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Probabilistic Modeling of Two-dimensional Consolidation Using the Spectral Feynman-Kac Approach

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Abstract

Consolidation analysis of saturated soils remains a critical challenge in geotechnical engineering because of the inherent spatial variability of soil properties and the complexities associated with the boundary conditions encountered in real-world scenarios. Traditional consolidation analysis often assumes soil to be homogenous, considering key properties such as coefficients of consolidation, permeability, and volume compressibility as constant throughout the soil domain. However, in reality, soils are inherently heterogeneous, and these soil properties exhibit significant spatial variability. Addressing these uncertainties accurately is essential, particularly for a reliable prediction of the excess pore water pressure (EPWP) profiles and associated settlement behavior.

To incorporate these uncertainties in a probabilistic framework, this article proposes a novel meshless approach based on the spectral Feynman-Kac (SF-K) framework to address the two-dimensional consolidation problem of saturated soils. The proposed SF-K approach extends the classical Feynman-Kac formula to accommodate the spatial variability of soil properties, employing the Karhunen-Loeve (KL) expansion technique. The coefficients of horizontal and vertical consolidation (c_h and c_v) are modeled as 2D random fields to realistically capture the spatial heterogeneity of soils. These random fields of c_h and c_v are represented using a truncated KL expansion to ensure efficient spectral characterization of spatial variability. Corresponding to the governing partial differential equation (PDE) for 2D consolidation, the SF-K framework reformulates the problem into two independent stochastic differential equations (SDEs) driven by Brownian motions that generate stochastic processes in the horizontal and vertical directions. The generator SDEs are simulated using the Euler-Maruyama method within a Monte Carlo (MC) simulation framework. The solutions of EPWP profiles are approximated by taking the ensemble average of the trajectories traced by the Brownian particles, depending on the interaction of the particles with the domain boundaries.

In the present work, mixed boundary conditions representing various drainage scenarios are considered to assess their influence on the EPWP dissipation profiles and subsequent settlement of the 2D consolidation problem. Four distinct cases of boundary conditions designated as Type I, Type II, Type III, and Type IV are considered in this study. In the Type I boundary condition, the top ($z=L_z$) and the right ($x=L_x$) boundaries are pervious, facilitating drainage of pore water, while the bottom ($z=0$) and the left ($x=0$) boundaries are impervious, restricting the flow of pore water. The Type II

boundary condition deals with the pervious top, bottom, and right boundaries, enabling drainage of pore water, with only the left boundary remaining impervious. In the Type III boundary condition, the drainage of pore water is allowed by the pervious top, left, and right boundaries, while the impervious bottom boundary inhibits the flow of pore water out of the soil domain. The Type IV boundary condition allows the pore water to drain through all the pervious top, bottom, left, and right boundaries, representing a fully drained condition.

Boundary conditions are incorporated stochastically in the simulations through the interaction of the Brownian particles with the domain boundaries and by classifying the boundaries as either a Dirichlet or Neumann boundary. This mechanism mimics the physical behavior of EPWP dissipation and settlement under mixed boundary conditions. For Dirichlet boundaries corresponding to pervious or drained conditions, particles reaching the boundary exit the domain, and the trajectory of the particle is terminated. This reflects the free outflow of pore water at a Dirichlet boundary in the stochastic simulations. In contrast, Neumann boundaries representing impervious or no-flux conditions are handled by reflecting the particles reaching the boundary back into the domain. This ensures the restriction of the drainage of pore water at Neumann boundaries in stochastic simulations.

To compare and validate the accuracy of the proposed SF-K framework, a spectral finite difference method (SFDM) is developed to solve the 2D consolidation problem, incorporating the spatial variability of c_h and c_v through the KL expansion technique. The SFDM models the random fields of c_h and c_v with identical statistical properties, similar to the SF-K framework. The variations of the EPWP profiles and the settlement profiles of the 2D consolidation problem obtained from the SF-K framework are then compared against those obtained from the SFDM under the four distinct drainage cases (Type I-IV) and a uniform initial condition. The results demonstrate excellent agreement between the frameworks for all cases of drainage boundary conditions, as evidenced by low values of root mean square error (RMSE). This highlights the robustness of the RF-K framework in accurately handling the spatial variability of soils and addressing various drainage boundary cases of the 2D consolidation phenomenon. These results demonstrate the robustness of the proposed SF-K framework in accurately capturing the spatial variability of soils and handling various drainage boundary conditions within a stochastic framework that addresses the 2D consolidation problem.

Grid-based numerical methods, such as the SFDM, require a predefined spatial discretization and time step satisfying the stability criteria to ensure accuracy of the solutions. Failure to satisfy this criterion leads to numerical dispersion of the EPWP profiles and settlement profiles and inaccurate solutions. In contrast, the proposed SF-K framework is meshless, eliminating pre-processing steps such as grid generation, mesh optimization, and refinement, thereby reducing computational complexity. Moreover, the proposed SF-K framework relies on independent simulations of driftless SDEs, which further reduces the computational cost while maintaining accuracy. The study demonstrates the accuracy and computational efficiency of the proposed SF-K framework in solving the 2D consolidation of saturated soils without being constrained by strict stability conditions.