Renormalization and Extensions of the Standard Model

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GIAN Course on Electrowak Symmetry Breaking, Flavor Physics and BSM

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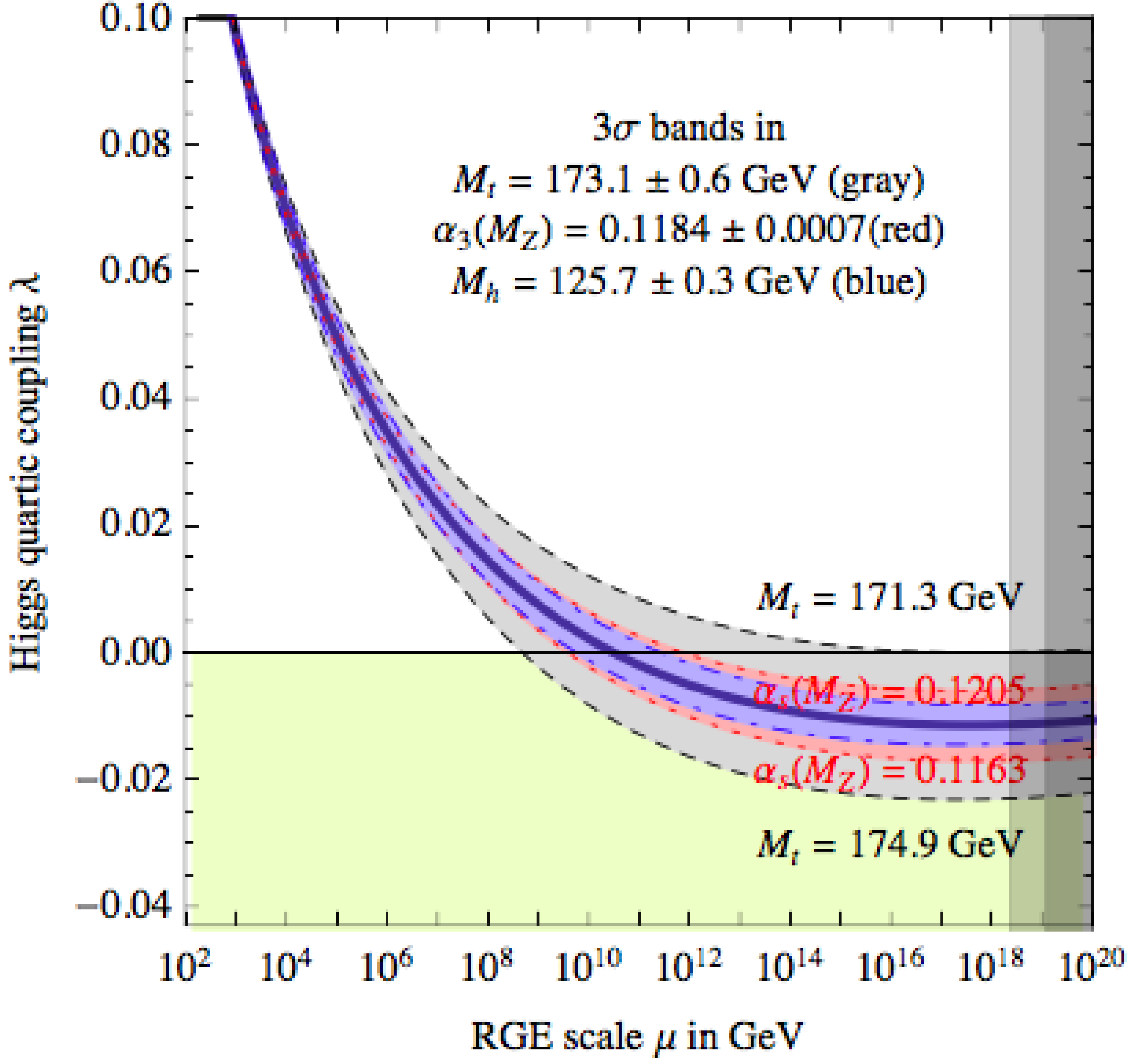
Stability of the Higgs Potential

- Stability of SM Higgs potential requires $\lambda > 0$
- When extrapolated to high energy, λ turns negative!
- This may be an indication that there is new physics below this scale
- How do we understand the running of the couplings?
- Answer relies on the renormalization group



De Grassi et. al. (2012)

Running of Coupling "Constants"

- In Quantum Field Theory, loop diagrams lead to divergences in amplitudes
- These divergences have logarithmic dependence on the cut-off scale Λ
- In renormalizable theories, these divergences can be absorbed into "bare coupling and mass parameters"
- The "renormalized couplings and masses" are devoid of infinities
- Physical parameters should be independent of cut-off scale Λ. This requirement leads to Log(μ) dependence of physical parameters
- The Log(μ) dependence arises from the "Renormalization Group"
- All that is needed to deduce running of couplings is to collect Log divergent piece in the amplitude

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}_e (i D_\mu \gamma^\mu - m) \psi_e$$

Example of Divergence Calculation

Let us evaluate this 4-point scalar loop diagram

$$-i\mathcal{M} = i.i.i.i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)^2}$$

$$= \frac{i}{16\pi^2} \int_0^{\Lambda^2} \frac{k_E^2 dk_E^2}{(k_E^2 + m^2)^2}$$

$$= \frac{i}{16\pi^2} Log\left(\frac{\Lambda^2}{m^2}\right)$$

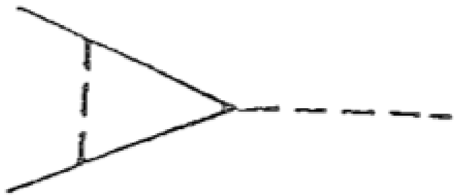
$$\mathcal{M} = \frac{-1}{8\pi^2} Log[\Lambda]$$

$$\mathcal{L} = \frac{1}{8\pi^2} Log[\Lambda]$$

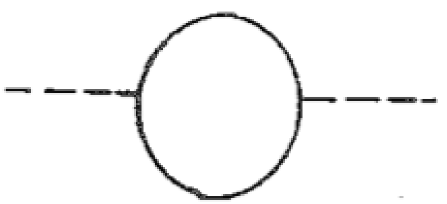
$\Rightarrow$ RGE factor is $\frac{1}{8\pi^2}$

Full RGE factors for one-loop diagrams

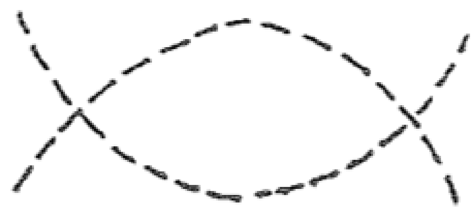
$-\frac{3}{2}$



1



$\frac{1}{2}$

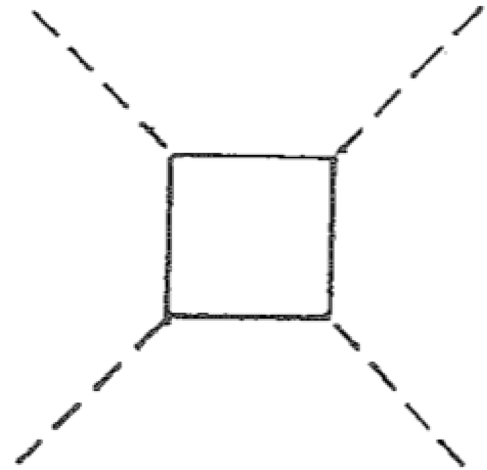


1

Must multiply by $\frac{1}{8\pi^2}$

Full RGE factors for one-loop diagrams

$\frac{1}{4}$



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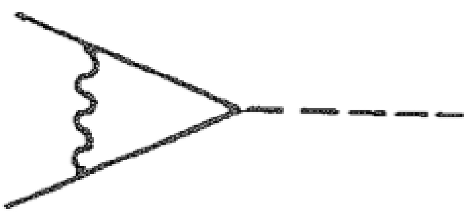

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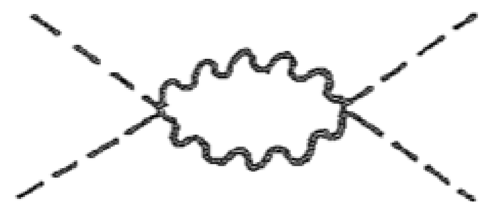


3

Must multiply by $\frac{1}{8\pi^2}$

RGE for Higgs Quartic Coupling

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2$$

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) = \left| (\partial_\mu - \frac{ig_1 B_\mu}{2} - \frac{ig_2 \sigma^a A^a_\mu}{2}) \right|^2$$

A^a_μ here is the $SU(2)_L$ gauge bosons (dented as W^a_μ earlier)

Tree-level couplings of the Higgs boson can be derived from here

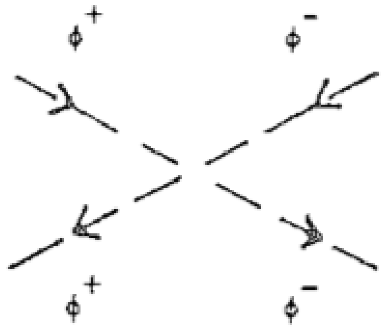
One can use a basis where Φ contains ϕ^+ and ϕ^0

Loop amplitude for quartic couplings can be constructed from here

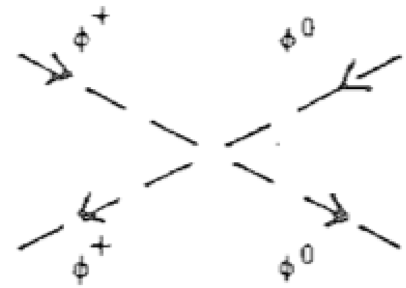
$$\phi^+ \phi^- \to \phi^+ \phi^-$$

$$\phi^+ \phi^+ \to \phi^+ \phi^+$$

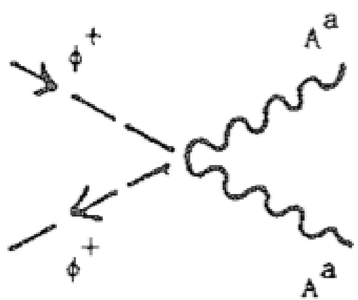
Higgs self interactions and with gauge bosons



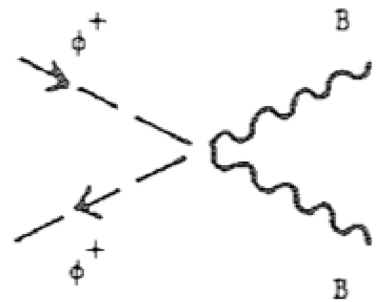
$\frac{1}{2}\lambda.2.2 = 2\lambda$



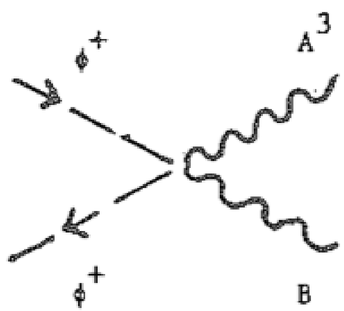
λ



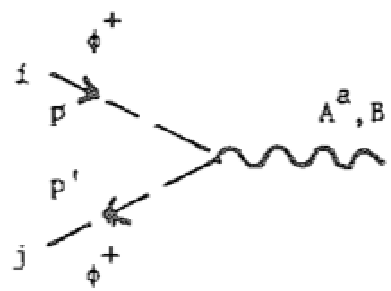
$2.\frac{1}{4}g_2^2 = \frac{g_2^2}{2}$



$2.\frac{1}{4}g_1^2 = \frac{g_1^2}{2}$

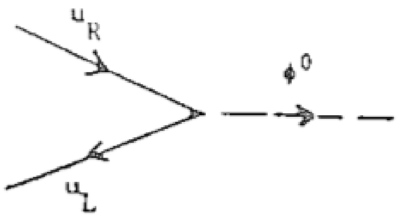


$\frac{1}{2}g_1g_2$

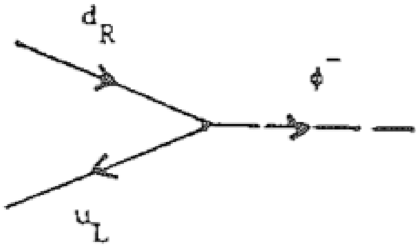


$ig_{2,1}(p+p')^{\mu}T^a_{ij}$

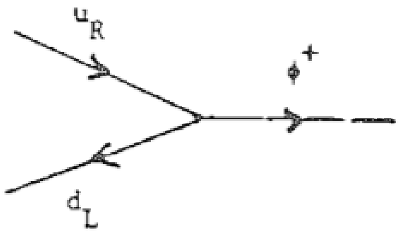
Higgs interactions with fermions



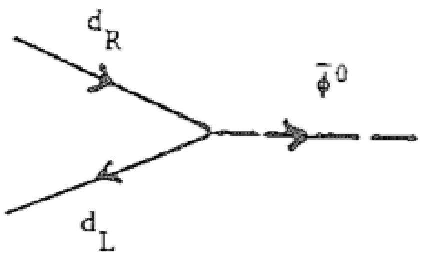
h_u



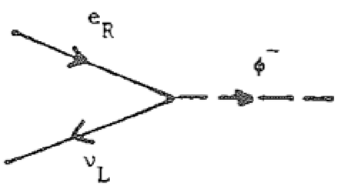
h_d



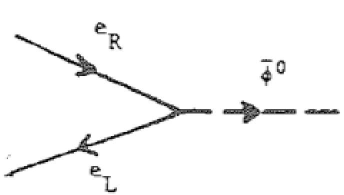
$-h_u$



h_d

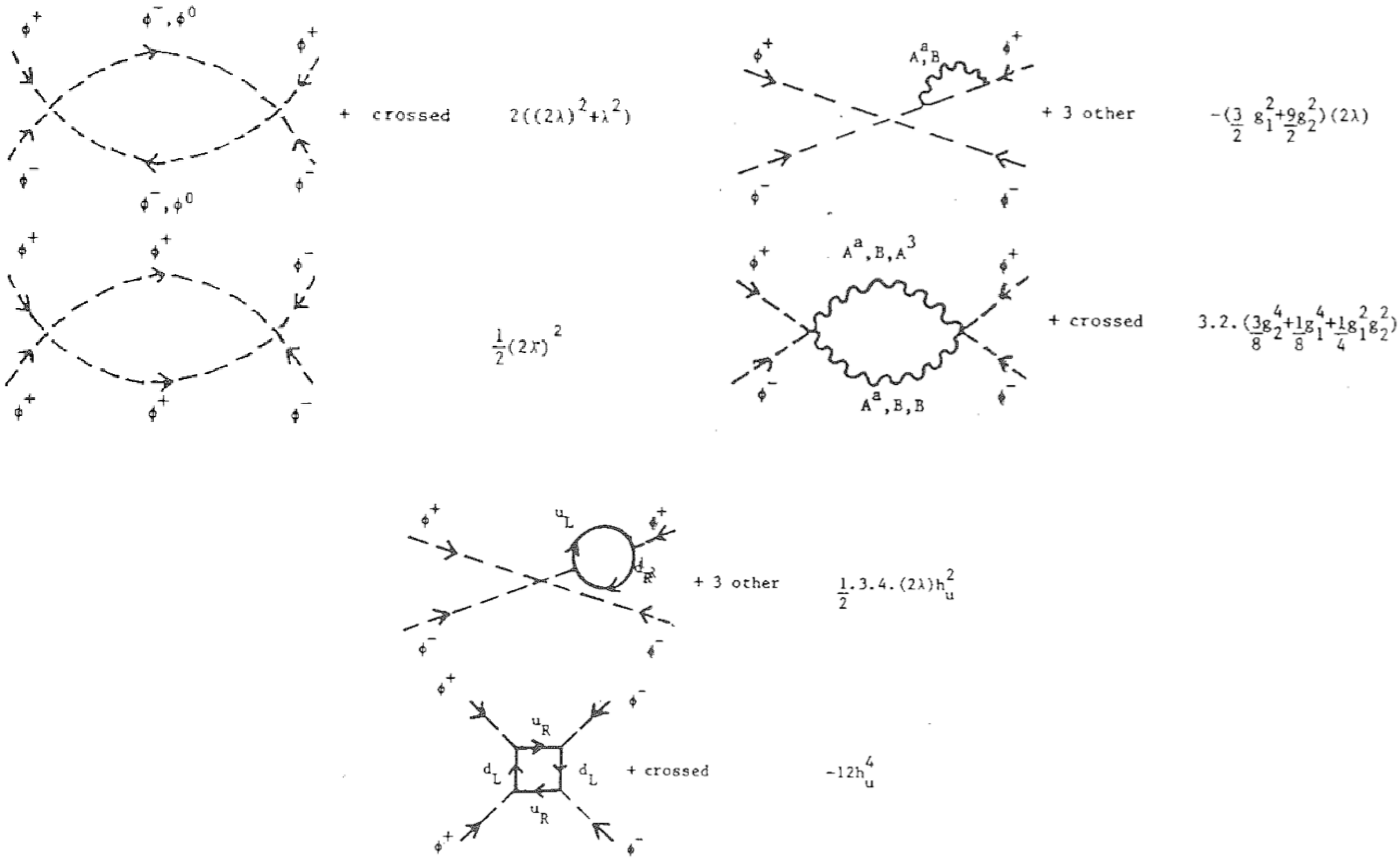


h_e



h_e

Loop corrections to Higgs self coupling



RGE for the Higgs self coupling

Combining all loop diagrams, we get

$$8\pi^2 \frac{d\lambda}{dt} = 6\lambda^2 - \lambda(\frac{3}{2}g_1^2 + \frac{9}{2}g_2^2) + \frac{3}{8}g_1^4 + \frac{9}{8}g_2^4 + \frac{3}{4}g_1^2 g_2^2 + 6h_u^2\lambda - 6h_u^4$$

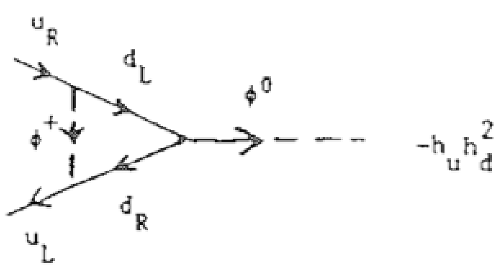
Here $t = Log(\mu)$

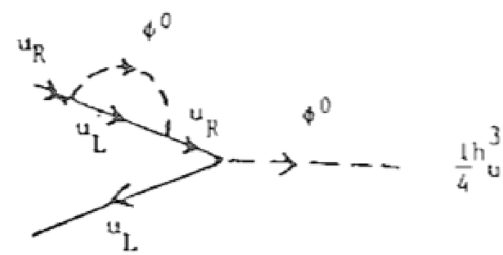
The $(-6h_u^4)$ term pulls λ to negative value at high energy!

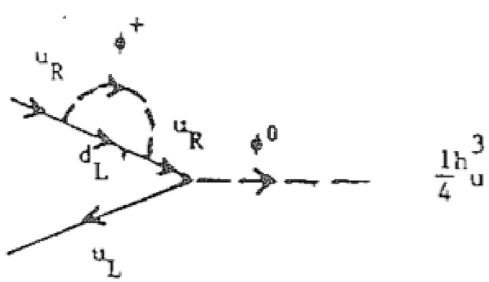
To solve this RGE, we need to know the RGE for $g_{1,2}$ and h_u

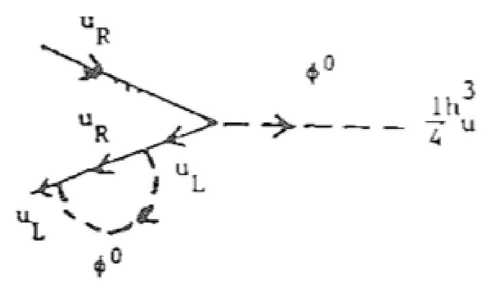
RGE for the Yukawa couplings

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= h_u(\overline{u}_L u_R \overline{\phi^0} - \overline{d}_L u_R \phi^-) + h_d(\overline{u}_L d_R \phi^+ + \overline{d}_L d_R \phi^0) \\ &+ h_e(\overline{\nu}_L e_R \phi^+ + \overline{e}_L e_R \phi^0) + H.C.\end{aligned}$$

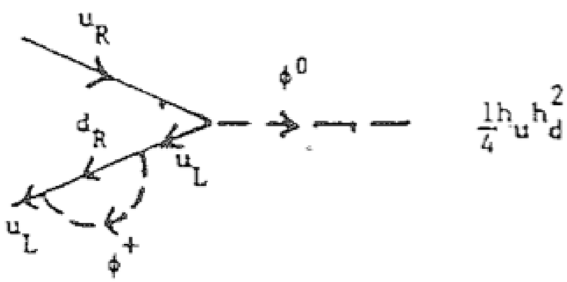


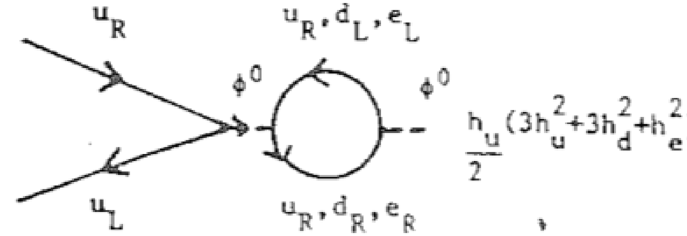


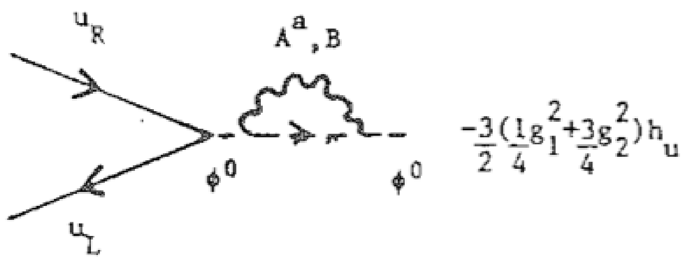


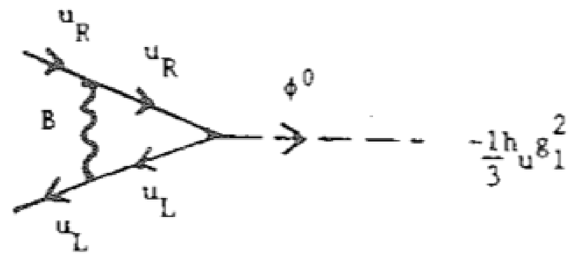


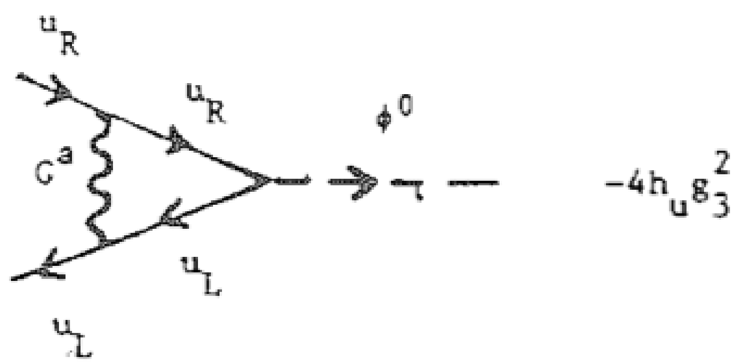
RGE for the Yukawa couplings (cont.)









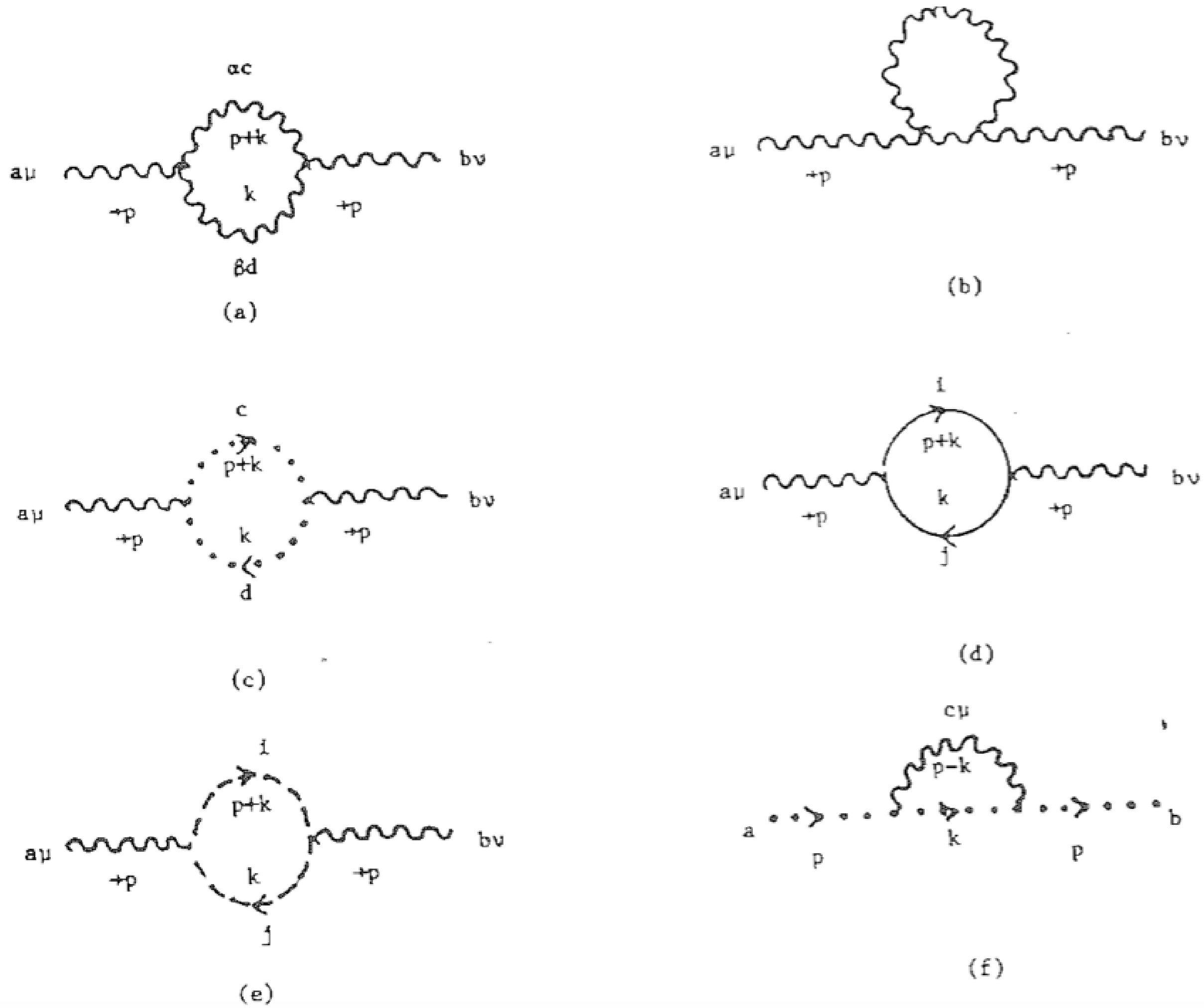


RGE for the Yukawa couplings (cont.)

$$
\begin{aligned}
8\pi^2 \frac{dh_u^2}{dt} &= h_u^2\left(\frac{9}{2}h_u^2 + \frac{3}{2}h_d^2 + h_e^2 - \frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right) \\
8\pi^2 \frac{dh_d^2}{dt} &= h_d^2\left(\frac{9}{2}h_d^2 + \frac{3}{2}h_u^2 + h_e^2 - \frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right) \\
8\pi^2 \frac{dh_e^2}{dt} &= h_5^2\left(\frac{9}{2}h_e^2 + 3h_u^2 + 3h_d^2 - \frac{15}{4}g_1^2 - \frac{9}{4}g_2^2\right)
\end{aligned}
$$

RGE for the gauge couplings

This only depends on gauge group and particle content



RGE for the gauge couplings (cont.)

$$\beta(g) = -\frac{g^3}{16\pi^2}\left(\tfrac{11}{3}C_2(G) - \tfrac{2}{3}T(R) - \tfrac{1}{3}T(S)\right)$$

G: Gauge boson; R: Fermion; S: Scalar

$$\begin{aligned} C_2(G) &= N \text{ for } SU(N), 0 \text{ for } U(1) \\ C_2(R) &= \frac{1}{2} \text{ for fundamental of } SU(N), y^2 \text{ for } U(1) \\ T(S) &= \frac{1}{2} \text{ for fundamental of } SU(N), y^2 \text{for } U(1) \end{aligned}$$

$$\begin{aligned} \beta_{g_1} &= \frac{g_1^3}{16\pi^2}\left(\frac{20}{9}n_G + \frac{1}{6}n_H\right) \\ \beta_{g_2} &= -\frac{g_2^3}{16\pi^2}\left(\frac{22}{3} - \frac{4}{3}n_G - \frac{1}{6}n_H\right) \\ \beta_{g_3} &= -\frac{g_3^3}{16\pi^2}\left(11 - \frac{4}{3}n_G\right) \end{aligned}$$

Standard Model Extensions

Motivations

- Neutrino masses
- Dark matter of the universe
- Explain the baryon asymmetry of the universe
- Explain the hierarchy problem
- Flavor puzzle
- The strong CP problem
- - - -

Neutrino Flavor Oscillations

Neutrino flavor eigenstates are admixtures of mass eigenstates

$$\begin{aligned} \nu_e &= \cos\theta\ \nu_1 + \sin\theta\ \nu_2 \\ \nu_\mu &= -\sin\theta\ \nu_1 + \cos\theta\ \nu_2 \end{aligned}$$

$$P(\nu_e \to \nu_\mu;\ L) = \sin^2 2\theta\ \ \sin^2(\frac{\Delta m^2 L}{4E})$$

$$= \sin^2 2\theta\ \ \sin^2\left\{1.27\left(\frac{\Delta m^2}{\mathrm{eV}^2}\right)\left(\frac{L}{\mathrm{km}}\right) / \left(\frac{E}{\mathrm{GeV}}\right)\right\}$$

$$\Delta m^2 = m_2^2 - m_1^2$$

For $\Delta m^2 \sim 10^{-3}$ eV^2, $\sin\theta \sim 1$, $L \sim 10^3$ km, $E \sim$ GeV, oscillation probability is of order one

A global fit to neutrino oscillation data

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 0.83$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.271 \to 0.345$
$\theta_{12}/^\circ$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$31.38 \to 35.99$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.385 \to 0.635$	$0.587^{+0.020}_{-0.024}$	$0.393 \to 0.640$	$0.385 \to 0.638$
$\theta_{23}/^\circ$	$41.6^{+1.5}_{-1.2}$	$38.4 \to 52.8$	$50.0^{+1.1}_{-1.4}$	$38.8 \to 53.1$	$38.4 \to 53.0$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.01934 \to 0.02392$	$0.02179^{+0.00076}_{-0.00076}$	$0.01953 \to 0.02408$	$0.01934 \to 0.02397$
$\theta_{13}/^\circ$	$8.46^{+0.15}_{-0.15}$	$7.99 \to 8.90$	$8.49^{+0.15}_{-0.15}$	$8.03 \to 8.93$	$7.99 \to 8.91$
$\delta_{CP}/^\circ$	261^{+51}_{-59}	$0 \to 360$	277^{+40}_{-46}	$145 \to 391$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.03 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.524^{+0.039}_{-0.040}$	$+2.407 \to +2.643$	$-2.514^{+0.038}_{-0.041}$	$-2.635 \to -2.399$	$\begin{bmatrix} +2.407 \to +2.643 \\ -2.629 \to -2.405 \end{bmatrix}$

Estaban, Gonzalez-Garcia, Maltoni, Martinze-Soler, Schwetz (2017)

Global data has a slight preference for $\delta_{CP} \sim 3\pi/2$, with $\delta_{CP} = 0$ excluded at 90% CL by recent T2K results

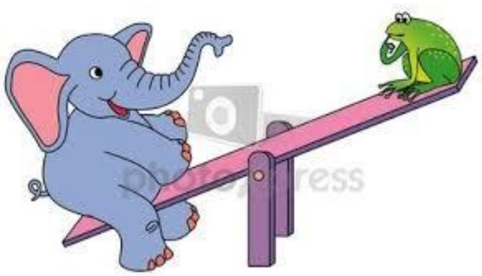
Seesaw Mechanism for Neutrino Masses

Add right-handed neutrino to the SM

(ν, ν^c) mass matrix: $\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$

With $M_R \gg m_D$, the mass eigenvalues are:

$$m_\nu \simeq -\frac{m_D^2}{M_R}, \qquad M_{\text{heavy}} \simeq M_R$$



Minkowski (1977)
Gell-Mann, Ramond, Slansky (1979)
Yanagida (1979)
Mohapatra, Senjanovic (1979)

$$m_D \approx 10^2\ \text{GeV}, M_R \approx 10^{14}\ \text{GeV} \Rightarrow m_\nu \approx 0.05\ \text{eV}$$

$$m_D \approx \text{MeV}, M_R \approx \text{TeV} \Rightarrow m_\nu \approx \text{eV}$$

ν^c with this mass can also produce baryon asymmetry of universe

Evolution of Higgs mass parameter in SM

$$V(\phi) = m_\phi^2 \phi^\dagger \phi + \frac{\lambda}{2}(\phi^\dagger \phi)^2$$

$$16\pi^2 \frac{dm_\phi^2}{dt} = m_\phi^2 \left(6\lambda + 2\mathrm{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e) - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2\right)$$

If m_ϕ^2 is positive at a high energy, it will remain so at low energy

If usual type-I seesaw mechanism is implemented, m_ϕ^2 becomes more positive at low energy

$$16\pi^2 \frac{dm_\phi^2}{dt} = 16\pi^2 \left(\frac{dm_\phi^2}{dt}\right)_{\mathrm{SM}} - 4\mathrm{Tr}(Y_\nu Y_\nu^\dagger M_R^\dagger M_R)$$

M_R cannot exceed 10^7 GeV, or else the Higgs mass is fine-tuned

Vissani (1997)

New TeV scale scalars can turn Higgs (mass)^2 negative

Variety of Seesaw Mechanisms

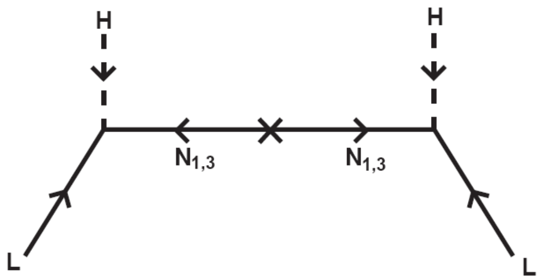
Effective neutrino mass operator:

$$\mathcal{L}_{\text{eff}} = \frac{LLHH}{M} \;\Rightarrow\; m_\nu \sim \frac{v^2}{M}$$

Neutrino oscillations can probe $M \sim 10^{14}$ GeV

Type (I,III) seesaw

Minkowski (1977)
Yanagida (1979)
Gell-Mann, Ramond, & Slansky (1980)
Mohapatra & Senjanovic (1980)

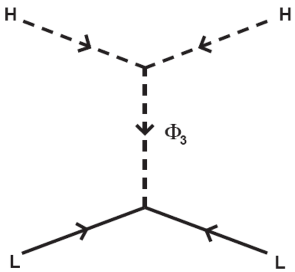


$N_1 : (1,1,0)$, $N_3 : (1,3,0)$

Foot, Lew, He, & Joshi (1989)

Type II seesaw

Mohapatra & Senjanovic (1980)
Schechter & Valle (1980)
Lazarides, Shafi, & Wetterich (1981)



$\Phi_3 : (1,3,+1)$

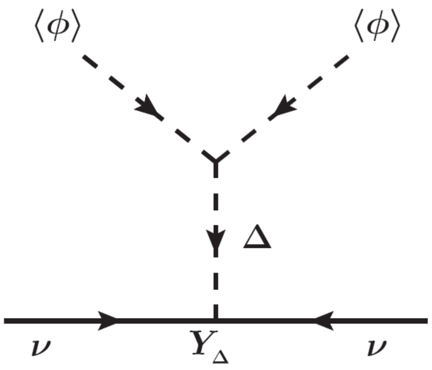
Type-II Seesaw Model

Introduces a weak triplet $\Delta(1,3,+2)$

$$\Delta = \frac{\sigma_i}{\sqrt{2}}\,\Delta_i = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^0 & \frac{\Delta^+}{\sqrt{2}} \end{pmatrix}$$

Neutrino Yukawa couplings:

$$\mathcal{L}_Y \supset -\frac{(\mathbf{Y}_\Delta)_{ij}}{\sqrt{2}}\,\ell_i^T C\, i\sigma_2\, \Delta\, \ell_j + h.c.$$



Δ^0 acquries an induced VEV

$$\langle\Delta\rangle = \frac{\mu v^2}{\sqrt{2}\mu_\Delta^2} \ll v \qquad \mathbf{m}_\nu \simeq \mathbf{Y}_\Delta\,\frac{\mu\, v^2}{2\mu_\Delta^2}$$

Same sign dilepton decay of Δ^{++} best test

Type-II seesaw models: Higgs potential

A scalar triplet $\Delta(1,3,1)$ acquires a small VEV and induces tiny Majorana neutrino masses directly.

$$\mathcal{L}_{\text{Yuk}} = -Y_{ij}^{\Delta}\ell_i^T C i\tau_2 \Delta \ell_j + h.c.$$

Higgs potential:

$$\begin{aligned} V &= m_\phi^2 \phi^\dagger\phi + m_\Delta^2 \text{Tr}(\Delta^\dagger\Delta) + \frac{\lambda}{2}(\phi^\dagger\phi)^2 + \frac{\Lambda_1}{2}(\text{Tr}(\Delta^\dagger\Delta))^2 \\ &+ \frac{\Lambda_2}{2}\left[(\text{Tr}(\Delta^\dagger\Delta))^2 - \text{Tr}(\Delta^\dagger\Delta\Delta^\dagger\Delta\right] + \Lambda_4\phi^\dagger\phi\text{Tr}(\Delta^\dagger\Delta) \\ &+ \Lambda_5\phi^\dagger[\Delta^\dagger,\Delta]\phi + \{\frac{\mu}{\sqrt{2}}\phi^T i\tau_2\Delta^\dagger\phi + h.c.\} \end{aligned}$$

Boundedness conditions:

$$\lambda > 0, \quad \Lambda_1 > 0, \quad \Lambda_1 + \frac{\Lambda_2}{2} > 0, \quad \Lambda_4 \pm \Lambda_5 + 2\sqrt{\lambda\Lambda_1} > 0$$

$$\Lambda_4 \pm \Lambda_5 + 2\sqrt{\lambda\left(\Lambda_1 + \frac{\Lambda_2}{2}\right)} > 0$$

Arhrib et al (2001)
Babu,Gogoldze, Khan (2016)

Evolution of mass parameters in type-II seesaw models

$$16\pi^2 \frac{dm_\phi^2}{dt} = m_\phi^2\left(-\frac{9}{10}g_1^2 - \frac{9}{2}g_2^2 + 2\mathrm{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e)\right) + m_\Delta^2(6\lambda_4) + 6\mu^2$$

$$16\pi^2 \frac{dm_\Delta^2}{dt} = m_\Delta^2\left(-\frac{18}{5}g_1^2 - 12g_2^2 + 2\mathrm{Tr}(Y_\Delta^\dagger Y_\Delta) + 8\Lambda_1 + 2\Lambda_2\right) + m_\phi^2(4\Lambda_4) + 2\mu^2$$

$$16\pi^2\dot{\lambda} = 6\lambda^2 - 3\lambda\left(3g_2^2 + \frac{3}{5}g_1^2\right) + 3g_2^4 + \frac{3}{2}\left(\frac{3}{5}g_1^2 + g_2^2\right)^2 + 4\lambda T - 8H + 12\Lambda_4^2 + 8\Lambda_5^2$$

$$16\pi^2\dot{\Lambda}_1 = -\frac{36}{5}g_1^2\Lambda_1 - 24g_2^2\Lambda_1 + \frac{108}{25}g_1^4 + 18g_2^4 + \frac{72}{5}g_1^2g_2^2 + 14\Lambda_1^2 + 4\Lambda_1\Lambda_2 + 2\Lambda_2^2 + 4\Lambda_4^2 + 4\Lambda_5^2 + 4\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\Lambda_1 - 8\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\Delta^\dagger Y_\Delta\right)$$

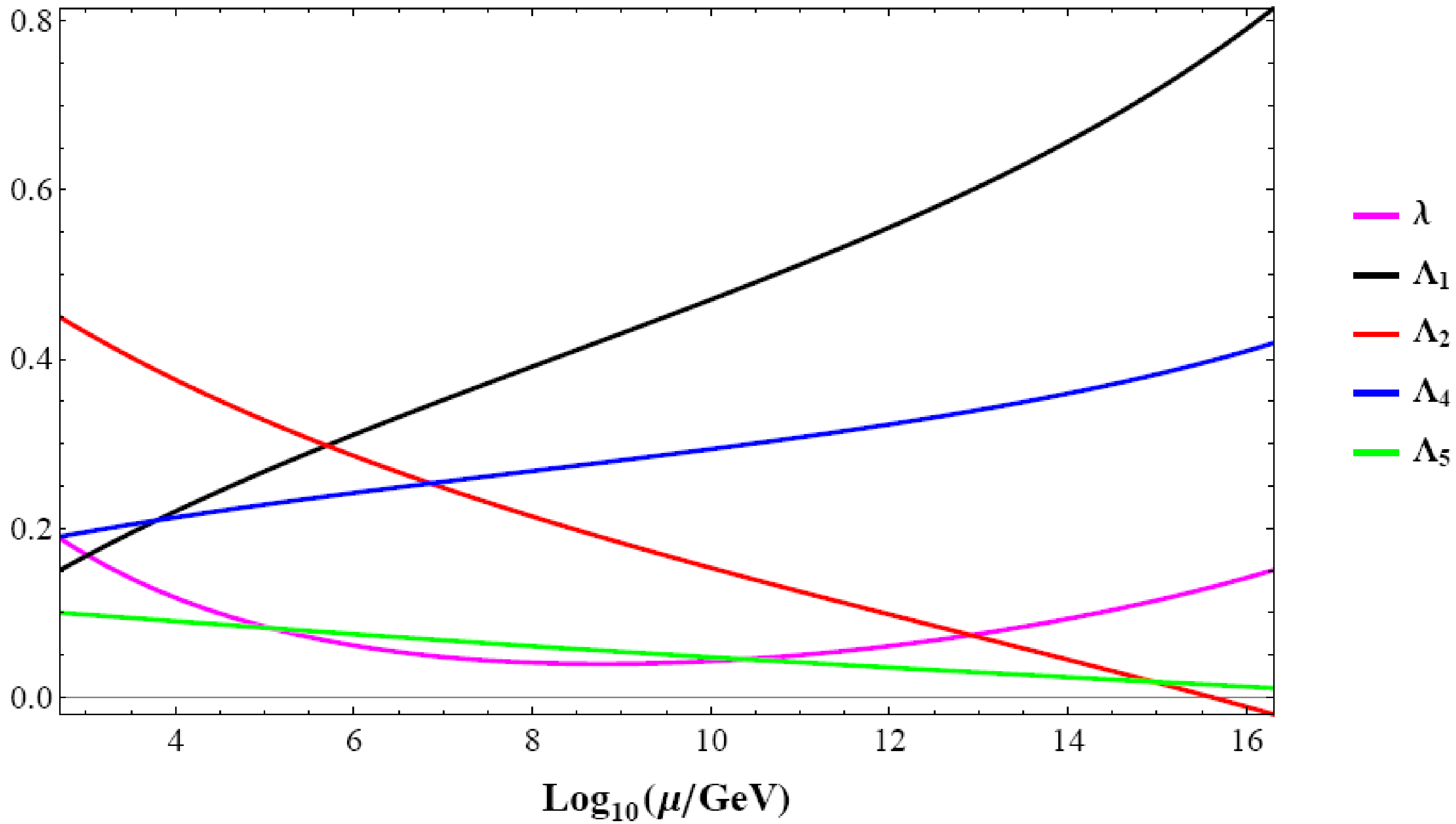
$$16\pi^2\dot{\Lambda}_2 = -\frac{36}{5}g_1^2\Lambda_2 - 24g_2^2\Lambda_2 + 12g_2^4 - \frac{144}{5}g_1^2g_2^2 + 3\Lambda_2^2 + 12\Lambda_1\Lambda_2 - 8\Lambda_5^2 + 4\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\Lambda_2 + 8\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta Y_\Delta^\dagger Y_\Delta\right)$$

$$16\pi^2\dot{\Lambda}_4 = -\frac{9}{2}g_1^2\Lambda_4 - \frac{33}{2}g_2^2\Lambda_4 + \frac{27}{25}g_1^4 + 6g_2^4 + 8\Lambda_1 + 2\Lambda_2 + 3\lambda + 4\Lambda_4 + 2T + 2\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\Lambda_4 + 8\Lambda_5^2$$

$$16\pi^2\dot{\Lambda}_5 = -\frac{9}{2}g_1^2\Lambda_5 - \frac{33}{2}g_2^2\Lambda_5 - \frac{18}{5}g_1^2g_2^2 + 2\Lambda_1 - 2\Lambda_2 + \lambda + 8\Lambda_4 + 2T + 2\,\mathrm{tr}\left(Y_\Delta^\dagger Y_\Delta\right)\Lambda_5$$

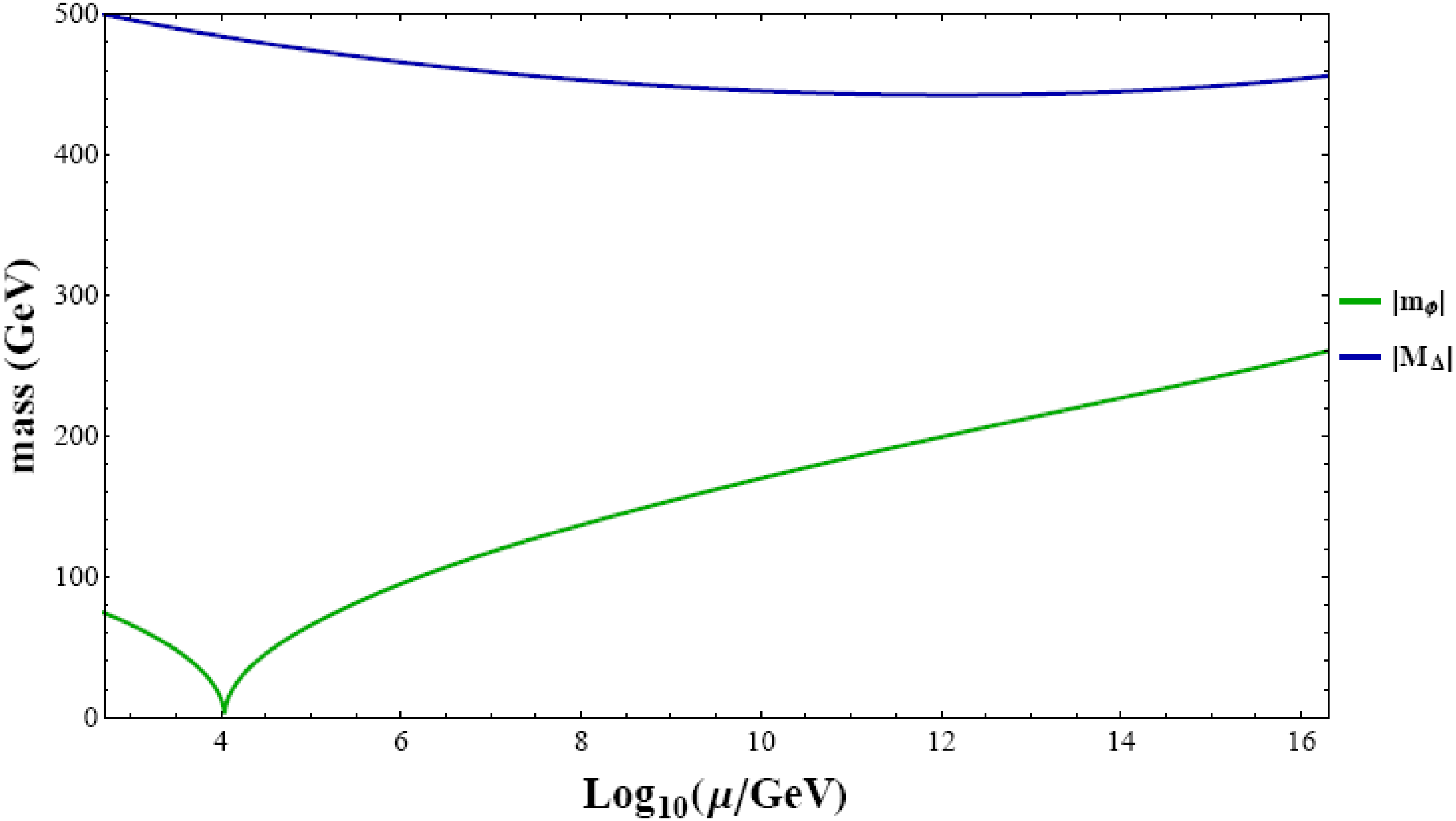
M. Schmidt (2007)
Babu, Gogoladze, Khan (2016)

Bounded quartic coupling evolution in type-II seesaw models



$M_\triangle = 500\,\mathrm{GeV}, \quad \Lambda_1(M_\triangle) = 0.15, \quad \Lambda_2(M_\triangle) = 0.45, \quad \Lambda_4(M_\triangle) = 0.19,$
$\Lambda_5(M_\triangle) = 0.1, \quad \mu(M_\triangle) = 10^{-5}\,\mathrm{GeV}, \quad \lambda(M_Z) = 0.258, \quad \lambda(M_\triangle) = 0.1887$

Turning of Higgs mass-squared in type-II seesaw models



Masses of Δ fields should be below a TeV

Singlet extension of SM for dark matter

A real scalar singlet S is introduced with a discrete Z_2 symmetry that remains unbroken

S is the dark matter, which annihilates through the Higgs portal

Silveira, Zee (1984), McDonald (1994), Bento, Bertolamii, Rosenfeld, Teodoro (2000), Burgess, Pospelov, ter Veldhuis (2001), Davoudiasl, Kitano, Li, Murayama (2005), Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy (2008), He, Ho, Tandem, Tsai (2010)

Higgs potential:

$$V = m_\phi^2 \phi^\dagger \phi + \frac{m_S^2}{2} S^2 + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{8} S^2 + \frac{\lambda_3}{2} \phi^\dagger \phi S^2$$

Boundedness conditions:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}$$

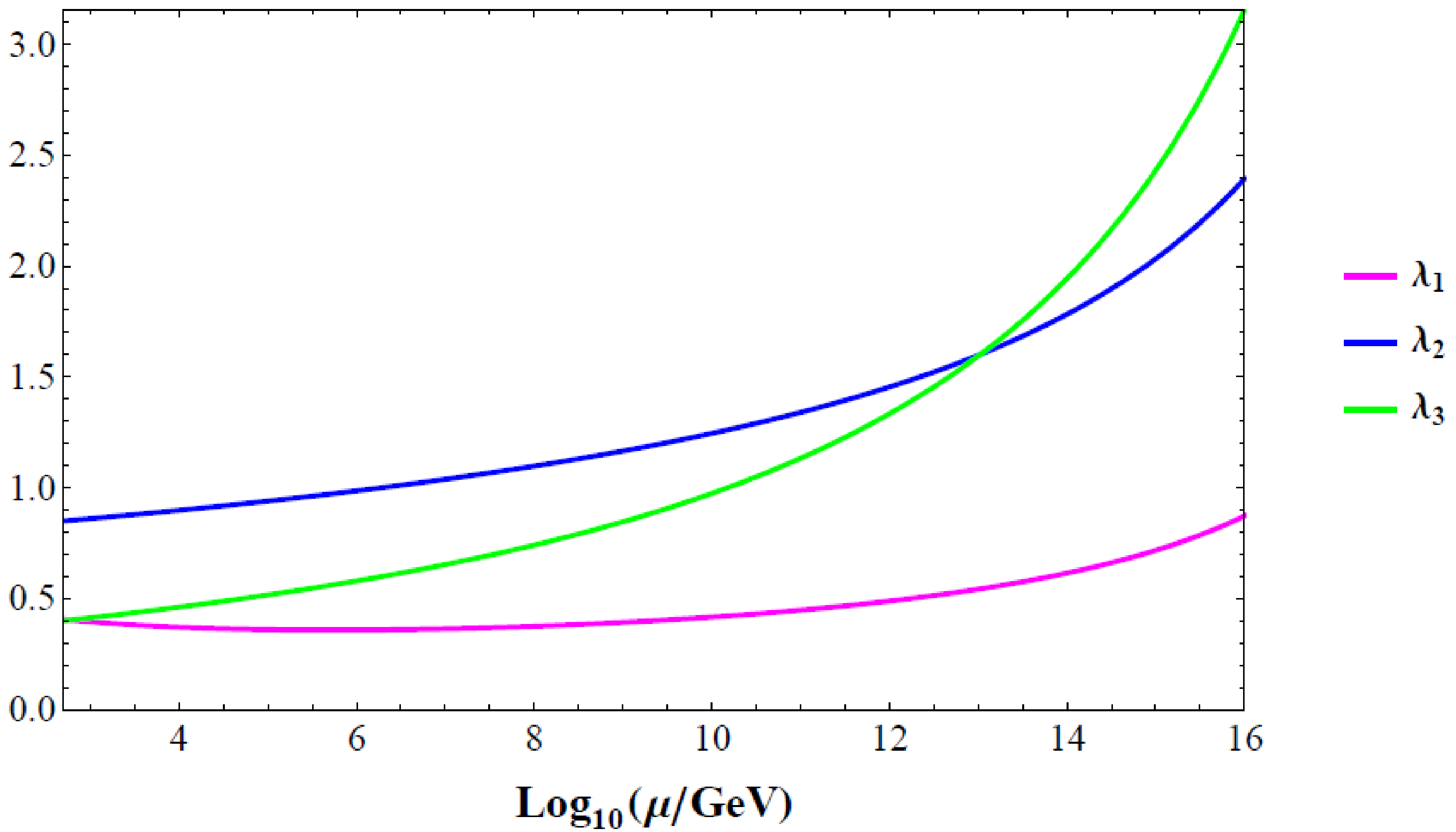
Evolution of masses:

$$16\pi^2 \frac{dm_\phi^2}{dt} = m_\phi^2 \left(6\lambda + 2\mathrm{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e) - \frac{9}{2} g_2^2 - \frac{9}{10} g_1^2\right) + \lambda_3 m_S^2$$

$$16\pi^2 \frac{dm_S^2}{dt} = 3\lambda_2 m_S^2 + 4\lambda_3 m_\phi^2$$

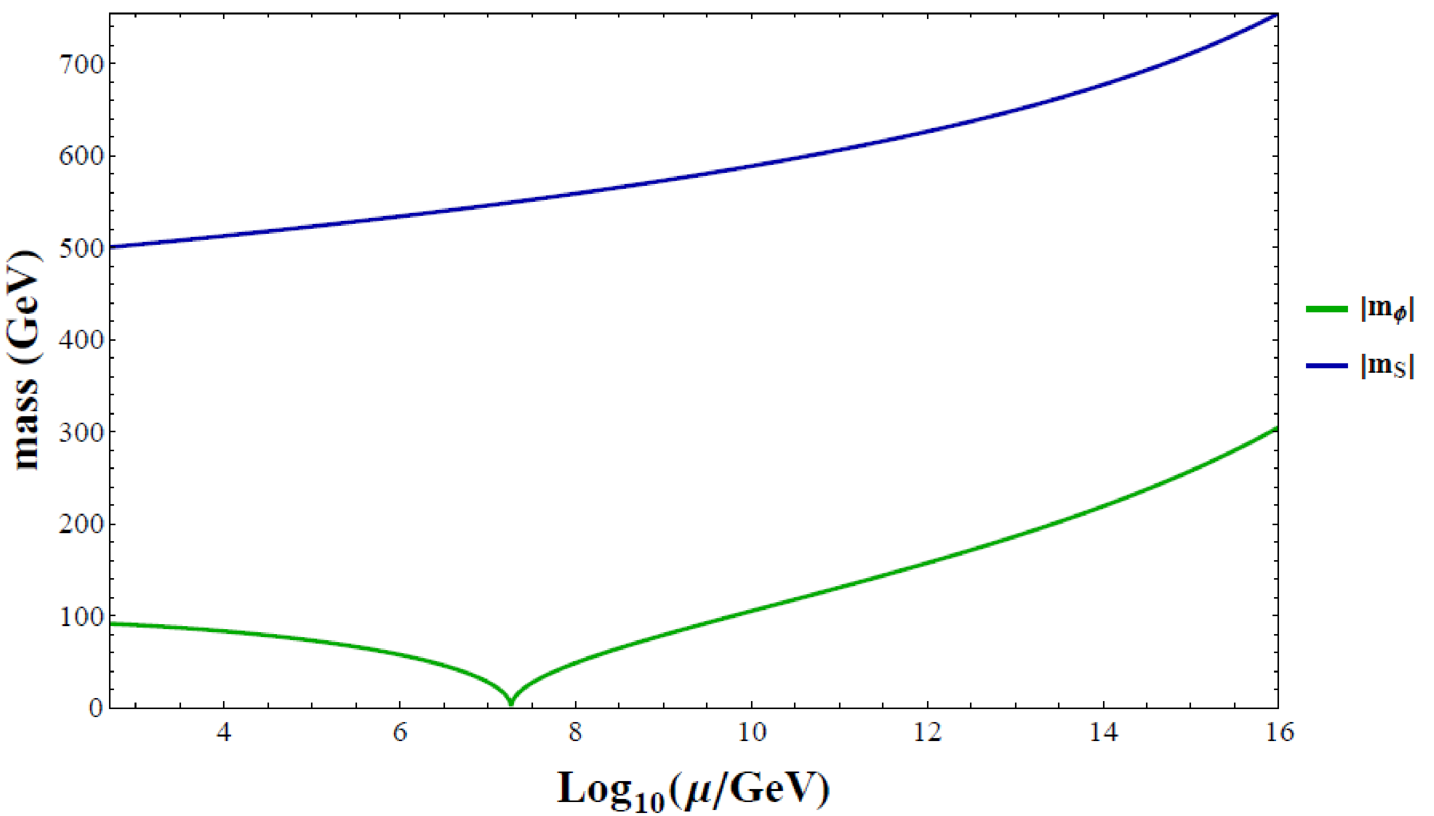
Kadastik, Kannike, Raidal (2009), Mambrini, Nagata, Olive, Zheng (2016)

Bounded quartic coupling evolution in singlet dark matter model



$$m_S = 500\,\text{GeV}, \quad \lambda_2(m_S) = 0.85, \quad \lambda_3(m_S) = 0.5$$

Turning of Higgs mass-squared in singlet dark matter models



m_S^2 remains positive, while m_ϕ^2 turns negative

Radiative EW symmetry breaking in Inert doublet models

A scalar doublett η which is Z_2 odd serves as dark matter

Higgs potential:

$$
\begin{aligned}
V &= m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta + \frac{\lambda_1}{2}(\phi^\dagger \phi)^2 + \frac{\lambda_2}{2}(\eta^\dagger \eta)^2 \\
&+ \lambda_3 (\phi^\dagger \phi)(\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta)(\eta^\dagger \phi) + \{\frac{\lambda_5}{2}(\phi^\dagger \eta)^2 + h.c.\}
\end{aligned}
$$

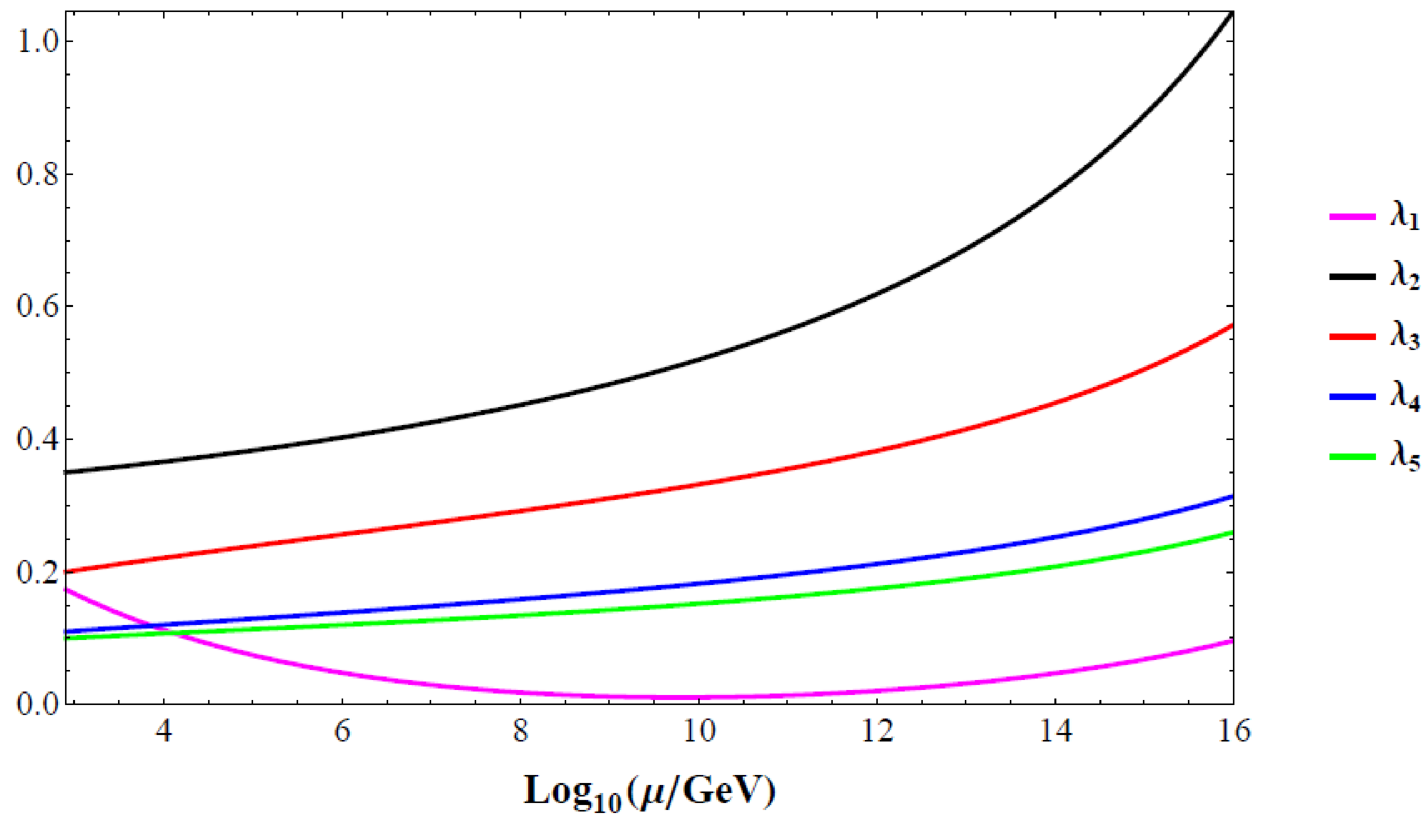
Mass evolution:

$$16\pi^2 \frac{dm_\phi^2}{dt} = m_\phi^2 \left(6\lambda_1 + 2\mathrm{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e) - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right) + m_\eta^2 (4\lambda_3 + 2\lambda_4)$$

$$16\pi^2 \frac{dm_\eta^2}{dt} = m_\eta^2 \left(6\lambda_2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right) + m_\phi^2 (4\lambda_3 + 2\lambda_4)$$

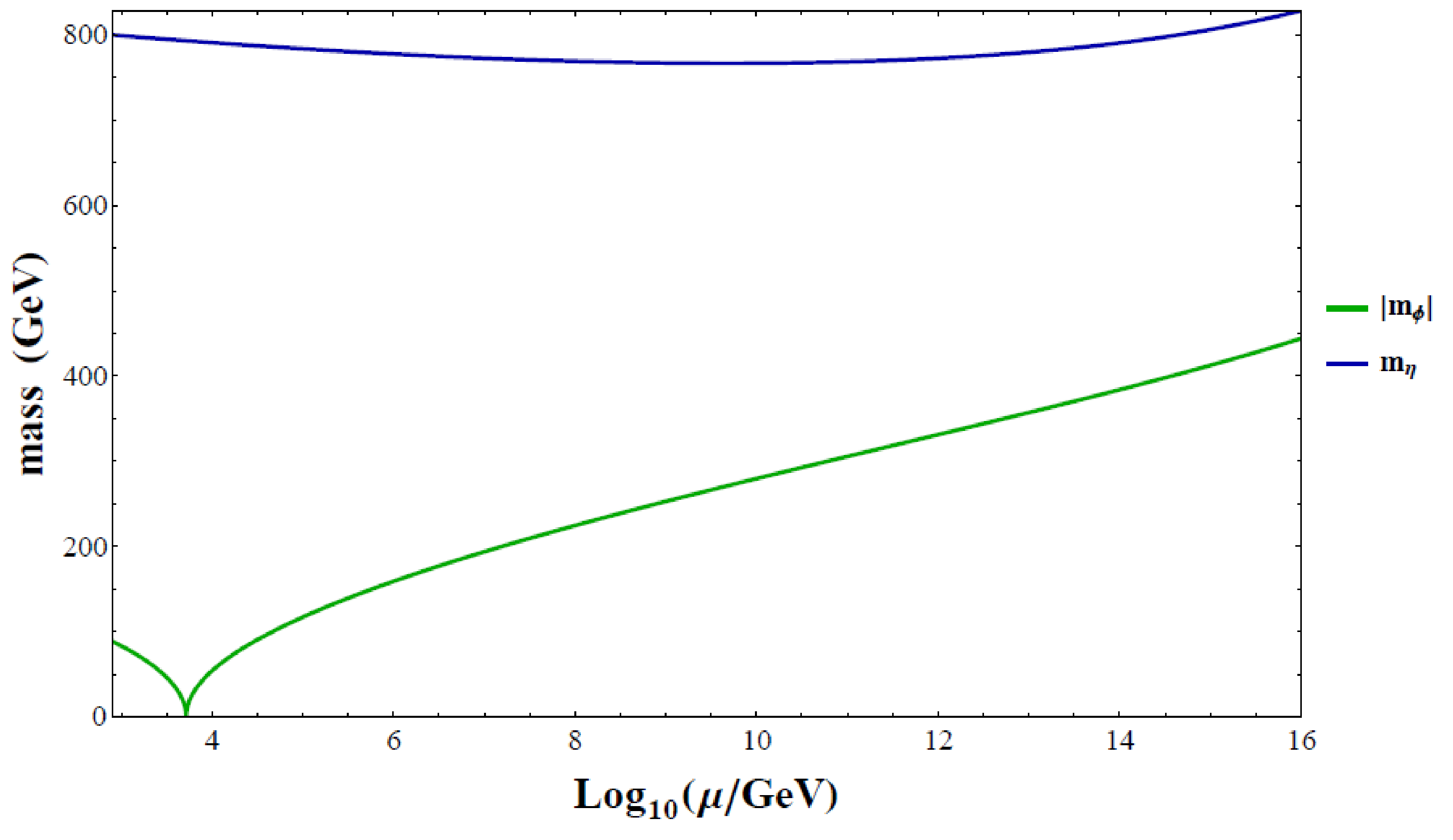
Ma (2006)
Barbieri, Hall, Rychkov (2006)

Bounded quartic coupling evolution in inert doublet model



$$m_\eta = 800\,\text{GeV}, \quad \lambda_2(m_\eta) = 0.35, \quad \lambda_3(m_\eta) = 0.2,$$
$$\lambda_4(m_\eta) = 0.11, \quad \lambda_5(m_\eta) = 0.1$$

Turning of Higgs mass-squared in inert doublet models



m_η^2 remains positive, while m_ϕ^2 turns negative