# CE 607 HOME-WORK

## Question-1)

Evaluate the triple integral: $\int_{t_0}^{t}\int_{t_0}^{s_1}\int_{t_0}^{s_2} dB(s_3)dB(s_2)dB(s_1)$

where $dB(t)$ signifies incremental Wiener process

## Question-2)

Consider the Ito-Taylor expansion for the solution of the diffusion equation $dx(t) = a[x(t)]dt + b[x(t)]dB(t), \;\; x(t_0) = x_0$;

$$\boldsymbol{x(t) = x(t_0) + \int_{t_0}^{t}\left\{a[x(t_0)] + \int_{t_0}^{s_1}\mathcal{L}^0\, a[x(s_2)]ds_2 + \int_{t_0}^{s_1}\mathcal{L}^1\, a[x(s_2)]dB(s_2)\right\}ds_1}$$

$$\boldsymbol{+ \int_{t_0}^{t}\left\{b[x(t_0)] + \int_{t_0}^{s_1}\mathcal{L}^0\, b[x(s_2)]ds_2 + \int_{t_0}^{s_1}\mathcal{L}^1\, b[x(s_2)]dB(s_2)\right\}dB(s_1)}$$

where $dB(t)$ signifies incremental Wiener process. Derive a numerical scheme in terms of **discrete time & discrete wiener increments** that includes two $\boldsymbol{O}\{\int\int\}\,\boldsymbol{\mathcal{L}^1}$ remainder terms ?

## Question-3)

During a race, a cyclist has average speed **m**. However, his speed varies in time. Sometimes the cyclist exceeds the speed **m**, but he gets tired after a while and slows down. If the cyclist's speed decreases under the mean **m**, then he recuperates the muscle power and is able to speed up again. The cyclist's instantaneous velocity $v_t$ satisfies a mean reverting process described by the equation: $dv_t = a(m - v_t)dt + \sigma\, dW_t$, where $a$ and $\sigma$ are constants and $dW_t$ is the Wiener increment.

The solution of this equation: $v_t = m + (v_0 - m)e^{-at} + \sigma e^{-at}\int_0^t e^{as}\, dW_s$.

*Find the mean and the variance of* $\boldsymbol{v_t}$ *?*

## Question-4)

Investigate whether the stochastic process $X(t)$ with autocorrelation function $R_{XX}(t_1, t_2) = \sigma^2 \exp(-\alpha|t_1 - t_2|)$ possesses mean square derivative or not.

**Question-5)**

Find the covariance of $W(s)$ and $\int_0^t W(r)dW(r)$ for $s \leq t$

**Question-6)**

Let {X(t), t ≥ 0} be a random process with stationary independent increments, and assume that $X(0) = 0$.

Show that Cov[X(t), X(s)] = $K_X(t, s) = \sigma^2 \min(t, s)$

**Question-7)**

Find the autocorrelation function $R_X(t, s)$ and the autocovariance function $K_X(t, s)$ of a Poisson process X(t) with rate A.

**Question-8)**

Find the variance of $W(1) - \int_0^1 W(s)ds$

**Question-9**

Brownian Bridge *is defined as* $X(t) = B(t) - tB(1)$; where $B(t) = \sigma W(t)$ and $X(0) = X(1) = 0$. Find the autocovariance function of a Brownian bridge ?