759
$\delta_{MNS}[^\circ]$	318.465	-	14.5386	-
$\alpha_1[^\circ]$	21.5053	-	345.645	-
$\alpha_2[^\circ]$	215.128	-	141.905	-
$r_{R(L)}$	5.62×10^{-14}	-	2.09×10^{-10}	-
χ^2		0.710777		54.1197

Joshipura, Patel (2011)

Large Neutrino Mixing From Lopsided Matrices

Quark and Lepton Mass hierarchy:

$$m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \epsilon_1 : \epsilon_2 : \epsilon_3$$

$$m_u : m_c : m_t \sim \epsilon_1^2 : \epsilon_2^2 : \epsilon_3^2$$

This motivates:

$$\begin{aligned} U &= H^T U_0 H \\ D &= D_0 H \\ L &= H^T L_0 \\ N &= N_0 \end{aligned}$$

$$H = \text{Diag}(\epsilon_1,\ \epsilon_2,\ \epsilon_3) \qquad \epsilon_1 << \epsilon_2 << \epsilon_3$$

10_i of $SU(5)$ carry flavor charge, $\bar{5}_i$ do not. Leads to large left-handed charged lepton mixing and large right-handed down quark mixing.

K.S. Babu, S. Barr, 1995

Albright, Babu , Barr, 1998

Sato and Yanagida, 1998

Irges, Lavignac, Ramond, 1998

Altarelli, Feruglio, 1998

Flavor $U(1)$ charges:

$10_i : (3,\, 2,\, 0) \qquad \bar{5}_i : (p,\, p,\, p) \qquad 1_i : (q,\, q,\, q)$

$$U_{ij} = \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} H_u, \qquad D_{ij} = \begin{pmatrix} \epsilon^3 & \epsilon^3 & \epsilon^3 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix} \epsilon^p H_d,$$

$$L_{ij} = \begin{pmatrix} \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \epsilon^p H_d, \qquad \nu^D_{ij} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \epsilon^{p+q} H_u$$

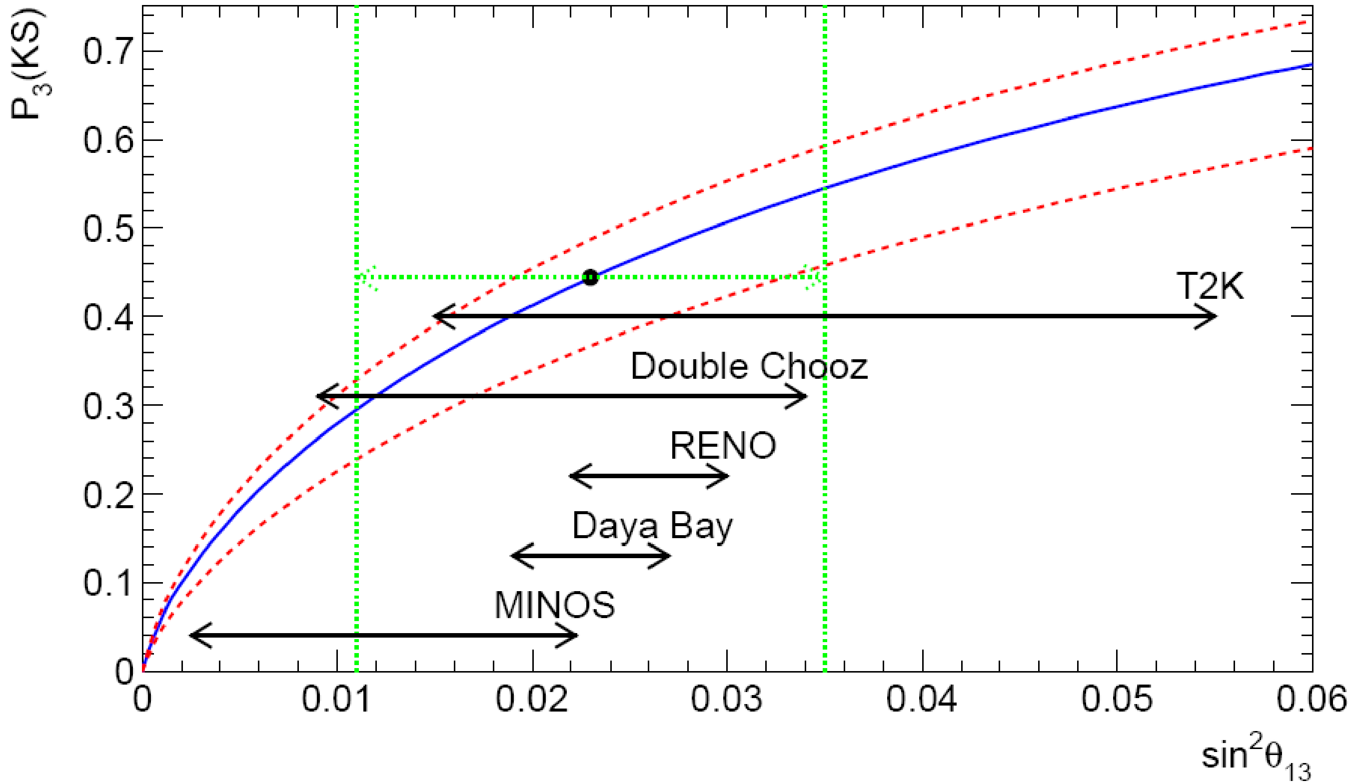
$$(M_\nu)_{ij} \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad \epsilon \sim 0.2$$

All features of fermion masses and CKM mixing reproduced

No particular hierarchy in neutrino masses

θ_{13} predicted to be large

Neutrino Mass Anarchy



De Gouvea, Murayama (2012)

Lepton mixing matrix described by random draw of numbers in a unitary matrix

Radiative mass generation

If the mass of a fermion is zero due to a symmetry or the particle content in a renormalizable model, then either its mass remains zero, or it acquires a mass that is finite.

First example due to 't Hooft in 1971 in the classis paper showing renormalizability of spontaneously broken gauge theories.

A toy model where the electron mass was zero at tree–level due to a symmetry, but was induced and finite proportional to the muon mass as a one–loop radiative correction.

No counter-term for m_e is permitted in the model, so the induced m_e is finite and thus "calculable".

Application of this idea to neutrino mass generation has many interesting and testable features.

Effective Delta(L) = 2 operators for neutrino masses

Standard seesaw operator

$$\mathcal{O}_1 = L^i L^j H^k H^l \epsilon_{ik}\epsilon_{jl}$$

Operators with four fermions:

C.N. Leung, KSB (2003)

$$\begin{aligned}
\mathcal{O}_2 &= L^i L^j L^k e^c H^l \epsilon_{ij}\epsilon_{kl} \\
\mathcal{O}_3 &= \{L^i L^j Q^k d^c H^l \epsilon_{ij}\epsilon_{kl}, \quad L^i L^j Q^k d^c H^l \epsilon_{ik}\epsilon_{jl}\} \\
\mathcal{O}_4 &= \{L^i L^j \bar{Q}_i \bar{u^c} H^k \epsilon_{jk}, \quad L^i L^j \bar{Q}_k \bar{u^c} H^k \epsilon_{ij}\} \\
\mathcal{O}_5 &= L^i L^j Q^k d^c H^l H^m \bar{H}_i \epsilon_{jl}\epsilon_{km} \\
\mathcal{O}_6 &= L^i L^j \bar{Q}_k \bar{u^c} H^l H^k \bar{H}_i \epsilon_{jl} \\
\mathcal{O}_7 &= L^i Q^j \bar{e^c} \bar{Q}_k H^k H^l H^m \epsilon_{il}\epsilon_{jm}
\end{aligned}$$

Choi, Jeong, Song (2002)
De Gouvea, Jenkins (2008)
Angel, Volkas (2012)

$$\begin{aligned}
\mathcal{O}_8 &= L^i \bar{e^c} \bar{u^c} d^c H^j \epsilon_{ij} \\
\mathcal{O}_9 &= L^i L^j L^k e^c L^l e^c \epsilon_{ij}
\end{aligned}$$

Review: Cai, Herrero-Garcia, Schmidt, Vicente, Volkas (2017)

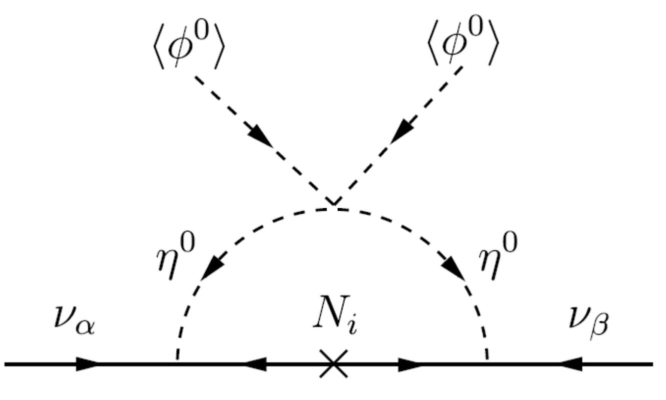
Dimension 5 operator through loops

$\mathcal{O}_1 = L^i L^j H^k H^l \epsilon_{ik}\epsilon_{jl}$ may arise at loops

without light fermion mass factor

Example: Inert Doublet Model – Scotogenic Model

E. Ma (2006)



$$m_\nu \approx \frac{Y_\nu^2 \lambda_5 v^2}{16\pi^2} \frac{M_R}{M_\eta^2 + M_R^2}$$

From m_ν alone, $M_R, M_\eta \sim 10^{12}$ GeV will work

Dark matter would require $M_R \sim M_\eta \sim$ TeV

Zee Model of Neutrino Mass

A. Zee (1980)

Simplest example of using $d = 7$ operator

Neutrino masses induced at one–loop

Loop and chiral suppression $\Rightarrow$ scale can be low

No right-handed neutrinos introduced, so the seesaw operator $\mathcal{O}_1 = L^i L^j H^k H^l \epsilon_{ik}\epsilon_{jl}$ absent at tree-level

Effective operator $\mathcal{O}_2 = L^i L^j L^k e^c H^l \epsilon_{ij}\epsilon_{kl}$ induces neutrino mass

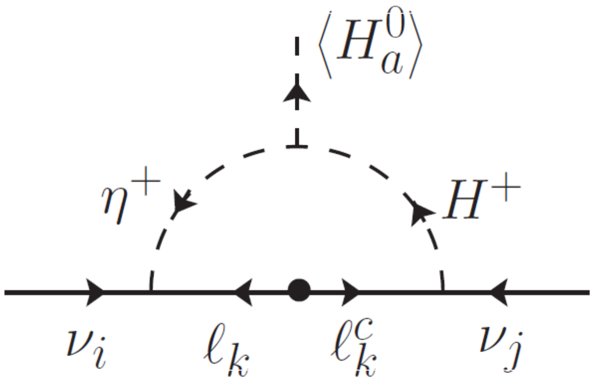
Introduces a second Higgs doublet and a charged singlet scalar η^+

Zee Model of Neutrino Mass (cont.)

$H_a(1,2,-\frac{1}{2}),\ \eta^+(1,1,1)$

$$\mathcal{L}_{\text{Yuk}} = f_{ij} L_i L_j \eta^+ + Y'_{ij} L_i e^c_j H_2 + h.c.$$

$$V = \mu H_1 H_2 \eta^+ + h.c. + \ldots$$



$f_{ij} = -f_{ji}$

$$M_\nu = \kappa \left(\hat{f} M_\ell^{\text{diag}} \hat{Y}^T + \hat{Y} M_\ell^{\text{diag}} \hat{f}^T \right)$$

$$\kappa = \frac{\sin 2\gamma}{16\pi^2} \log\left(\frac{M_1^2}{M_2^2}\right)$$

γ: $\eta^+ - H^+$ mixing angle, $M_{1,2}$: charged Higgs masses

In the Zee model, both Higgs doublets couple to leptons $\Rightarrow$ Flavor changing neutral currents at tree-level mediated by Higgs bosons

$$M_\nu = \kappa \left(\widehat{f} M_\ell^{\text{diag}} \widehat{Y}^T + \widehat{Y} M_\ell^{\text{diag}} \widehat{f}^T \right)$$

$\widehat{Y}$ is arbitrary, so quantitative predictions difficult

Wolfenstein suggested a discrete Z_2 symmetry that allows only one Higgs doublet to couple to leptons

L. Wolfenstein (1980)

FCNC avoided, $\widehat{Y} = \frac{M_\ell^{\text{diag}}}{v}$

Zee-Wolfenstein model very predictive for neutrinos

Smirnov, Tanimoto (1997)
Jarlskog, Matsuda, Skaldhauge, Tanimoto (1999)
Frampton, Glashow (1999)

Zee-Wolfenstein model:

$$M_\nu = \frac{\kappa}{v}\left[\widehat{f}(M_\ell^{\text{diag}})^2 + (M_\ell^{\text{diag}})^2\widehat{f}^T\right]$$

$$M_\nu = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

3 real parameters explain all neutrino oscillation data

Compatible with bimaximal neutrino mixing

KamLand and solar neutrino data excluded this possibility

Koide (2001)
X.G. He (2004)

A Predictive Variant

– Complete absence of FCNC too restrictive –

– General Zee model too arbitrary –

– Choose an intermediate scenario – Babu, Julio (2013)

Impose a Z_4 symmetry that is family-dependent

$$L_i : (-i,\, i,\, i); \qquad e_i^c : (-i,\, -i,\, -i);$$
$$H_1 : +1; \quad H_2 : -1; \quad \eta^+ : -1$$

$$Q_i : (-i,\, -i,\, -i),\ u_i^c : (i,\, i,\, i),\ \text{and}\ d_i^c : (i,\, i,\, i)$$

Predictions:

- Neutrino mass hierarchy is inverted
- $\delta_{CP} = 0$ or π
- Majorana phases 0 or π
- $|U_{\tau 1}| = |U_{\tau 2}|$
- $m_3 = \frac{1}{2} \frac{\Delta m^2_{\text{solar}}}{|\Delta m^2_{\text{atm}}|^{1/2}} \simeq 7.5 \times 10^{-4}$ eV

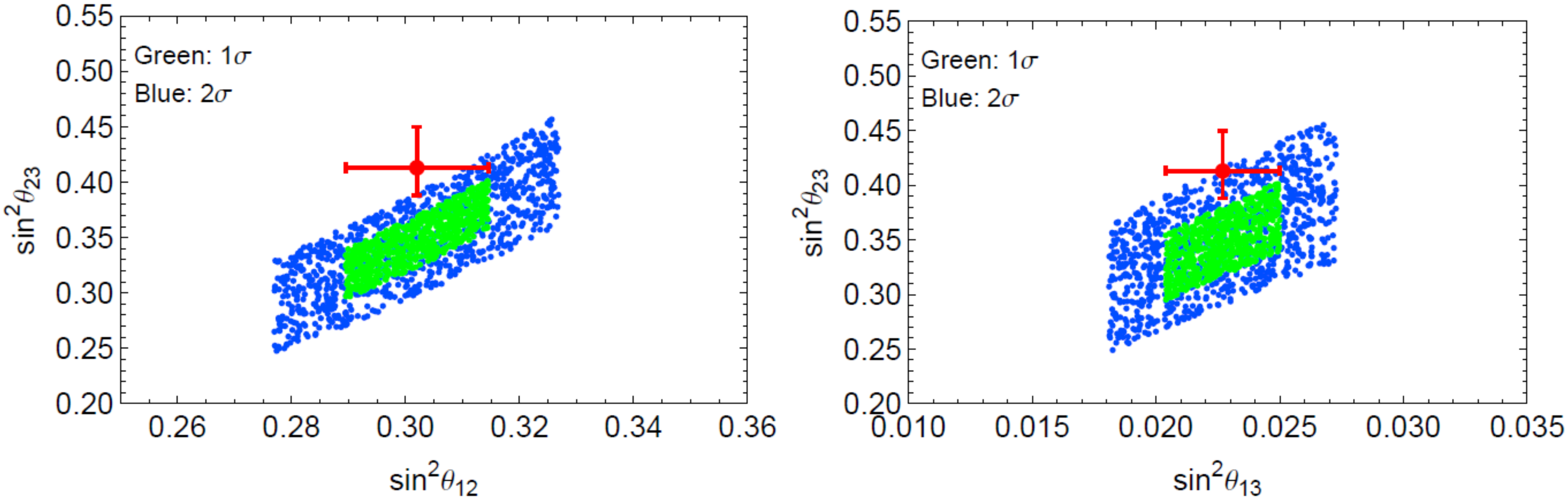
$$U_{\text{PMNS}} = \begin{pmatrix} \frac{1}{\sqrt{2}}(C_\chi C_\psi + S_\psi) & \frac{1}{\sqrt{2}}(C_\chi C_\psi - S_\psi) & -S_\chi C_\psi \\ \frac{1}{\sqrt{2}}(C_\chi S_\psi - C_\psi) & \frac{1}{\sqrt{2}}(C_\chi S_\psi + C_\psi) & -S_\chi S_\psi \\ \frac{S_\chi}{\sqrt{2}} & \frac{S_\chi}{\sqrt{2}} & C_\chi \end{pmatrix}$$

$|U_{\tau 1}| = |U_{\tau 2}|$ relation:

$$s_{13} = t_{23}\frac{1-t_{12}}{1+t_{12}}, \quad \text{or} \quad s_{13} = -t_{23}\frac{1+t_{12}}{1-t_{12}} \qquad (\delta_{CP} = \pi)\ ;$$

$$s_{13} = t_{23}\frac{1+t_{12}}{1-t_{12}}, \quad \text{or} \quad s_{13} = -t_{23}\frac{1-t_{12}}{1+t_{12}} \qquad (\delta_{CP} = 0)$$

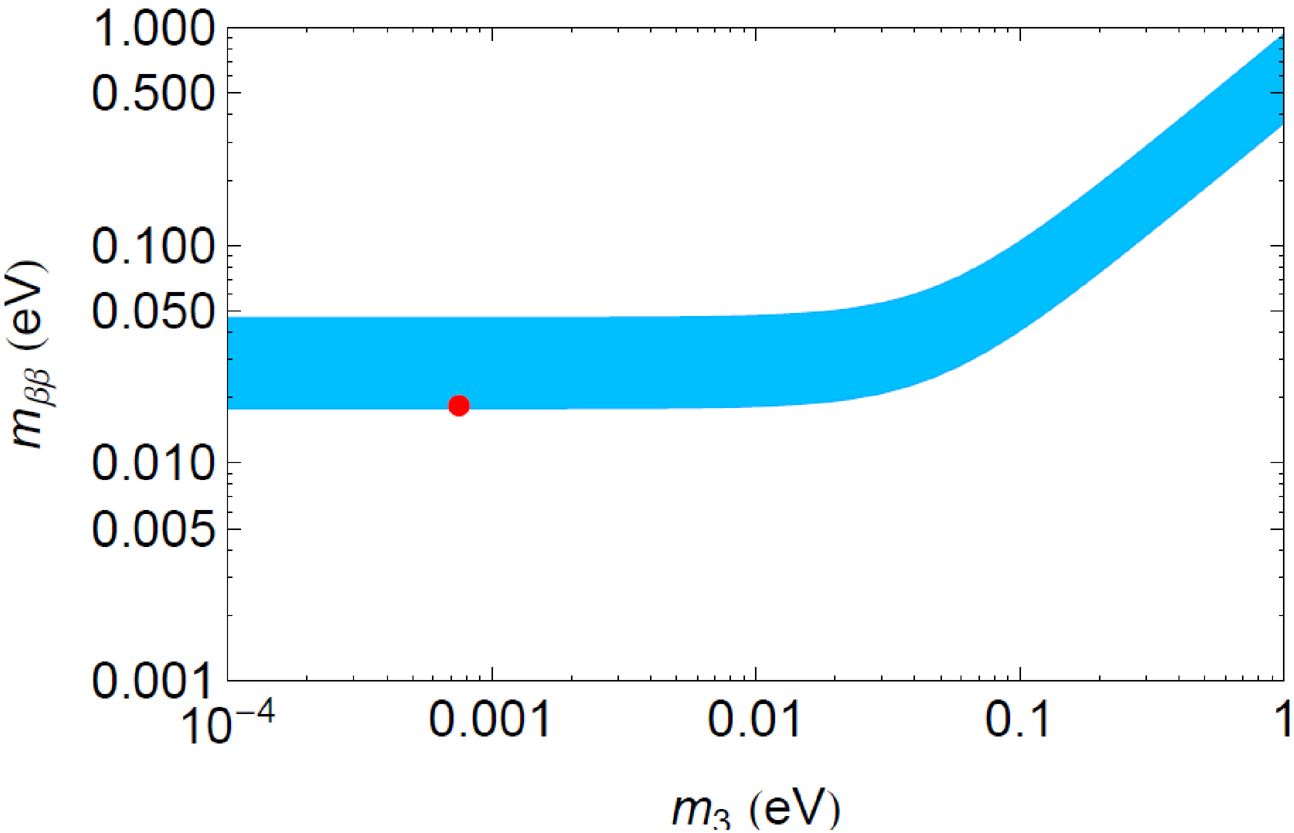
$\delta_{CP} = \pi$ required



Gonzalez-Garcia et al (2012)
Fogli et al (2012)

Effective mass in neutrinoless double beta decay:

$$m_{\beta\beta} \equiv |\sum_{i=1-3} U_{ei}^2 m_i| = (17.6 - 18.5)\ \text{meV}$$

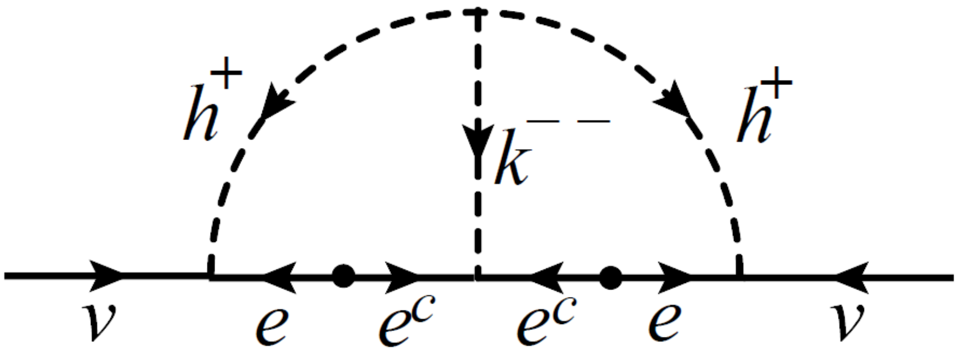


Two–loop neutrino mass generation via $\mathcal{O}_9$

$$\mathcal{L} = f_{ij} L_i^a L_j^b h^+ \epsilon_{ab} + g_{ij} e_i^c e_j^c k^{--} + \mu h^+ h^+ k^{--} + \text{h.c.}$$

$$\Downarrow$$

$$\mathcal{O}_9 = L^i L^j L^k e^c L^l e^c \epsilon_{ij} \epsilon_{kl}$$



A. Zee, (1985)
Babu (1988)

Consistent with all neutrino oscillation data

Predicts doubly charged Higgs boson with TeV mass

One neutrino is nearly massless

Two–loop neutrino mass model

$$(\mathcal{M}_\nu)_{ab} = 16\mu f_{ac} m_c g^*_{cd} I_{cd} m_d f_{bd}$$

$$I_{cd} = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(k^2 - m_c^2)} \frac{1}{(k^2 - m_h^2)} \frac{1}{(q^2 - m_d^2)} \frac{1}{(q^2 - m_h^2)} \frac{1}{(k-q)^2 - m_k^2}.$$

$$I_{cd} \simeq I = \frac{1}{(16\pi^2)^2} \frac{1}{m_h^2} \tilde{I}\left(\frac{m_k^2}{m_h^2}\right)$$

$$\tilde{I}(r) = -\int_0^1 dx \int_0^{1-x} dy \frac{1-y}{x + (r-1)y + y^2} \log \frac{y(1-y)}{x + ry}$$

$$\tilde{I}(r) = \begin{cases} 1 + \frac{3}{\pi^2}(\log^2 r - 1) & \text{for } r \gg 1 \\ 1 & \text{for } r \to 0 \end{cases}$$

$$M_\nu = \xi f \omega f^T$$

$$f_{ab} = -f_{ba}, \quad \omega_{ab} = g_{ab} m_a m_b$$

Two–loop neutrino mass model

f has an eigenvector with zero eigenvalue:

$$v_0^T = (1, -\epsilon, \epsilon'); \quad f v_0 = 0 \ .$$

$$\epsilon = f_{e\tau}/f_{\mu\tau}, \quad \epsilon' = f = e\mu/f_{\mu\tau}$$

v_0 is an eigenvector of M_ν with zero eigenvalue: $M_\nu v_0 = 0$

$$\epsilon = \frac{m_{12}m_{33} - m_{13}m_{23}}{m_{22}m_{33} - m_{23}^2} \qquad \epsilon' = \frac{m_{12}m_{23} - m_{13}m_{22}}{m_{22}m_{33} - m_{23}^2}$$

$$\epsilon \ = \ \tan\theta_{12} \, \frac{\cos\theta_{23}}{\cos\theta_{13}} \ + \ \tan\theta_{13} \, \sin\theta_{23} \ e^{-i\delta}$$

Normal hierarchy

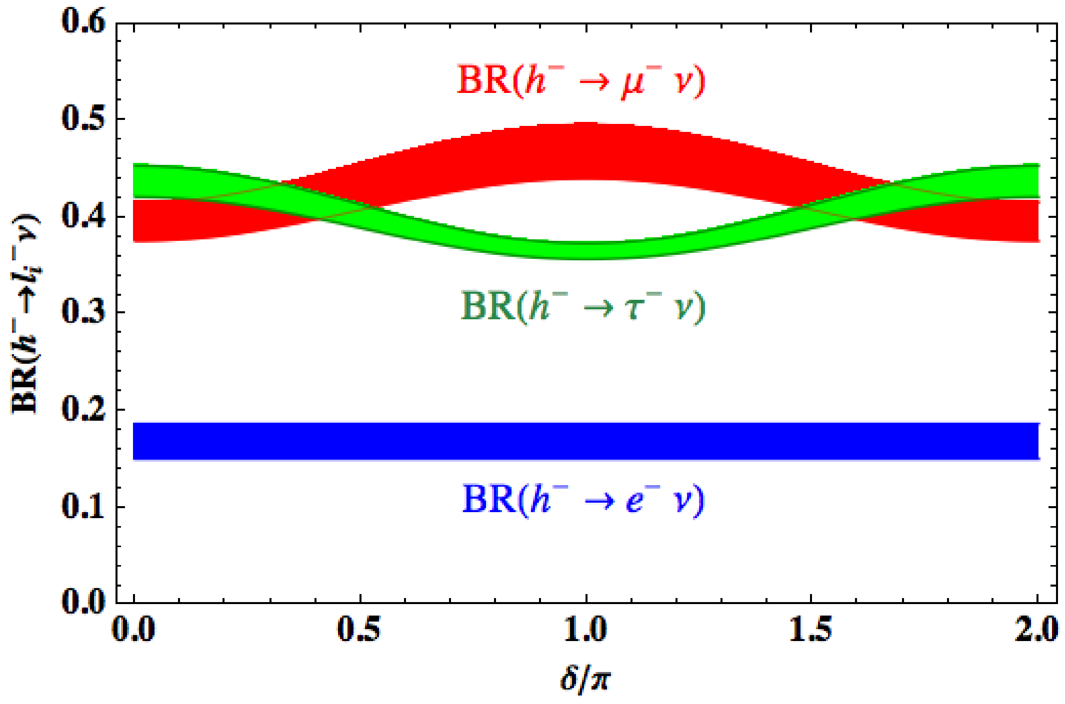
$$\epsilon' \ = \ \tan\theta_{12} \, \frac{\sin\theta_{23}}{\cos\theta_{13}} \ - \ \tan\theta_{13} \, \cos\theta_{23} \ e^{-i\delta}$$

Two–loop neutrino mass model (cont.)

Inverted hierrarchy:

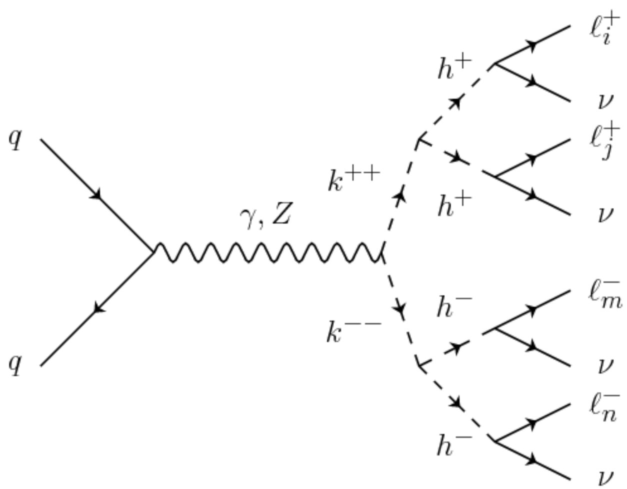
$$\epsilon = -\sin\theta_{23} \cot\theta_{13}\ e^{-i\delta} \quad , \quad \epsilon' = \cos\theta_{23} \cot\theta_{13}\ e^{-i\delta}$$

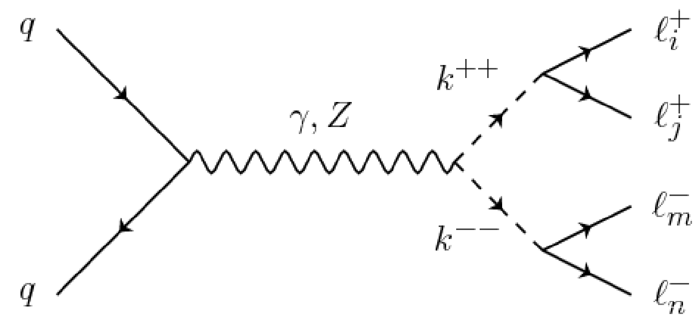
$h^- \to \ell^- \nu$ branching ratios fixed:



Producion of $h^+h^+h^-h^-$

$pp \to k^{++}k^{--}$ with $k^{++} \to h^+h^+$

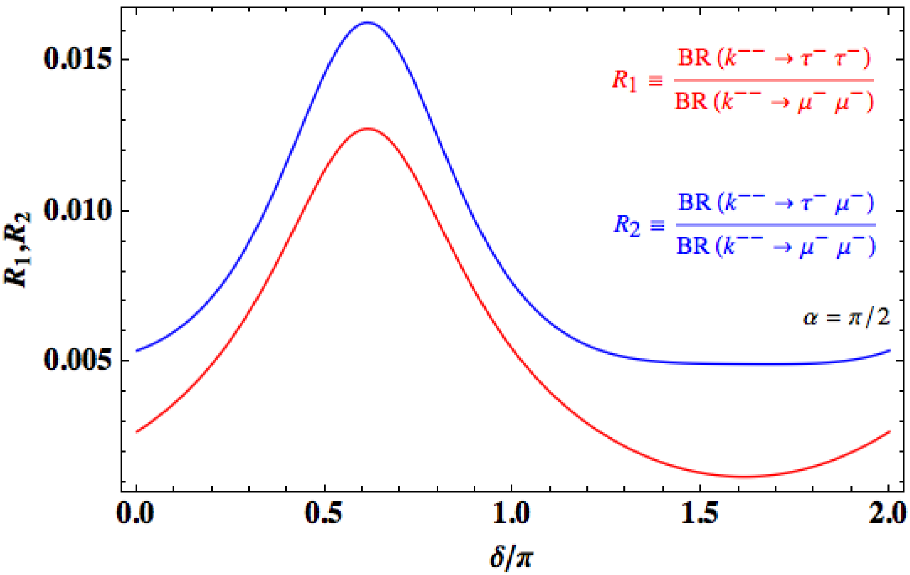


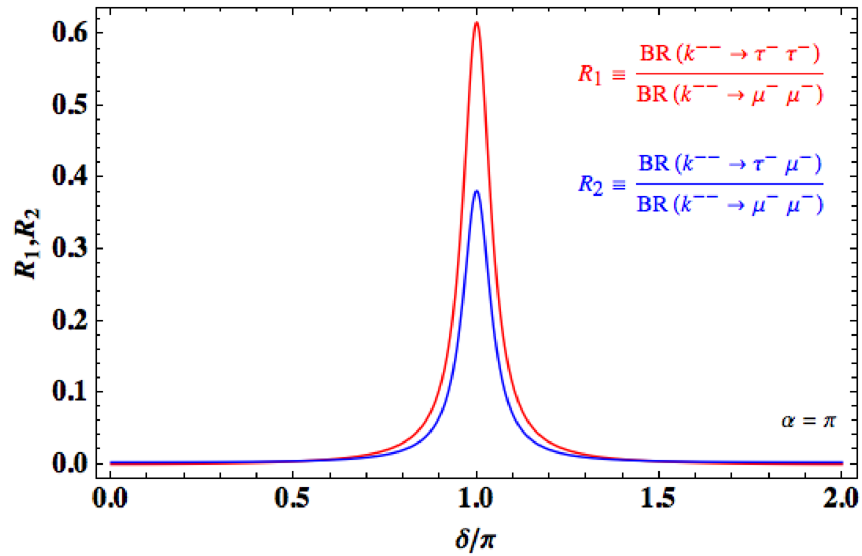


CMS and ATLAS limits of $m_{k^{++}} > 750$ GeV not applicable for $k^{++} \to h^+h^+$ decay

Branching ratios of $k^{++} \to \ell^+\ell^+$ predicted from neutrino data

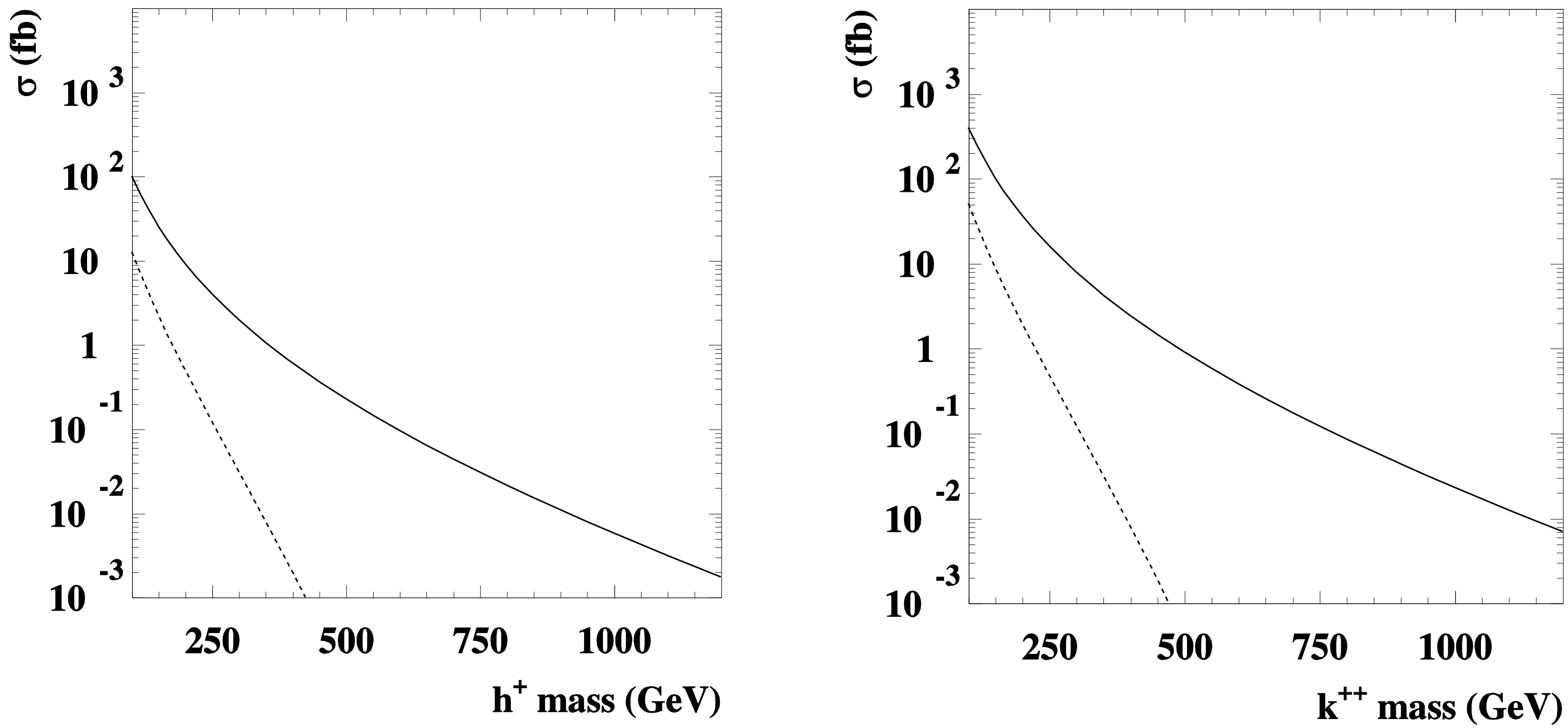
Branching ratio for $k^{++} \to \ell^+ \ell^+$





Relative branching ratio gives insight into Majorana phases

Cross section for h^+ and k^{++} at LHC and Tevatron



LHC: Solid line, Tevatron: dashed

Detailed study:

K.S. Babu, C. Macesanu (2005)
D. Sierra, M. Hirsch (2006)
M. Nebot, J. Oliver, D. Paolo, A. Santamaria (2008)

Lepton flavor violation constraints

Process	Experiment (90% CL)	Bound (90% CL)
$\mu^- \to e^+e^-e^-$	BR$< 1.0 \times 10^{-12}$	$\lvert g_{e\mu}g^*_{ee}\rvert < 2.3 \times 10^{-5}\,(m_k/\mathrm{TeV})^2$
$\tau^- \to e^+e^-e^-$	BR$< 3.6 \times 10^{-8}$	$\lvert g_{e\tau}g^*_{ee}\rvert < 0.010\,(m_k/\mathrm{TeV})^2$
$\tau^- \to e^+e^-\mu^-$	BR$< 2.7 \times 10^{-8}$	$\lvert g_{e\tau}g^*_{e\mu}\rvert < 0.006\,(m_k/\mathrm{TeV})^2$
$\tau^- \to e^+\mu^-\mu^-$	BR$< 2.3 \times 10^{-8}$	$\lvert g_{e\tau}g^*_{\mu\mu}\rvert < 0.008\,(m_k/\mathrm{TeV})^2$
$\tau^- \to \mu^+e^-e^-$	BR$< 2.0 \times 10^{-8}$	$\lvert g_{\mu\tau}g^*_{ee}\rvert < 0.008\,(m_k/\mathrm{TeV})^2$
$\tau^- \to \mu^+e^-\mu^-$	BR$< 3.7 \times 10^{-8}$	$\lvert g_{\mu\tau}g^*_{e\mu}\rvert < 0.008\,(m_k/\mathrm{TeV})^2$
$\tau^- \to \mu^+\mu^-\mu^-$	BR$< 3.2 \times 10^{-8}$	$\lvert g_{\mu\tau}g^*_{\mu\mu}\rvert < 0.010\,(m_k/\mathrm{TeV})^2$
$\mu^+e^- \to \mu^-e^+$	$G_{M\bar{M}} < 0.003 G_F$	$\lvert g_{ee}g^*_{\mu\mu}\rvert < 0.2\,(m_k/\mathrm{TeV})^2$

Two–loop neutrino mass model (cont.)

Fits to neutrino masses and mixing angles, consistent with perturbativity and boundedness of potential as well as FCNC limits sets constraints on h^+ and k^{++} masses:

$$\text{NH}: \quad 306 \text{ GeV} < m_{k^{++}} < 177 \text{ TeV}; \quad 779 \text{ GeV} < m_{h^+} < 63 \text{ TeV}$$
$$\text{IH}: \quad 997 \text{ GeV} < m_{k^{++}} < 25 \text{ TeV}; \quad 2.3 \text{ TeV} < m_{h^+} < 7.9 \text{ TeV}$$

Since couplings are essentially fixed from neutrino masses, charged lepton flavor violation can be predicted

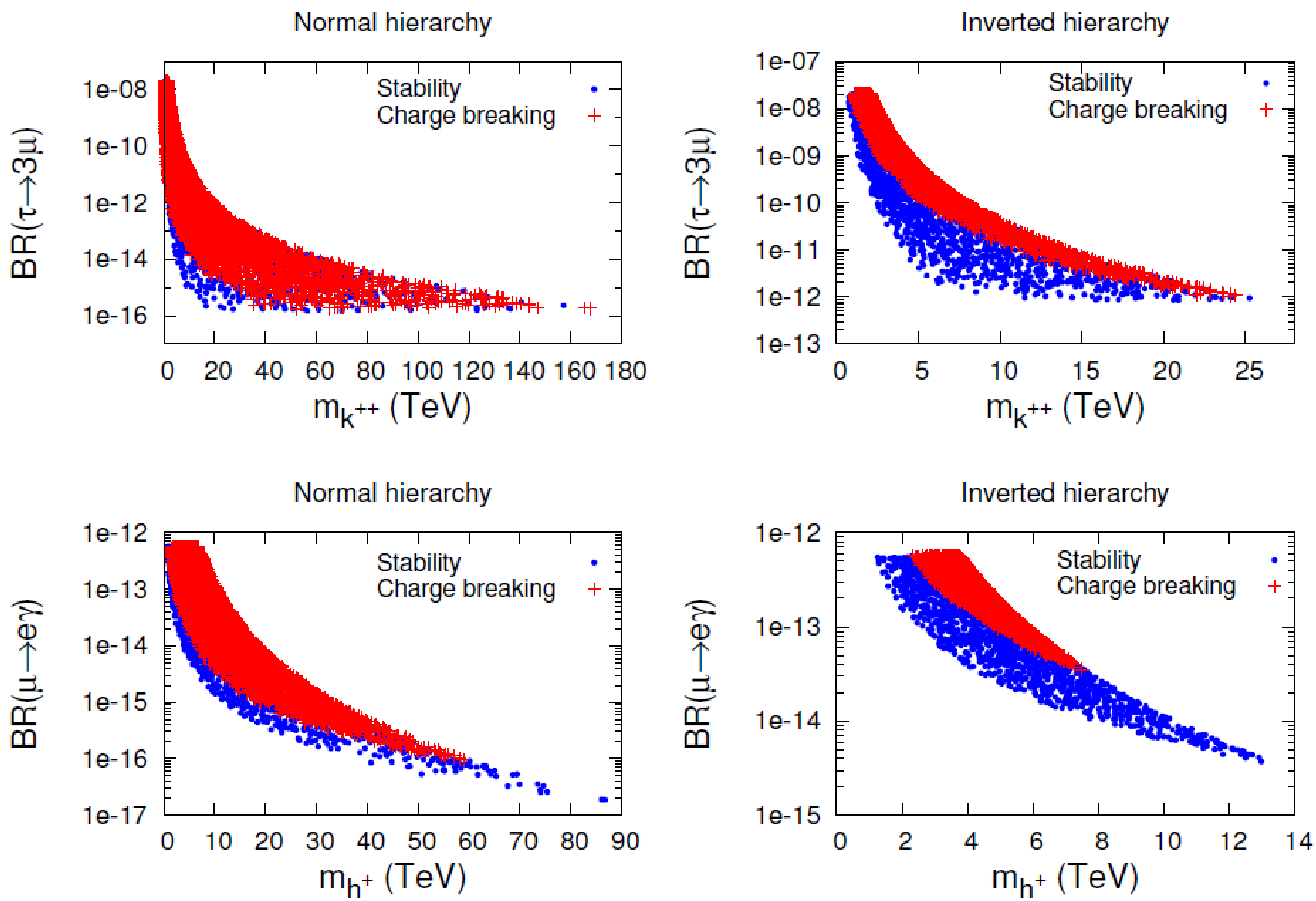
$\mu \to e\gamma$ and $\tau \to 3\mu$ limits used as input

Lower limits on branching ratios for $\mu \to 3e$, $\mu - e$ conversion in nuclei, as well as $\mu \to e\gamma$ and $\tau \to 3\mu$ follow

K.S. Babu, J. Julio (2013)
D. Schmitz, T.Schwetz, H. Zhang (2014)
J. Herrero-Garcia, M. Nebot, N. Rius, A. Santamaria (2014)

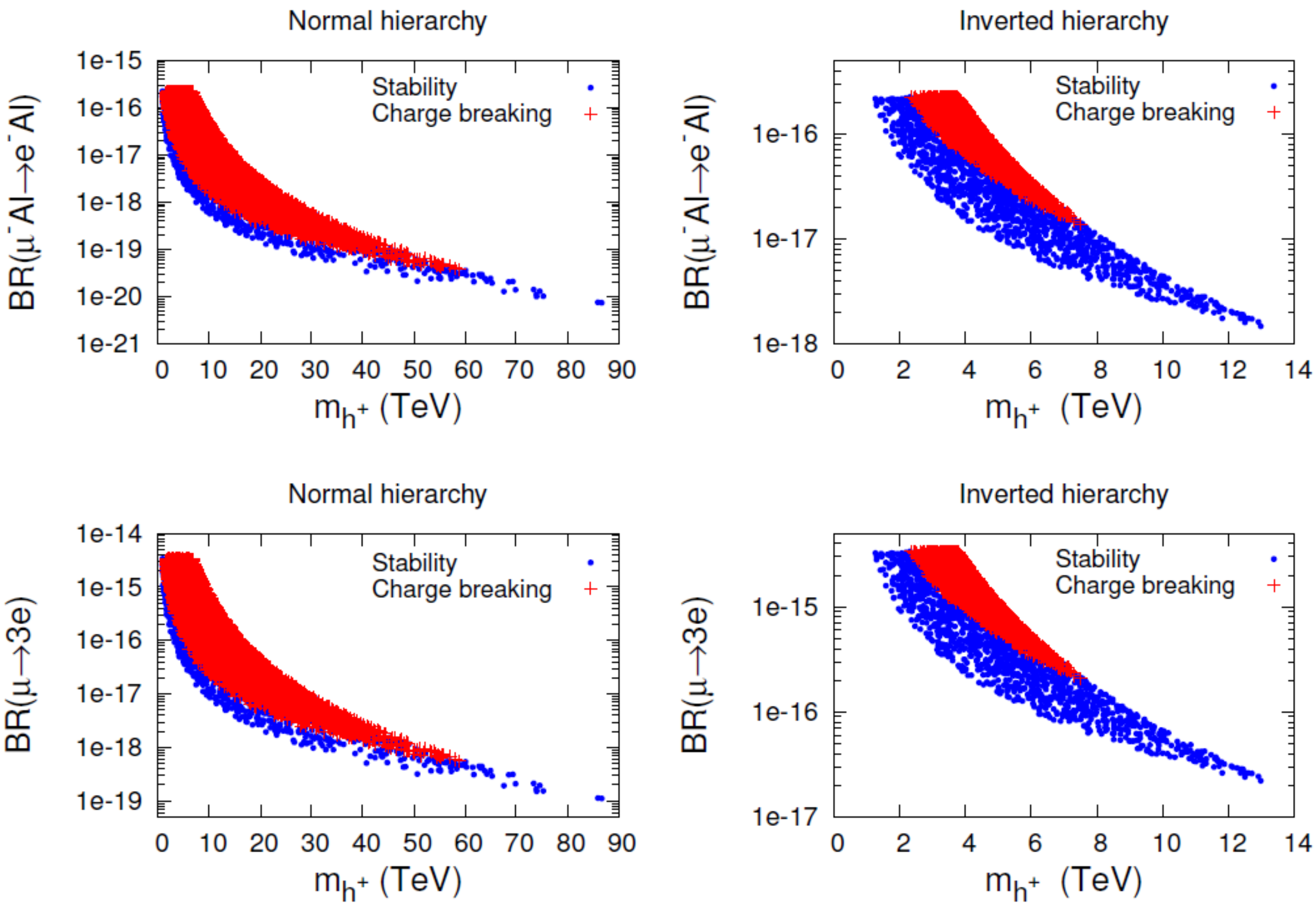
K.S. Babu, C. Macesanu (2005)
D. Sierra, M. Hirsch (2006)
M. Nebot, J. Oliver, D. Paolo, A. Santamaria (2008)

LFV in Radiative Neutrino Mass Model



K.S. Babu, J. Julio (2013)

LFV in Radiative Neutrino Mass Model (cont.)



Zeros in fermion mass matrices

CKM mixing angles may be related to quark mass ratios

A two family example:

$$M_u = \begin{pmatrix} 0 & A_u \\ A_u^* & B_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & A_d \\ A_d^* & B_d \end{pmatrix}.$$

$$\Rightarrow \quad |\sin\theta_C| \simeq \left| \sqrt{\frac{m_d}{m_s}} - e^{i\psi}\sqrt{\frac{m_u}{m_c}} \right|$$

Weinberg (1977)
Wilczek, Zee (1977)
Fritzsch (1977)

Here ψ is a free phase, but still θ_C is constrained

Two symmetries needed to enforce this structure:

(i) Parity symmetry for hermiticity

(ii) Family $U(1)$ symmetry for the zero in (1,1) entries

Texture zeros for neutrinos

$$A_1 : \begin{pmatrix} 0 & 0 & X \\ 0 & X & X \\ X & X & X \end{pmatrix} \qquad A_2 : \begin{pmatrix} 0 & X & 0 \\ X & X & X \\ 0 & X & X \end{pmatrix}$$

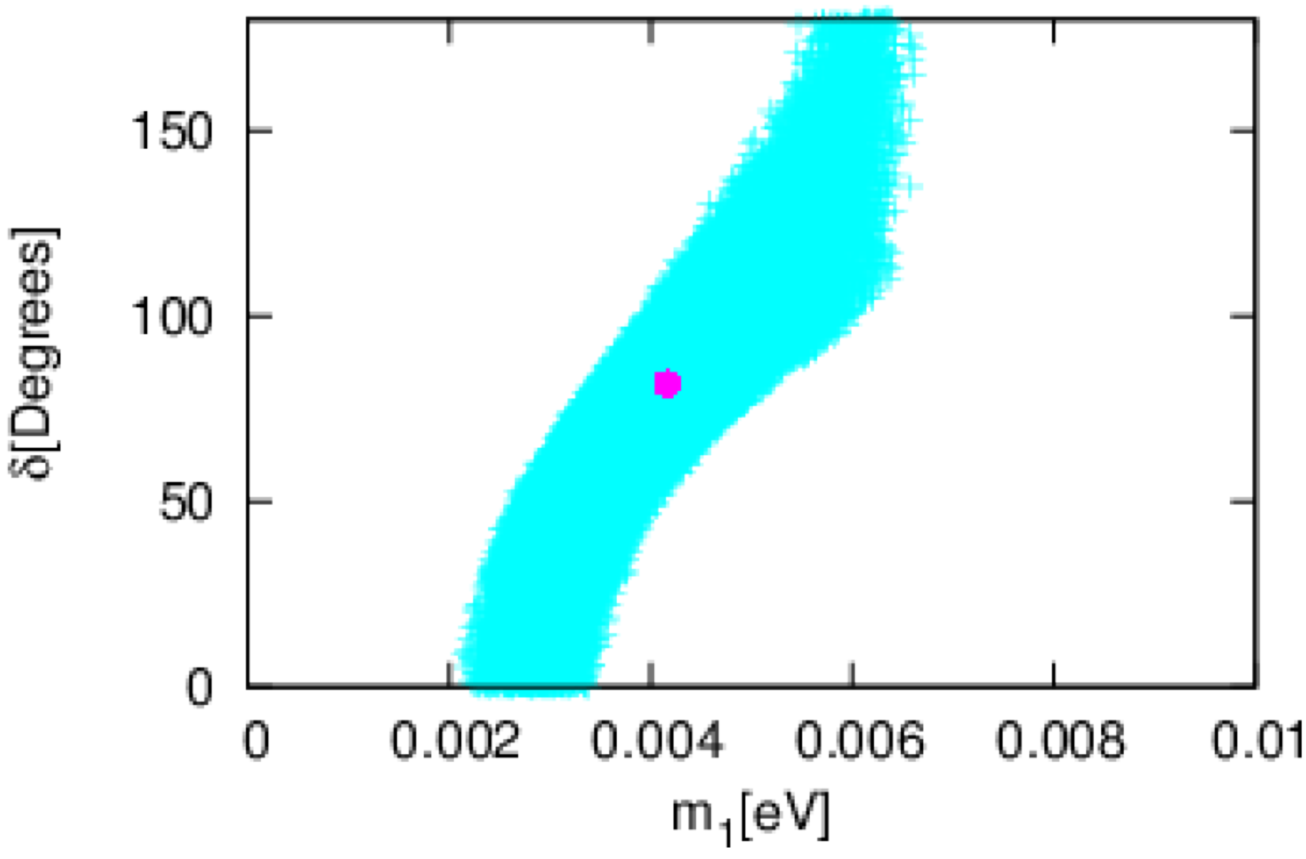
$$B_1 : \begin{pmatrix} X & X & 0 \\ X & 0 & X \\ 0 & X & X \end{pmatrix} \qquad B_2 : \begin{pmatrix} X & 0 & X \\ 0 & X & X \\ X & X & 0 \end{pmatrix}$$

$$B_3 : \begin{pmatrix} X & 0 & X \\ 0 & 0 & X \\ X & X & X \end{pmatrix} \qquad B_4 : \begin{pmatrix} X & X & 0 \\ X & X & X \\ 0 & X & 0 \end{pmatrix}$$

$$C : \begin{pmatrix} X & X & X \\ X & 0 & X \\ X & X & 0 \end{pmatrix}$$

Frampton, Glashow, Marfatia (2002)
Xing (2002)
Merle, Rodejohann (2006)
Goswami et. al (2006)

Predictions for Model A1



Z. Devi, S. Goswami (2014)
J. Liao, D. Marfatia, K. Whisnant (2014), (2015)

Non-standard Neutrino Interactions

Neutrinos may have nonstandard interactions with electrons and/or u, d quarks

$$\mathcal{L}^{NSI} = -2\sqrt{2}G_F(\overline{\nu}_\alpha\gamma_\mu\nu_\beta)\left[\epsilon^{fL}_{\alpha\beta}(\overline{f}_L\gamma^\mu f_L) + \epsilon^{fR}_{\alpha\beta}(\overline{f}_R\gamma^\mu f_R)\right] + h..c$$

Matter potential for neutrino oscillaion becomes

$$H = \sqrt{2}G_F n_e \begin{pmatrix} 1+\epsilon_{ee} & \epsilon^*_{e\mu} & \epsilon^*_{e\tau} \\ \epsilon_{e\mu} & \epsilon_{\mu\mu} & \epsilon^*_{\mu\tau} \\ \epsilon_{e\tau} & \epsilon_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix}$$

L. Wolfenstein (1978)

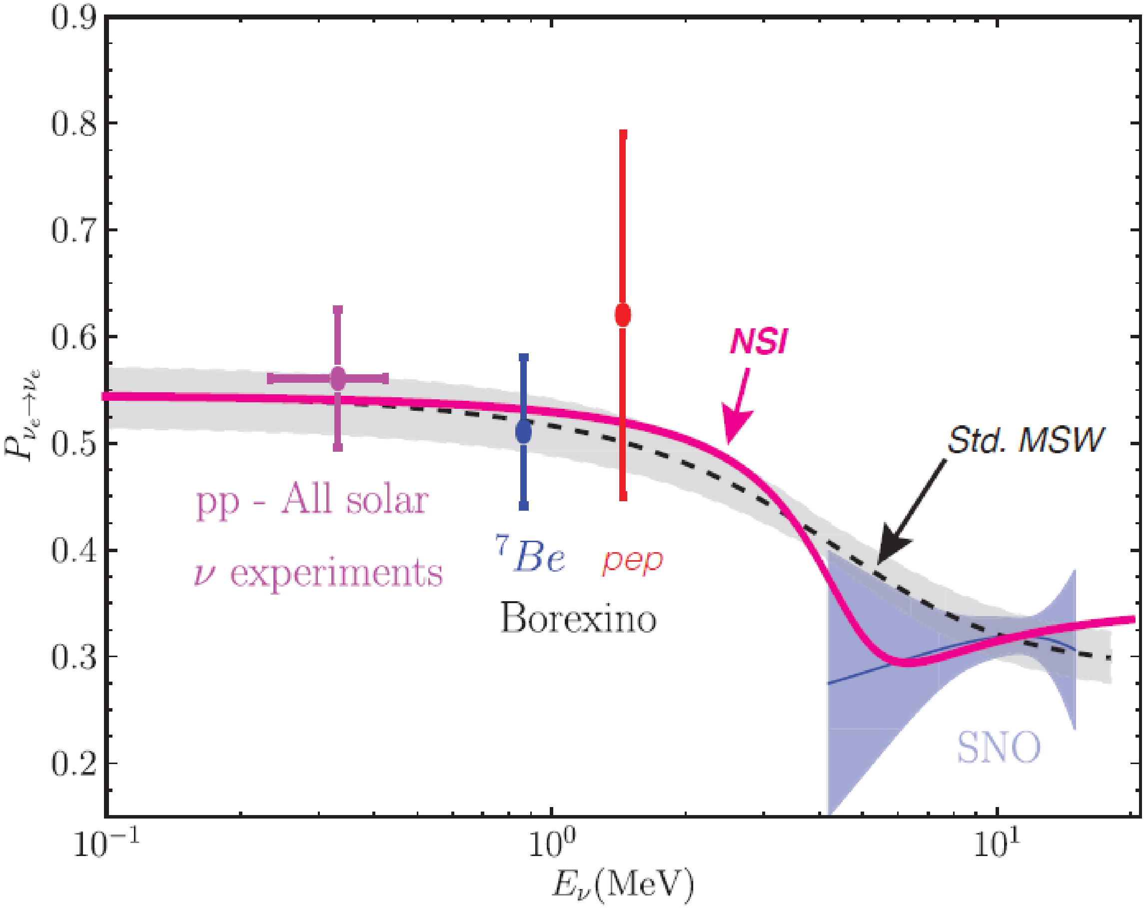
ϵ's constrained by accelerator data, charged lepton flavor violation, and by neutrino oscillations

Global Constraints on NSI

		90% CL		3σ	
Param.	best-fit	LMA	LMA $\oplus$ LMA-D	LMA	LMA $\oplus$ LMA-D
$\varepsilon^u_{ee} - \varepsilon^u_{\mu\mu}$	$+0.298$	$[+0.00, +0.51]$	$\oplus\ [-1.19, -0.81]$	$[-0.09, +0.71]$	$\oplus\ [-1.40, -0.68]$
$\varepsilon^u_{\tau\tau} - \varepsilon^u_{\mu\mu}$	$+0.001$	$[-0.01, +0.03]$	$[-0.03, +0.03]$	$[-0.03, +0.20]$	$[-0.19, +0.20]$
$\varepsilon^u_{e\mu}$	-0.021	$[-0.09, +0.04]$	$[-0.09, +0.10]$	$[-0.16, +0.11]$	$[-0.16, +0.17]$
$\varepsilon^u_{e\tau}$	$+0.021$	$[-0.14, +0.14]$	$[-0.15, +0.14]$	$[-0.40, +0.30]$	$[-0.40, +0.40]$
$\varepsilon^u_{\mu\tau}$	-0.001	$[-0.01, +0.01]$	$[-0.01, +0.01]$	$[-0.03, +0.03]$	$[-0.03, +0.03]$
ε^u_{D}	-0.140	$[-0.24, -0.01]$	$\oplus\ [+0.40, +0.58]$	$[-0.34, +0.04]$	$\oplus\ [+0.34, +0.67]$
ε^u_{N}	-0.030	$[-0.14, +0.13]$	$[-0.15, +0.13]$	$[-0.29, +0.21]$	$[-0.29, +0.21]$
$\varepsilon^d_{ee} - \varepsilon^d_{\mu\mu}$	$+0.310$	$[+0.02, +0.51]$	$\oplus\ [-1.17, -1.03]$	$[-0.10, +0.71]$	$\oplus\ [-1.44, -0.87]$
$\varepsilon^d_{\tau\tau} - \varepsilon^d_{\mu\mu}$	$+0.001$	$[-0.01, +0.03]$	$[-0.01, +0.03]$	$[-0.03, +0.19]$	$[-0.16, +0.19]$
$\varepsilon^d_{e\mu}$	-0.023	$[-0.09, +0.04]$	$[-0.09, +0.08]$	$[-0.16, +0.11]$	$[-0.16, +0.17]$
$\varepsilon^d_{e\tau}$	$+0.023$	$[-0.13, +0.14]$	$[-0.13, +0.14]$	$[-0.38, +0.29]$	$[-0.38, +0.35]$
$\varepsilon^d_{\mu\tau}$	-0.001	$[-0.01, +0.01]$	$[-0.01, +0.01]$	$[-0.03, +0.03]$	$[-0.03, +0.03]$
ε^d_{D}	-0.145	$[-0.25, -0.02]$	$\oplus\ [+0.49, +0.57]$	$[-0.34, +0.05]$	$\oplus\ [+0.42, +0.70]$
ε^d_{N}	-0.036	$[-0.14, +0.12]$	$[-0.14, +0.12]$	$[-0.28, +0.21]$	$[-0.28, +0.21]$

Gonzalez-Garcia, Maltoni (2014)

NSI and solar neutrino oscillations



$\epsilon_{e\tau} = 0.4$

Friedland, Shoemaker (2012)

Neutrino NSI from light mediators

Effective operators are severely constrained once SU(2) invariance is imposed – from charged lepton flavor violation and universality

Light vector boson mediators can avoid this problem

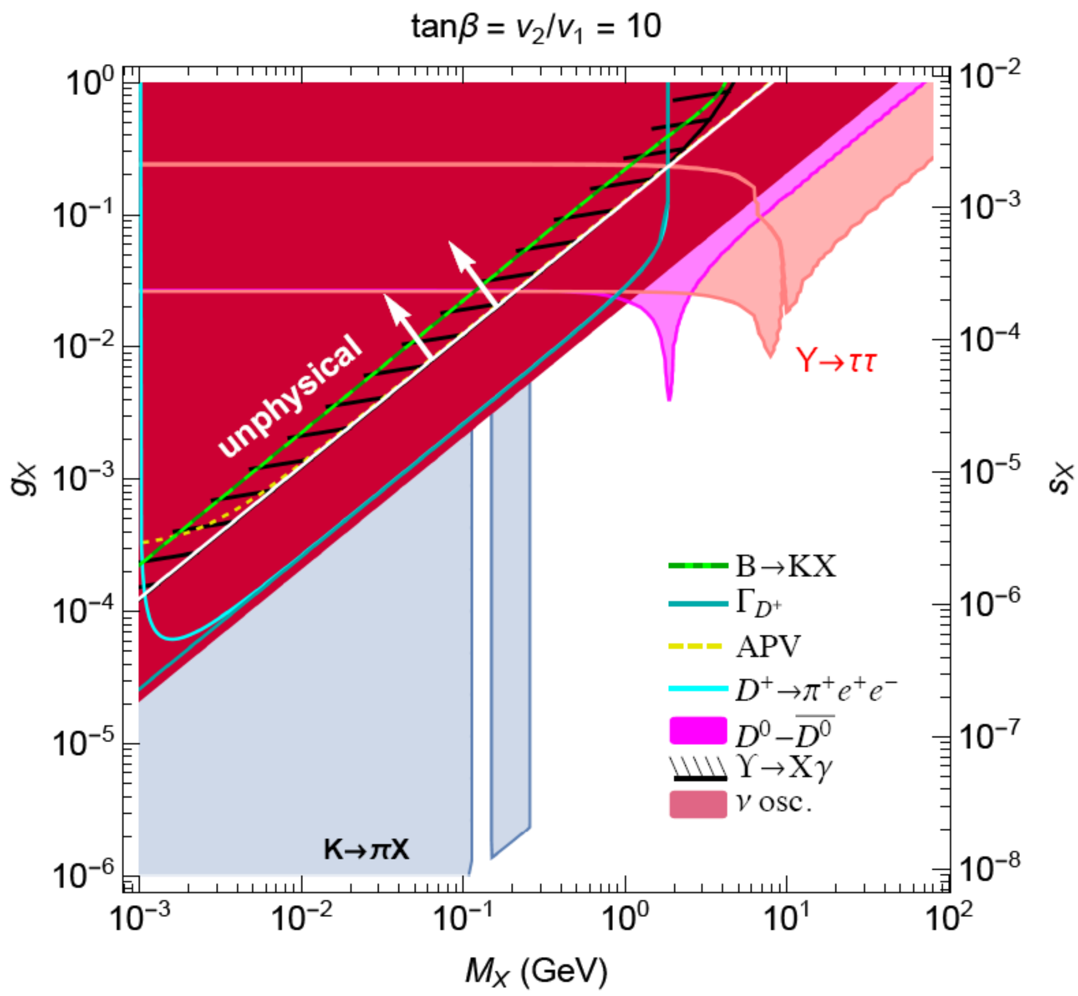
Explicit Model: Gauged $B - L$ for third family

For $g_X \sim 10^{-3}$ and $M_X \sim 100$ MeV, large $\epsilon_{\tau\tau}$ can be induced

Matter potential in neutrino oscillations is a $q^2 = 0$ effect

Friedland, Machado, Mocioiu, KSB (2017)

Consistency checks of light mediator



Summary and Conclusions

- Variety of seesaw mechanisms, some testable in colliders
- A class of GUTs naturally predict large neutrino mixings, including large θ_{13}
- New particles at TeV expected in many scenarios of radiative neutrino mass generation
- Lepton flavor violation provides complementary information on neutrino mass generation
- Zeros in neutrino mass matrix quite consistent and leads typically to large θ_{13}
- Neutrino CP violation, $m_{\beta\beta}^{0\nu}$, and mass hierarchy measurements will greatly enhance our fundamental understanding of Nature

Perspective

- We have taken a long journey through particle physics. Many new ideas were introduced, many more are waiting to be discovered (by you!)
- We went over the established mechanism of electroweak symmetry breaking; but we explored many new ideas and models that modify the standard paradigm
- Flavor sector is very rich; we still need to understand the observations better. Some well-motivated attempts were introduced
- Grand Unified Theories are beautiful, we studied them and tried to connect to experiments
- Neutrino oscillations will play an important role in particle physics. We may be up for some surprises here
- I hope these lectures and discussions will inspire you to think “out of the box” and come up with revolutionary ideas that may be Nature’s choice

Thank You!