CE 513

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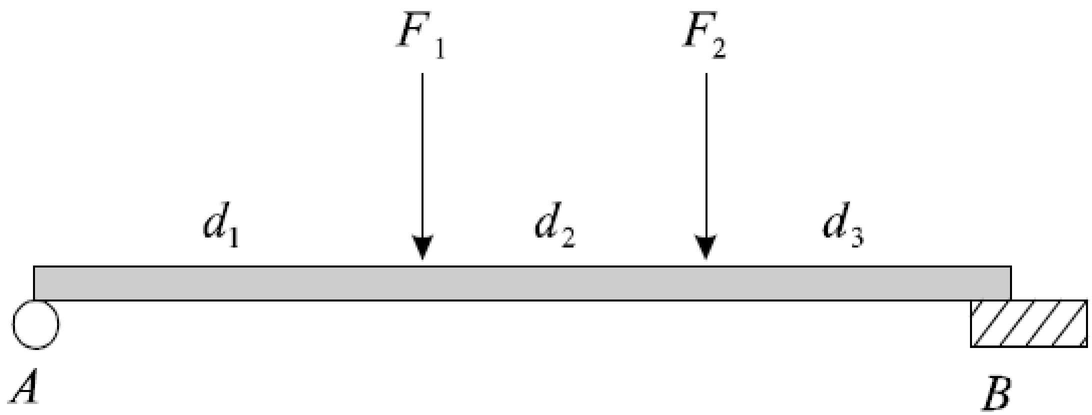
Example-1



Given the following loading statistics:

$$\mu_1 = E\{F_1\}$$

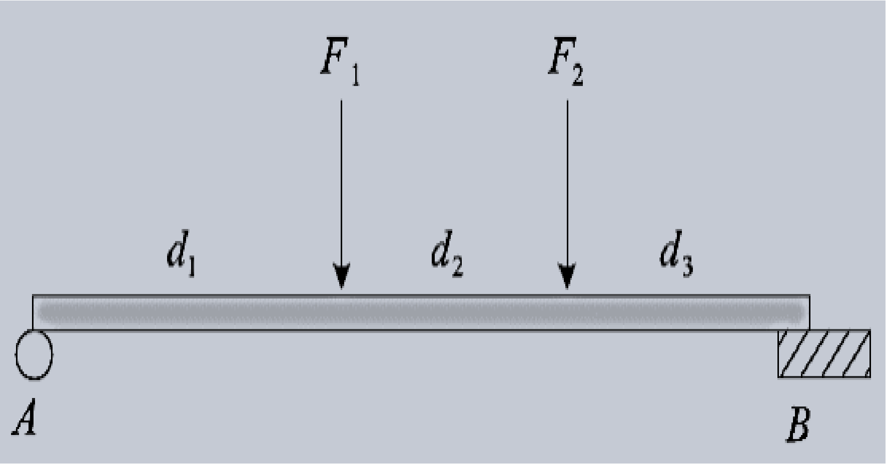
$$\sigma_1 = \sqrt{E\{F_1^2\} - \mu_1^2}$$

$$\mu_2 = E\{F_2\}$$

$$\sigma_2 = \sqrt{E\{F_2^2\} - \mu_2^2}$$

Derive the statistics of the reactions at A and B, R_A *and* R_B, assuming that the forces are statistically independent.

Example-1



$$R_A = \frac{(F_1+F_2)d_3 + F_1 d_2}{L} = \frac{F_1(d_3 + d_2) + F_2 d_3}{L}$$

$$R_B = \frac{(F_1+F_2)d_1 + F_2 d_2}{L} = \frac{F_2(d_1 + d_2) + F_1 d_1}{L}$$

$$\mu_A = \frac{(\mu_1 + \mu_2)d_3 + \mu_1 d_2}{L}$$

$$\mu_B = \frac{(\mu_1 + \mu_2)d_1 + \mu_2 d_2}{L}$$

$$\sigma_A^2 = \frac{1}{L^2}[(d_3 + d_2)^2\sigma_1^2 + d_3^2\sigma_2^2]$$

$$\sigma_B^2 = \frac{1}{L^2}[d_1^2\sigma_1^2 + (d_1 + d_2)^2\sigma_2^2]$$

$$E\{R_A R_B\} = E\left\{\frac{[F_1(d_3+d_2)+F_2 d_3][F_2(d_1+d_2)+F_1 d_1]}{L^2}\right\}$$

$$= \frac{1}{L^2} E\left\{\begin{matrix}(2d_1d_3+d_2d_3+d_1d_2+d_2^2)F_1F_2 \\ +d_1(d_3+d_2)F_1^2+d_3(d_1+d_2)F_2^2\end{matrix}\right\}$$

$$= \frac{1}{L^2}\left[\begin{matrix}(2d_1d_3+d_2d_3+d_1d_2+d_2^2)E\{F_1F_2\} \\ +d_1(d_3+d_2)E\{F_1^2\}+d_3(d_1+d_2)E\{F_2^2\}\end{matrix}\right]$$

Since F_1 and F_2 are statistically independent $E\{F_1F_2\} = E\{F_1\}\, E\{F_2\}$

$$E\{F_1^2\} = \sigma_1^2 + \mu_1^2$$
$$E\{F_2^2\} = \sigma_2^2 + \mu_2^2$$

Then, $E\{R_A\, R_B\} = a_1\mu_1\mu_2 + a_2(\sigma_1^2+\mu_1^2) + a_3(\sigma_2^2+\mu_2^2)$

The covariance is given by, $Cov\{R_A, R_B\} = E\{R_A R_B\} - \mu_A\mu_B$

and $\rho_{R_A R_B} = \frac{Cov\{R_A, R_B\}}{\sigma_A\sigma_B}$

Which can be used for numerical evaluation. Consider the case where $\sigma_1 = \sigma_2 = \sigma$,

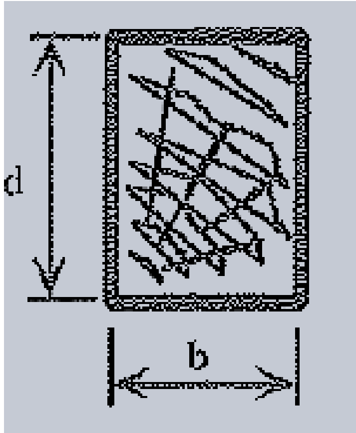
$$\sigma_A^2 = \frac{1}{L^2}[(d_3 + d_2)^2\sigma^2 + d_3^2\sigma^2] = \frac{\sigma^2(2d_3^2 + 2d_3d_2 + d_2^2)}{L^2}$$

$$\sigma_B^2 = \frac{1}{L^2}[d_1^2\sigma^2 + (d_1 + d_2)^2\sigma^2] = \frac{\sigma^2(2d_1^2 + 2d_1d_2 + d_2^2)}{L^2}$$

and

$$\rho_{R_A}\rho_{R_B} = \frac{L^2[a_1\mu_1\mu_2 + a_2(\sigma^2 + \mu_1^2) + a_3(\sigma^2 + \mu_2^2) - \mu_A\mu_B]}{\sigma^2\sqrt{(2d_3^2 + 2d_3d_2 + d_2^2)(2d_1^2 + 2d_1d_2 + d_2^2)}}$$

Example-2



Consider the design of a beam c/s using bending-stress as a performance criterion. That is:

$$Y = F_b - \frac{6M}{bd^2}$$
$$Y = F\,(F_b, M, b, d)$$

Goal: Calculate the mean and the variance of Y

$\mu_M = 100{,}000\ lb - in$ $\quad V_M = 0.12 \quad \Rightarrow \sigma_M = V_M \mu_M = 12{,}000\ ln - in$

$\mu_{F_b} = 100{,}000\ lb - in$ $\quad V_{F_b} = 0.32 \quad \Rightarrow \sigma_{F_b} = V_{F_b} \mu_{F_b} = 512\ psi$

$\mu_B = 5.6\ in$ $\quad V_B = 0.04 \quad \Rightarrow \sigma_B = V_B \mu_B = 0.224\ in$

$\mu_D = 11.4\ in$ $\quad V_D = 0.03 \quad \Rightarrow \sigma_D = V_D \mu_D = 0.342\ in$

The goal is to calculate the mean and variance of Y

<u>Solution.</u> Since Y is a nonlinear function, we must linearize the function about the design point values. We will use the mean values. The linearized form of Y will look like,

$$Y \approx \left[\mu_{F\,b} - \frac{6\mu_M}{\mu_B(\mu_D)^2}\right] + \left(F_b - \mu_{F\,b}\right)\frac{\partial f}{\partial F_b}\bigg|_* + (M - \mu_M)\frac{\partial f}{\partial M}\bigg|_*$$

$$+(b - \mu_B)\frac{\partial f}{\partial B}\bigg|_* + (d - \mu_D)\frac{\partial f}{\partial D}\bigg|_*$$

Where the partial derivatives are evaluated at the deign point values (mean values) of the random variables. The partial derivatives are,

$$\frac{\partial f}{\partial F_b} = 1 \quad \Rightarrow \quad \left.\frac{\partial f}{\partial F_b}\right|_* = 1$$

$$\frac{\partial f}{\partial M} = -\frac{6}{bd^2} \quad \Rightarrow \quad \left.\frac{\partial f}{\partial M}\right|_* = -\frac{6}{\mu_B \mu_D^2}$$

$$\frac{\partial f}{\partial B} = \frac{6M}{b^2 d^2} \quad \Rightarrow \quad \left.\frac{\partial f}{\partial B}\right|_* = \frac{6\mu_M}{\mu_B^2 \mu_D^2}$$

$$\frac{\partial f}{\partial D} = \frac{12M}{bd^3} \quad \Rightarrow \quad \left.\frac{\partial f}{\partial D}\right|_* = \frac{12\mu_M}{\mu_B \mu_D^3}$$

Substituting theses derivatives into the linearized equation, plugging in the mean values of the variables, and rearranging gives the following linearizes form of Y:

$$Y = F_b - 0.008244M + 147.2b + 144.6d - 2473$$

$$\sigma_Y^2 = (1)^2 \sigma_{F_b}^2 - (0.008244)^2 \sigma_M^2 + (147.2)^2 \sigma_B^2 + (144.6)^2 \sigma_D^2$$