CE 513: STATISTICAL METHODS IN CIVIL ENGINEERING (2019)

Lecture-1: Introduction & Overview

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Schedule of Lectures

- 4-5 extra classes on weekends: **5 marks** bonus for full attendance
- Grading scheme:
    - Midterm 30 %
    - End term: 40%
    - Surprise Quizzes : 20 %
    - Programming assignments: 10%
- Lectures: **Wed** (1-20 to 2:50) **5101**;

  Thurs (12-40 to 2-00 pm) **5101**

INTROCUTION

- Principle aim of design: SAFETY
- Often this objective is non-trivial
- On occasions, structures fail to perform their intended function
- RISK is inherent
- Absolute safety can never be guaranteed for any engineering system; a probabilistic notion

A motivating example

F = 1 KN
EI = 10000000 Nm^2
L = 2 m

$$\Delta = \frac{FL^3}{3EI}$$

$$= 0.2667\, mm$$

Uncertainties

- Can we be always certain about EI ?
- For RCC, fixing a point or a single value of E

  is fraught with risks
- Are we always sure about I ? Or the dimensional

properties ? Can be risky again
- In lot of practical applications, even F cannot be known

for certain ?

Let's consider the cantilever beam example again, **now with some uncertainties**

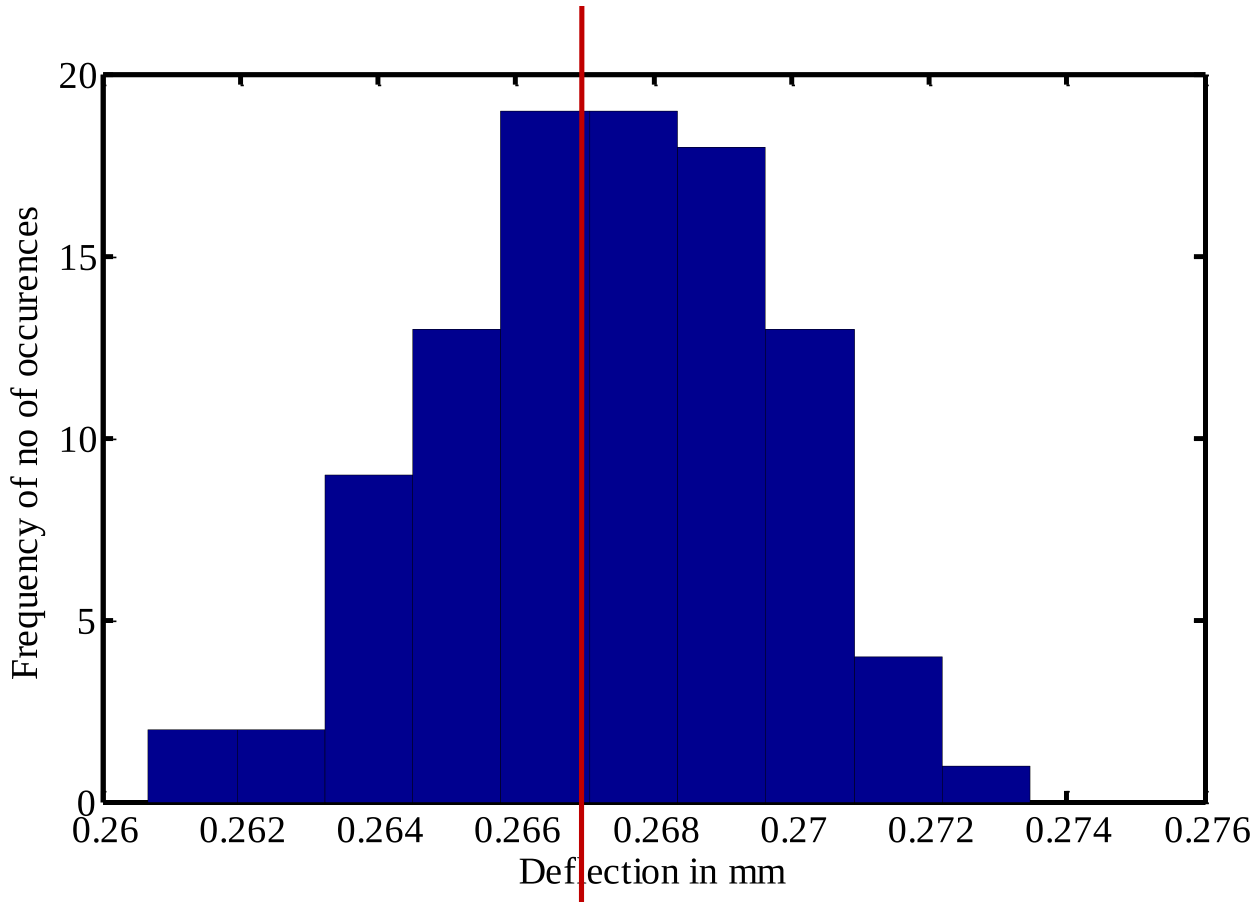
```
F=1+0.1*randn(100,1) ;    % F is normally distributed

EI=10^7+1000*randn(100,1);  % EI is normally distributed

L=2;

 for i=1:100
       delta(i)= F(i)*L^3/(3*EI(i));
end
```

The displacement becomes uncertain too

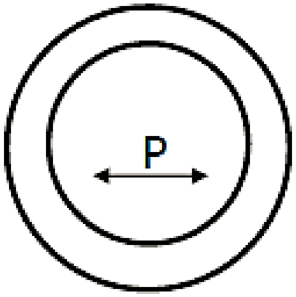


Mean value = 0.2674 mm

Practical example: Reliability based design

- Consider a pipe section with diameter D, thickness W, that is subjected to internal pressure P
- The hoop stress S in the pipe section is given as

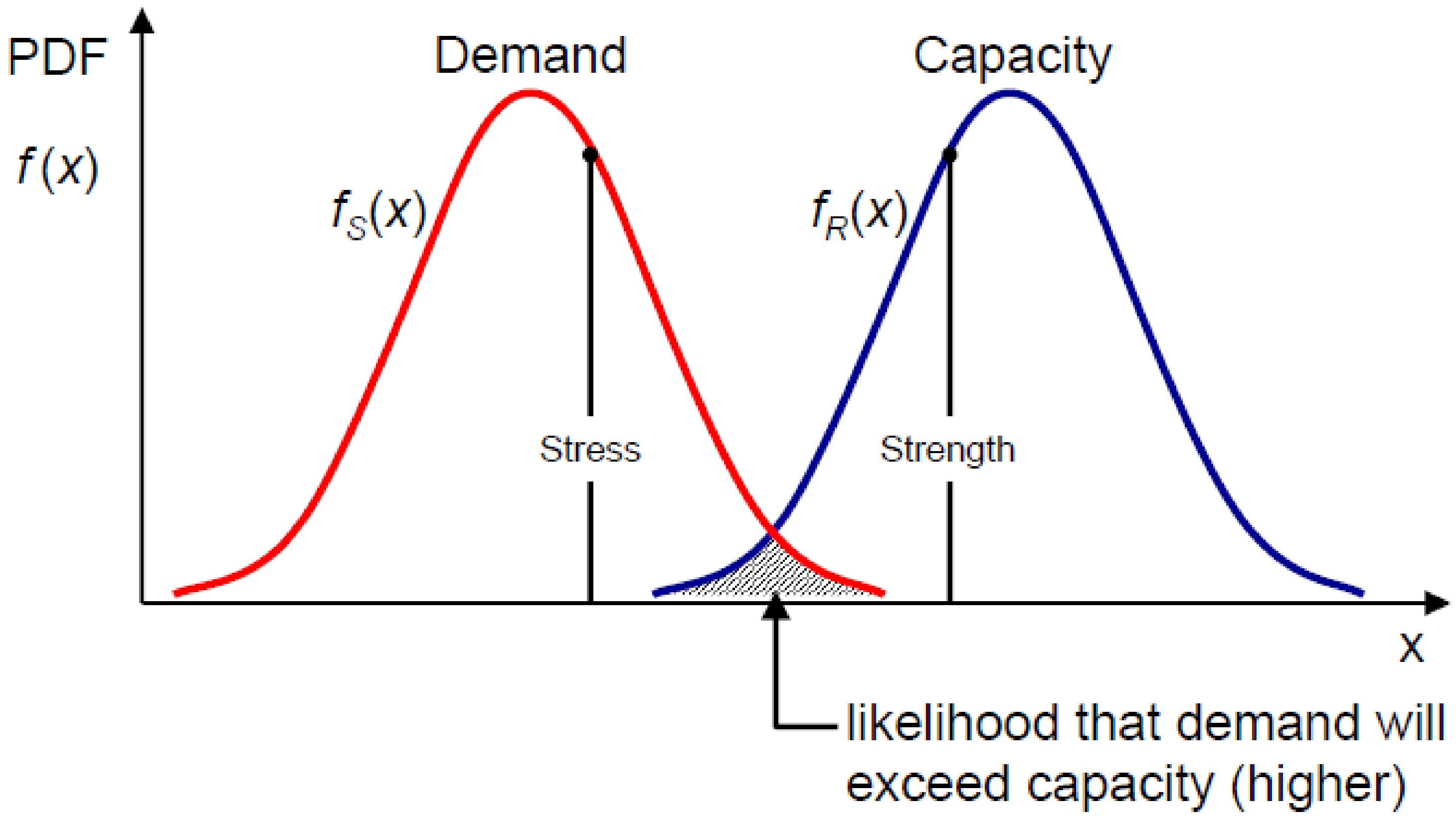
$$S = \frac{PD}{2W}$$



- The yield strength of the material is denoted by R
- Failure:

Stress (S) > Yield strength (R)

Reliability example



Probability

- Random variables are defined through the concept of probability
- Probability quantifies the variability in the **outcome** of an event, whose **exact** outcome cannot be predicted with certainty

> ***Probability:*** *The likelihood of occurrence of an event relative to a set of alternative events.*

- Need to identify
  - A set (or range) of all possibilities (outcomes or sample space)
  - The event of interest

Classical definition

- Need to determine the likelihood associated with the occurrence of each event (= probability)

- Definition

> *For a game that has n equally likely outcomes, of which s outcomes correspond to "success" or winning, the probability of winning is given by s/n.*

- All events have an equal likelihood of occurring
  - i.e. all outcomes are equally likely

- The classical probability concept is applicable to games of chance (i.e. gambling)

Sample space

- Consider a coin toss or rolling a die

Sample Space = 2 Events

"Heads"
or
"Tails"

Sample Space = 6 Events

{1,2,3,4,5,6}

- Or, tossing a quarter and a dime

H_Q T_Q H_D T_D

Sample Space = 4 Events

Example-1

When we roll a pair of balanced dice, what are the probabilities of getting (a) 3, (b) 2 or 12, or (c) 7?

Solution:

(a) There are two such outcomes (1,2) and (2,1). Thus, the probability is equal to 2/36 = **1/18**

(b) There are two such outcomes (1,1) and (6,6). Thus, the probability is equal to 2/36 = **1/18**

(c) There are six such outcomes (1,6), (2,5), (3,4), (6,1), (5,2) and (4,3). Thus, the probability is equal to 6/36 = **1/6**

Issues with classical definition

- What is "equally likely"?
- What if not equally likely?

e.g.: what is the probability that sun would rise tomorrow ?

- No room for experimentation.

Frequency def

Frequency definition

If a random experiment has been performed *n* number of times and if *m* outcomes are favorable to event A, then the probability of event A is given by

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

Issues

What is meant by limit here?

One cannot talk about probability without conducting an experiment

What is the probability that someone meets with an accident tomorrow?

Axioms of probability

Notions lacking definition

- Experiments
- Trials
- Outcomes

• An experiment is a physical phenomenon that is repeatable. A single performance of an experiment is called a trial. Observation made on a trial is called outcome.

• Axioms are statements commensurate with our experience. No formal proofs exist. All truths are relative to the accepted axioms.

Sample space

Sample space (Ω)

Set of all possible outcomes of a random experiment.

Examples

(1) Coin tossing: $\Omega=(h \quad t)$; Cardinality=2; finite sample space.

(2) Die tossing: $\Omega=(1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6)$; Cardinality=6; finite sample space.

(3) Die tossing till head appears for the first time:

$\Omega=(\text{h} \quad \text{th} \quad \text{tth} \quad \text{ttth} \quad \text{tttth} \quad \cdots)$; Cardinality=$\infty$; countably infinite sample space.

(4) Maximum rainfall in a year: $\Omega=(0 \leq X < \infty)$;

Cardinality=∞; uncountably infinite sample space.

Axioms of probability

1. The probability of an event is a real non-negative number

$$0 \le P(E) \le 1$$

2. The probability of a certain (or inevitable) event S is 1.0 (contains all the sample points in the sample space; i.e. is the sample space itself)

$$P(S) = 1.0$$

(conversely, the probability of an impossible event ϕ is zero)

3. Law of Addition

$$\sum_{k=1}^{N} P(E_k) = 1$$

Rigorously speaking the axioms of probability requires the knowledge of measure theory & sigma algebra

Problems that we will study

- **Random Processes:** Wind, earthquake & wave loads on engineering structures. Hydro-climatological processes like temperature & rainfall data
- **Stochastic Calculus:** If $F(x) = x^2 dF \neq 2xdx$; where F(x) is a stochastic function of a random process **x**
- **Stochastic Differential Equations**: $\frac{dx}{dt} = f(x,t) + L(x,t)w(t)$; $w(t)$ is a realization of white noise
- **Monte Carlo Simulation**

Reference material

1. Probability, reliability and statistical methods in engineering design

   by **A. Halder & S. Mahadevan**

1. Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering **by** **Alfredo H-S. Ang & Wilson H. Tang**

2. Probability, Statistics, and Reliability for Engineers and Scientists by **Bilal M. Ayyub & Richard H. McCuen**

4. Probability, Random Variables, and Stochastic Processes: **Papoulis and Pillai**

5. Probabilistic models for dynamical systems: **Benaroya, Han & Nagruka**