RANDOM FINITE ELEMENT AND LIMIT EQUILIBRIUM METHODS BASED PROBABILISTIC STABILITY ANALYSES OF A CUT SLOPE

Rubi Chakraborty, Arindam Dey

Rubi Chakraborty

Assistant Professor, Department of Civil Engineering, National Institute of Technology Meghalaya, India.

Email: rubi@nitm.ac.in

Arindam Dey*

Associate Professor, Department of Civil Engineering and Centre for Disaster Management & Research, Indian Institute of Technology Guwahati, Assam, India. Contact No.: +918011002709, ORCID ID: 0000-0001-7007-2729

Email: arindam.dey@iitg.ac.in

* Corresponding author

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Abstract

This paper elucidates the efficacy of advanced Random Finite Element Method (RFEM) to assess the excavation induced slope instability over the conventional Limit Equilibrium Method (LEM). Both the studies have been carried out in a probabilistic framework in which the shear strength parameters of the slope material are considered as random variables. While the probabilistic LEM is used for the conventional one-dimensional (1-D) spatial variation with pre-assumed failure surfaces, the adopted probabilistic RFEM incorporates a more realistic two-dimensional (2-D) spatial variation of soil properties with the evolution of failure surfaces during the progress of the analysis. In absence of any previous studies employing RFEM to investigate the toe-excavation induced hillslope instability, the present study illustrates the importance of considering uncertainty and two-dimensional spatial variability in soil properties for a safe and economical cut slope design for hill-road constructions. The outcomes suggest that the chances of a cut-slope failure are strongly influenced by the coefficient of variation (CoV), cross-correlation coefficient between the shear strength parameters ($\rho_{c\varphi}$), and the non-dimensional correlation length (Θ) governing the spatial variability of soil properties. For a specific $\rho_{c\varphi}$ = +0.5 and CoV varying between 0.2-0.4, the probability of failure (P_f) of the cut slope exhibits a variation of approximately 6%. Based on the various possible correlation types exhibited by the soil parameters ($\rho_{c\varphi}$: -0.5, 0, +0.5), for a specific CoV=0.4, a similar variation in P_f of approximately 6% is observed. Based on comparative study with 2-D RFEM-based spatial variability, it is deciphered that adoption of 1-D LEM-based spatial variability can significantly undermine the failure probability of the cut-slope, especially

when the Θ ranges in 0.5-0.7. In this range, a cut slope section may even be adjudged safe ('above average' performance level) through 1-D spatial variability, while in reality the slope may have substantially higher chances of failure with a 'poor' performance level. The outcomes of the reported study indicate the inadvertent requirement of obtaining an appropriate site-specific spatial variation of soil parameters for the successfully assessing the probabilistic failure of cut slopes.

Keywords: Cut slope instability; Probabilistic analysis; Limit equilibrium method; Random finite element method; Coefficient of variation: Correlation coefficient; Correlation length

1. Introduction

In the hilly terrains of India, cutting of hillslopes for construction of roadways is a common practice. The growing necessity for establishing proper means of communication for rapidly developing urbanization in the hills has increased the requirement manifold. Stability assessment of such cut slopes in the hills are of prime importance, as their failure and collapse would have extremely serious social and economic consequences [1-7]. Several literatures have elucidated the reasons and consequences of such failures, thereby highlighting the necessity of a rigorous and thorough stability analysis of cut slopes. A landslide induced disaster to a habitation in San Francisco bay area, USA was reported by Stark et al. [3], wherein the slope cutting operation for broadening of the highway triggered the instability. As reported by Umrao et al. [8], various locations of the Rudraprayag District, Uttarakhand, India, along the NH-109, were significantly affected by rock slope failures triggered by slope cutting. The degradation of dry stable basaltic hillslopes with lateritic shear seams in Mahabaleshwar, India was reported by Kainthola et al. [9,10], wherein subsequent excavations triggered widespread failure of the saturated slopes. The repeated failures along the Highway G212, China was illustrated by Zhang et al. [11], which was initiated as a result of the toe-excavation of slopes, thereby subsequently exposing the weak bedding planes. A finite element based stability analysis was conducted by Mahanta et al. [12] to identify the deformation mechanisms of the vulnerable cut slopes in Kullu, Himachal Pradesh, India. Depending on the slope geometry and properties of the hillslope material, the influence of hydrogeological and seismic parameters on the stability of cut slopes was reported by Chakraborty and Dey [13], wherein the safe thresholds of toe excavations were proposed that would not jeopardize the hillslope stability.

All the above-mentioned studies adopted a deterministic approach to interpret the stability of cut slopes. However, because of the uncertainties in the slope geometry and geotechnical properties, the deterministic factor of safety can be hardly appreciated to be a very good interpreter of slope stability. Several factors contribute towards the uncertainties in the outcomes from the analyses, namely the geological anomalies, limited investigations in difficult terrains, spatial variability of soil characteristics, uncertainty in slope failure mechanisms and, lastly, adopting simplifying presumptions in geotechnical modeling. Nonetheless, very less attention is given to integrate the geotechnical uncertainties for authentic risk assessment of slope instability in hilly areas. Although probabilistic concepts have been applied in various geotechnical problems in the last few decades, there still exists an inhibition in applying such necessary techniques in practical considerations and economic road constructions involving hill cutting. Recently, through a comparative study between LEM-based deterministic and probabilistic slope stability analyses, the importance of probabilistic analysis in assessing the stability of cut slopes was also reported by Chakraborty and Dey [14]. The probabilistic study was further extended to assess the seismic response of a toe-excavated hillslope retained by sheet pile anchor system [15].

This article illustrates the assessment of cut slope instability by incorporating one-dimensional (1-D) spatial variation of soil shear strength parameters in a limit equilibrium method (LEM) based probabilistic framework. Further, the influences of a more realistic two-dimensional (2-D) spatial variation of soil shear strength parameters are investigated in a random finite element method (RFEM) based probabilistic framework developed by Griffiths and Fenton [16, 17]. While the former approach employs a presumed failure surface, the advantage in the latter approach lies in the evolution of failure surface during the analysis. The results obtained from both the approaches are compared, and the influence of the coefficient of variation (CoV), cross-correlation coefficient between the shear strength parameters ($\rho_{c\varphi}$) and the non-dimensional correlation length (Θ) on the probability of cut slope failure are illustrated.

2. Probabilistic Approach for Slope Stability Study

Deterministic slope stability analysis is based on presumed characteristic values of soil properties without incorporating any associated uncertainty. Probabilistic analysis renders a more pragmatic approach to assess the cut slope stability as it is capable to explicitly incorporate the uncertainty and variability in soil properties [14]. Unlike deterministic analysis, which provides a single measure of factor of safety (FoS), the probabilistic approach delivers a more rational measure in terms of the probability of failure (P_f). Several earlier studies have adopted probabilistic approaches to assess the stability of slopes [16-25]. However, none of the earlier studies addressed the probabilistic analysis of the to-excavation induced slope failure.

In this paper, two different probabilistic approaches are utilized to include the uncertainties associated with the soil shear strength parameters in assessing cut slope stability. The first approach utilizes a LEM-based slope stability analysis within a probabilistic framework that has been conducted using the Slope/W module of Geostudio v2018. However, this approach carries the inherent limitations of LEM in presuming the shape and location of the failure plane [14]. Furthermore, Slope/W is incapable of incorporating the 2-D spatial variation in soil properties [14]. The second approach is based on advanced RFEM that has been conducted using a computer model *Rslope2D* developed by Griffiths and Fenton [16, 17]. This approach can incorporate the spatial variation of soil properties in both horizontal and vertical directions, as well as it does not presume the shape or location of the failure surface [16, 17]. The failure plane is allowed to develop through the soil elements in which the shear strength is less than the applied shear stresses.

The former method, even though having its own inherent trade-offs, is yet very popular due to its simplicity in modelling and analysis. The latter method, although more pragmatic, demands the knowledge of RFEM, for which the expertise might not be always available. Nonetheless, for both the approaches, the framework of computing the probability of cut slope failure approximately remains the same, which are listed as follows:

1. A spatially variable 1-D or 2-D random field (RF) for the selected soil properties is simulated based on the specified statistical parameters.
2. Limit equilibrium (LE) or random finite element (RFE) based slope stability analysis is conducted for the simulated model to ascertain the stability of slope against the specified soil properties and slope geometry.
3. The steps 1 and 2 are repeated using the Monte Carlo simulation (MCS) process to estimate the probability of failure, P_f.

There are mainly two categorical approaches for probabilistic slope stability analyses. The first approach pertains to the approximate methods such as first order reliability method (FORM) [26], the mean value first order second-moment reliability method (MVFOSM) [27] and the second order reliability method (SORM) [28]. In the field of structural reliability, the Hasofer–Lind reliability index and the First Order Reliability index are commonly used for approximating the probability of failure [29, 30]. However, it is worth mentioning at this point that the 'Approximate Methods' are not able to incorporate the spatial variation of soil properties in the analysis, thereby resulting in either over-estimation or under-estimation of the probability of slope failure under different conditions [31]. This pitfall can be overcome through the second categorical approach that adopts the Monte Carlo Simulation (MCS) based methods [31, 32]. For the present study, Direct Simulation Monte Carlo, or DSMC, [33] has been used. It is a classical technique, wherein MCS generates a set of trial values of the random variable within a user-specified range that can be defined based on the site-specific investigations or recommendations from available literature [34, 35]. For each trial value, the random variable estimates the *limit state function*, which is a mathematical quantification of system failure and is usually represented by $g(x)$. A system is ascertained safe if $g(x)>0$, and in the otherwise case, it is specified to be in a state of failure. From all the simulations conducted using the trial values of the random variable, the total number of system failures are counted, and the failure probability is estimated as $P_f = \frac{n_f}{N}$, where, n_f is the number of failures that occurred in total N number of trials.

In the present study, Monte Carlo technique has been used with both LEM and RFEM-based approaches. For both the cases, MCS have been used to draw the magnitudes of shear strength parameters from their respective distributions. In a conventional MCS-based probabilistic slope stability analysis, from each draw, the magnitudes of the random variables are selected for a single simulation of the slope stability analysis. Thus, for each of the MCS-based analysis, the chosen magnitude of the random variable still represents a homogeneous slope. Hence, at the end of the MCS-based analyses, the probability of failure is estimated through a set of homogeneous slope stability analysis. However, in reality, the hillslope material would comprise materials having spatially varying properties. In such case, it is important to simulate the spatial variability through 1-D or 2-D random field modelling, as has been done in this study with the aid of amalgamation of MCS and LEM or RFEM, respectively. In such case, for each MCS simulation, a random field of the shear strength parameter would be generated. Based on the correlation coefficient, the random field would generate spatially variable shear strength properties along the entire domain of the slope. Thus, in this approach, the MCS is still used to draw the parameter magnitudes, but the spatial variability of the parameters is controlled by the 1-D or 2-D random field. Thus, this procedure is advantageous in modelling the spatial variability of parameter more realistically.

3. Numerical Model for Present Study

For obtaining more carriageway width during a road construction in a hilly terrain, vertical or near vertical excavations are commonly carried out in the hillside slope. Due to the presence of lateritic soils, the hillslopes in the north-eastern, and similar other regions, of India can withstand near-vertical cuts (even in the tune of 80º-90º slope inclinations) that are generally carried out at the site during the dry periods of the year. A vertical cut in a soil slope is clearly considered as the worst case of excavation scenario in hillslopes, and hence, only the stability of vertical cut slopes is addressed in this study.

As commonly encountered in the hilly terrains of north-east India, Fig. 1 shows a typical 40 m high slope having a gradient of 2H:1V. A vertical cut is presumed to be carried out at the toe of the slope, such that a carriageway width (b_t) of 10 m can be developed. The shear strength parameters (cohesion, c, and angle of internal friction, φ) of the hillslope soil material are considered as random variables, and are assumed to have their mean values as 40 kPa and 27.5º, respectively, while both of them are assumed having the coefficient of variation (CoV) as 0.4. The parameter magnitudes of both the random variables are fetched from a 'Rectified Normal distribution' that has zero as the lower bound. The chosen CoV falls within the typical ranges for cohesion (0.05-0.5) and internal friction angles (0.02-0.56) as can be found from the literature [34]. It may be noted that for both the LEM-based and the RFEM-based slope stability analysis conducted for this study, the primary strength-related input parameters are cohesion (c), friction angle (φ) and the soil unit weight (γ). As the variability of soil unit weight (γ) is generally less as compared to the shear strength parameters [35-39], it has little influence in probabilistic slope stability study. Hence the unit weight of hillslope material is considered deterministic for the present study, having a constant magnitude of 20 kN/m^3. The other parameters necessary to conduct the slope stability analysis are the elastic parameters (Young's Modulus *Es* and Poisson's ratio υ) and the dilation angle (ψ). It is reported in literature that the dilation angle and the elastic parameters have little influence on the predicted stability of a slope [40]. It is also worth mentioning that the uncertainties in slope geometry and water table location within the slope also affects the toe-excavation induced slope stability, and the same has been highlighted in earlier literature [41-43]. However, the primary aim of the reported study is to assess the influence of the variability in shear strength parameters on the slope stability assessment of cut slopes through both LEM and RFEM-based methods and subsequently find out their efficacy of assessment. Hence, for the present study, only the variability in the shear strength parameters c and φ are considered, and, thus, the same are characterised as random fields.

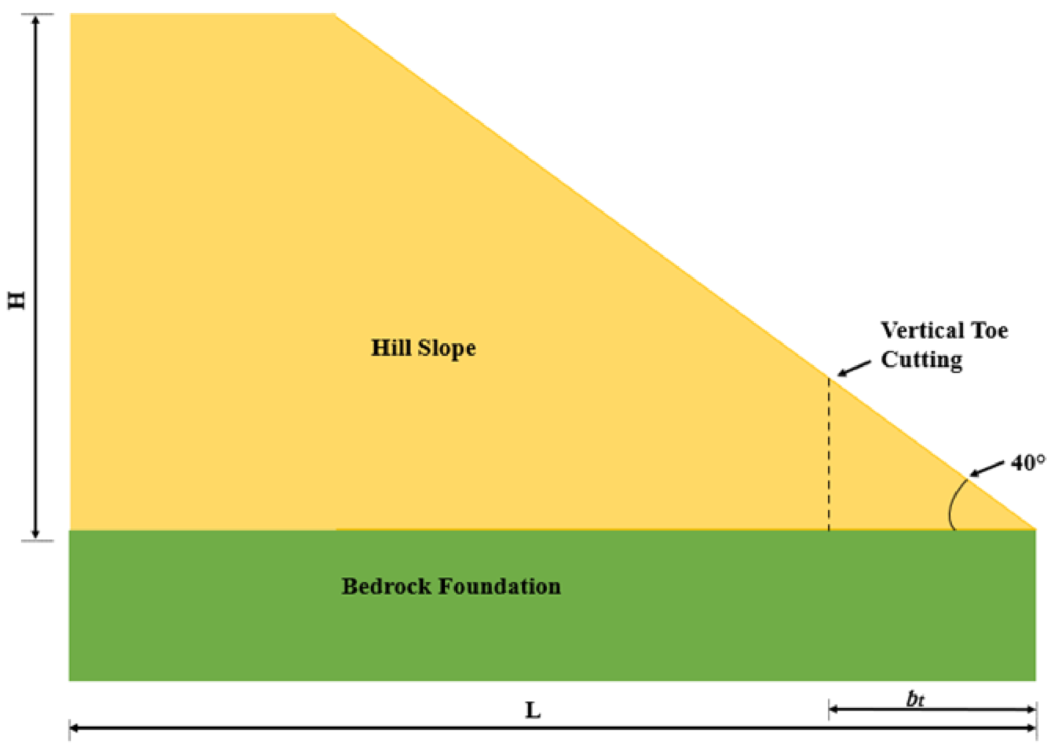


Fig. 1 Schematic diagram of cut slope geometry

4. LEM-based Probabilistic Analysis for Cut Slope

This section reports the failure probability of the chosen slope section estimated through LEM-based probabilistic approach whose details can be found in very recent literatures [14, 15]. In this part of the study, the spatial variation of c and φ is considered only in horizontal direction (represented by one-dimensional random field). The analyses are conducted using Morgenstern-Price LEM in Slope/W to assess the probability of failure. In such approach, it is necessary to identify the minimum numbers of MCS trials that would be required to generate a consistent estimate. Further increase in number of simulations beyond that required does not lead to significant refinement in the result, rather unfavourably increase the computational time. As shown in Fig. 2, it is understood that 2000 MCS trials is adequate to produce a consistent estimate of failure probability for the chosen problem, and the same is adopted for rest of the study.

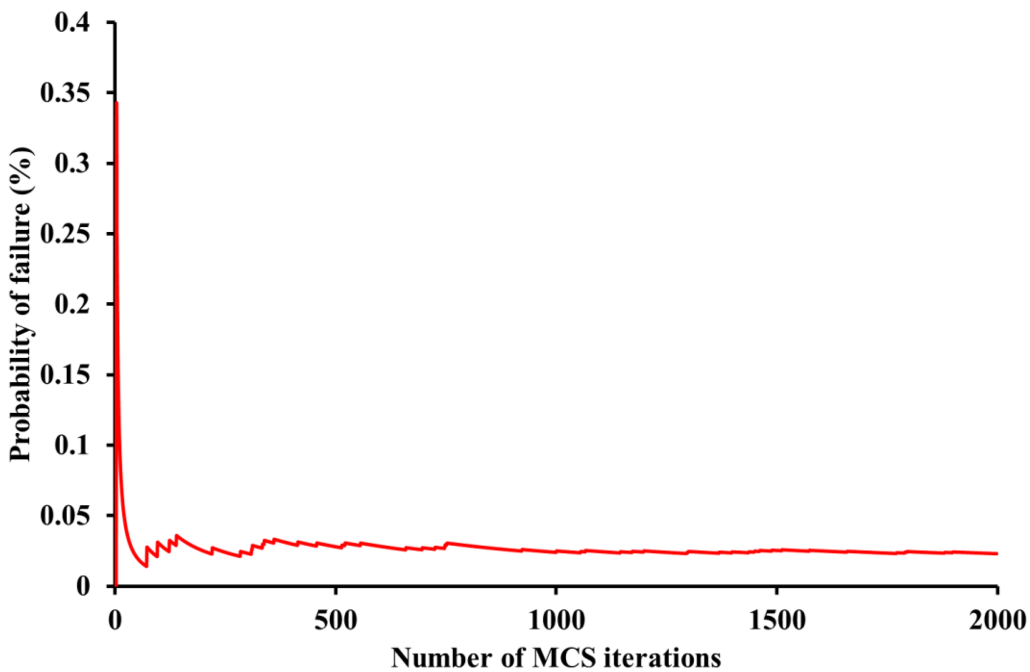


Fig. 2 Variation in the probability of failure with numbers of MCS iterations for Θ = 1

5. RFEM-based Probabilistic Analysis for Cut Slope

RFEM was developed in the 1990s [15, 44] and has proven to be a more appropriate tool for probabilistic analysis in geotechnical engineering, as it can comfortably incorporate the spatial variation of soil properties. However, the application of this advanced technique in regularly assessing the stability of cut slopes has yet not gained impetus. There are several instances of cut slope failure even after being designed using conventional tools. In this regard, this section attempts to highlight the importance of including RFEM as an effective measure of assessing the cut slope stability by duly incorporating the spatial variability of soil commonly encountered in hillslopes, thereby paving the possible way of safe and economical design.

In RFEM analysis, a soil property is considered to be a continuous random variable that is characterised using a probability density function (pdf) and is correlated with other random variables at adjoining locations. The set of random variables is characterised by the joint probability density function and is represented as a random field (RF). RFEM amalgamates the random field theory with the nonlinear FEM problem. In RFEM, a finite element mesh is overlaid by a random field and, thus, each mesh element (with its defined soil properties) behaves as a random variable. The random field is generated by Local average subdivision (LAS) technique that considers the spatial correlation among the random variables and also adopt a local averaging scheme. The spatial correlation between various soil properties (considered as random variables) is expressed by a spatial autocorrelation function. Based on a standard normal distribution function (i.e., having zero mean and unit variance) and a spatial correlation function, the LAS method generates correlated local averages of the soil property [45, 46]. Hence, the variance of any geotechnical property, spatially averaged over a particular soil domain, is lower than the variance at discrete locations. The variance decreases with an increase in the extent of the soil domain over which the property is averaged [16, 46-48]. For a random field whose mean and covariance is spatially varying, for mathematical convenience, it becomes essential to assume the random field to be statistically homogeneous or stationary whose characterising joint pdf is spatially invariant, i.e., its mean, variance and all the higher order moments remain constant at any location within the random field remains same. The correlation between any two random variables entirely depends on their separation distance, and not on their absolute locations. In order to characterize a stationary random field, it is important to have an a-priori knowledge of its mean, variance and the pattern of spatial variability characterized using correlation length.

5.1. Simulation of Random Field in *Rslope2D*

To simulate the spatial variation in a chosen slope domain, *Rslope2D* uses the random field theory developed by Vanmarcke [49, 50]. To conduct the elastoplastic FE slope stability analysis, *Rslope2D* uses various input parameters such as cohesion (c), friction angle (φ), dilation angle (ψ), Young's Modulus (E_s), Poisson's ratio (υ), and the soil unit weight (γ). For the present study, as explained earlier, only the shear strength parameters c and φ are considered random variables that are represented through a transformed log-normal pdf. The remaining input parameters are considered constant (γ = 20 kN/m^3, E_s = 100 MPa, υ = 0.3 and ψ = 0). In this part of the study, the slope geometry as well the mean and CoV of shear strength parameters are considered same as in the earlier discussed LEM-based probabilistic study. Subsequently, a two-dimensional random field for c and φ are assigned to the chosen slope section for the RFEM analysis. At this stage, for simplicity of the analyses, the cross-correlation coefficient between c and φ is considered zero, i.e. there is no correlation between the shear strength parameters. In this study, the two-dimensional spatial variation of the soil properties in the slope is characterized by an exponentially decaying (Markovian) correlation function, which implies that the covariance between two points decays exponentially with absolute separation distance between the points in the field.

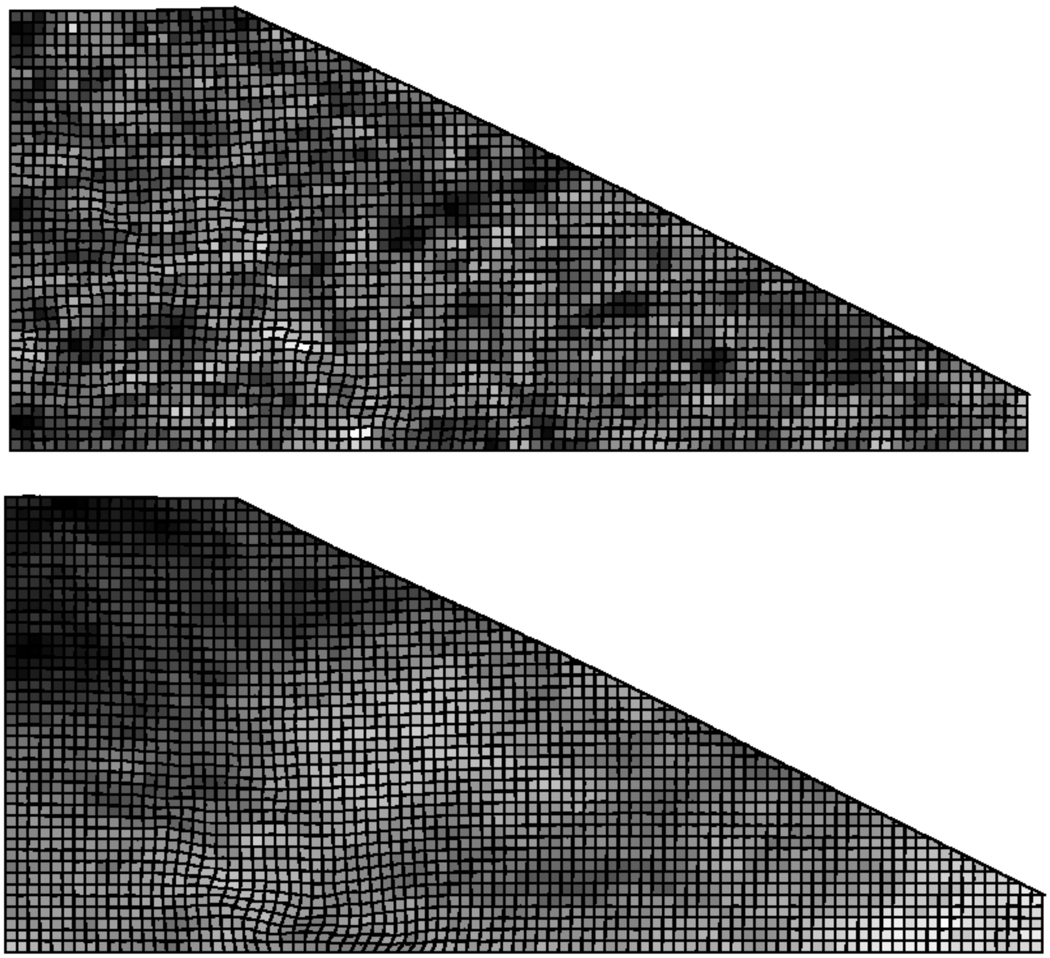
5.2. Estimation of Deterministic FoS and Probability of Failure

In the next stage, the random fields for c and φ are mapped onto the FE mesh of the slope geometry. To develop the FE model of the slope geometry, proper boundary conditions need to be assigned.

In this regard, for the FE models developed in *Rslope2D*, the bottom boundary of the slope is considered completely fixed from deformation in any direction. The vertical boundaries of the model are allowed to deform in the vertical direction, while the lateral deformation is restrained. Further, following a mesh convergence study, the FE model of the slope is optimally discretized into elements of size of 1 m × 1 m. The finite elements used in the model are represented by 8-noded quadrilateral elements with reduced integration for addressing various phases of the algorithm including generation of gravity loads, stiffness matrix evaluation and stress redistribution analyses [40, 48, 51, 52]. For plane strain FE modelling in *Rslope2D*, the constitutive behaviour of soil is represented through an elastic-perfectly plastic stress-strain law enveloped by a Mohr-Coulomb failure criterion. With the mean values of shear strength parameters (c and φ), the FE algorithm in *Rslope2D* utilizes strength reduction technique to estimate the deterministic FoS [53] that is defined as factor by which the original shear strength parameters need to be reduced for generating marginal stability condition for failure [54].

In the next stage of analysis, the probability of failure is estimated using MCS approach. As shown in Fig. 2, the consistency of the computed probability of failure (P_f) has a strong dependency on the number of MCS conducted. It was reported by Griffiths and Fenton [17] that 1000 numbers of MCS are sufficient for a 2H:1V undrained clay slope to generate converged estimates of P_f. However, based on several studies, it has been recommended to use 4000 number of MCS for a drained c-φ soil slope (soil) while using the *Rslope2D* computer model [48], and the same is adopted in the present study to estimate the probability of cut slope failure for various correlation lengths. For generalization, the correlation length is presented with a dimensionless correlation length (Θ) that is obtained by dividing the absolute correlation length (θ) by the width of the excavated cut slope (L-b_t), and is expressed as $\Theta = \theta/(L-b_t)$.

Figure 3 shows the typical random fields for cohesion assigned over the discretized soil domain for two different correlation lengths. Accordingly, the magnitudes of cohesion are assigned to each of the mesh element that behaves as a random variable for the probabilistic assessment of cut slope stability through an MCS approach. In Fig. 3, the light greyish cells represent relatively weaker sections of the slope comprising lesser cohesion values, whereas the darker cells represent the mesh elements with higher magnitudes of cohesion. Figure 3a represents a comparatively smaller correlation length ($\Theta = 0.05$), while Fig. 3b represents a larger correlation length ($\Theta = 1$). It should be noted that both the cohesion distributions are extracted from the same log-normal distribution assigned to the property, and only the spatial correlation length is different. Figure 3 clearly highlights that with the change in the correlation length, the initial distribution of the shear strength parameters might vary significantly, thereby resulting in recognizable difference in the failure probabilities. Indirectly, this figure also indicates the sheer importance of conducting a thorough field survey to capture the spatial variation in the soil properties to the best possible detail.



(a)

(b)

Fig. 3 Typical two-dimensional random field for cohesion magnitudes extracted from the same lognormal pdf using the dimensionless correlation length of (a) $\Theta = 0.05$ and (b) $\Theta = 1$

6. Comparison of LEM and RFEM-based Analyses

The deterministic study conducted on the chosen slope section using LEM in Slope/W and FEM in *Rslope2D* resulted in a FoS value of 1.63 and 2, respectively. Therefore, as per the deterministic analysis, for the chosen slope section, a 10 m wide toe cut can be recommended without jeopardizing the safety of the resulting vertical face of the cut slope. For different correlation lengths, Figure 4 presents the comparison of failure probabilities of the chosen slope section (b_t = 10 m) obtained by 1-D random field modeling (using LEM-based probabilistic approach) with that from the 2-D random field modeling (from RFEM-based analyses). The study shows that the LEM-based probabilistic study predicts the chosen slope section to be safe up to $\Theta = 0.7$, with a low probability of failure and having a performance level 'above average' [14]. However, on the contrary, the RFEM analysis exhibits the same feature up to $\Theta = 0.5$. Therefore, it is noticed that LEM-based approach considering 1-D random field underestimates the failure probabilities as compared to RFEM. *Rslope2D* can simulate the field uncertainty more realistically than the 1-D spatial variation, thereby increasing the failure probability. Moreover, LEM-based deterministic approach presumes the failure slip plane, whereas the RFEM allows the failure slip plane to generate along the best possible weakest plane in the soil domain in each realization which is governed by the local perturbations in the shear strength properties.

It is also noticed that estimation of probability of cut slope failure, in both the methods, are highly influenced by the spatial variation of the shear strength properties within the soil domain. For small correlation length, the failure probability is essentially zero; while for intermediate correlation lengths, the failure probability increases rapidly. For very small value of correlation length, the local averaging gets maximized. Therefore, in case of small correlation lengths, the random variables tend to attract values very near to their mean values and, thus, for each and every realization, the soil domain become essentially homogeneous. As the mean values provide a safe slope (FoS value is more than one for this particular slope geometry under specified soil properties), the failure probability is always zero. In case of large correlation lengths (as Θ approaches to 1), the entire soil domain becomes highly correlated. Therefore, the slope becomes essentially homogeneous within each realization, but different from one realization to the next. As a result, the probability of failure values for both the approaches considered in present study coincides for large correlation lengths (Fig. 4). However, for intermediate values of correlation lengths, the slope domain reflects heterogeneity in shear strength of soil and, thus, the estimation of failure probability is governed by the spatial variation characterized using correlation length.

The study reveals that, in both the approaches, the probability of cut slope failure is very sensitive to the choice of correlation length characterizing the spatial correlation model that is used to simulate the in-situ spatial variation of the soil shear strength parameters. Given the inherent heterogeneity of soil and spatial variation of soil properties, very small correlation lengths or large correlation lengths are of mere mathematical interest, which have very little practical importance. Geotechnical engineers are most likely to encounter intermediate correlation lengths, and the study shows that P_f changes rapidly in this intermediate range. It is also noteworthy that in this intermediate range of dimensionless correlation length, the two different approaches predict noticeably different probabilities of failure. Hence, it highlights the importance of ascertaining a proper and reliable spatial variation model for a particular site before designing an engineered cut slope. This realization calls for the necessity to conduct reasonable numbers of field investigations to ideate the in-situ spatial variation of the desired properties that can be suitably transpired to the realistic slope stability analysis of cut-slopes.

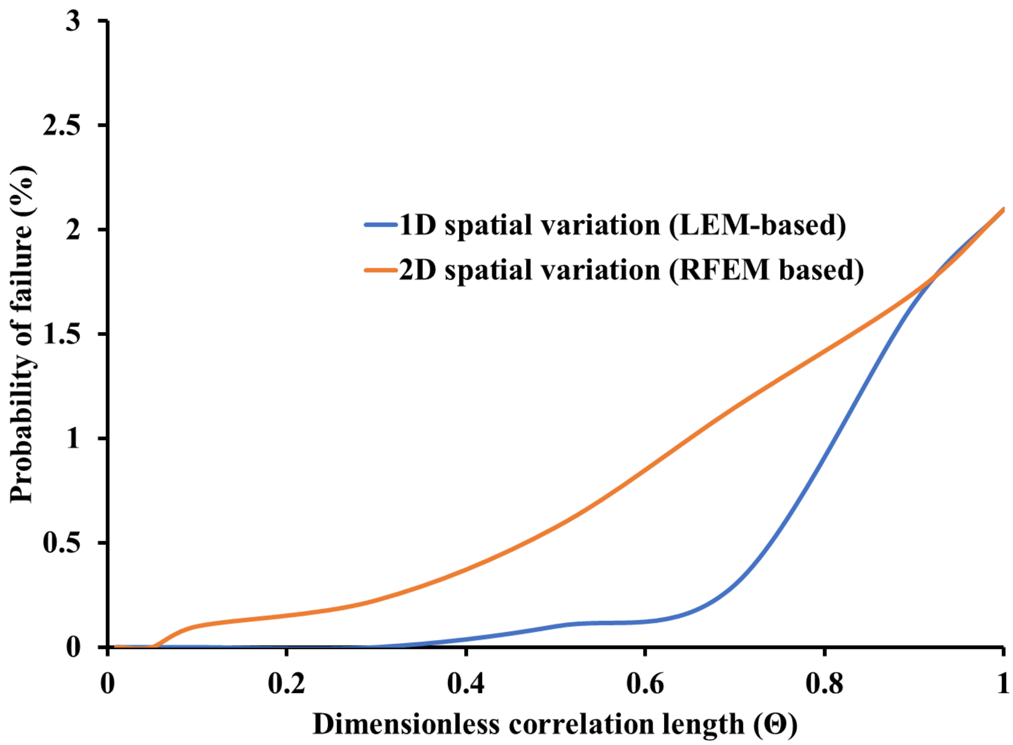


Fig. 4 Comparison of the probability of failure for various dimensionless correlation length (Θ) for the slope section chosen

7. Effect of Coefficient of Variation (CoV) and Cross-Correlation Coefficient in RFEM analysis

This section reports the influence of the coefficient of variation (CoV) and cross-correlation between the soil shear strength properties (cohesion, c, and angle of internal friction, φ) on the failure probability of the chosen slope section (H = 40 m, b_t = 10 m). The study is carried out by using RFEM in *Rslope2D*, wherein cross-correlation coefficient ($\rho_{c\varphi}$) between c and φ is implemented by 'covariance matrix decomposition' method [55].

To seek out the influence of CoV, the different values considered are 0.2, 0.3 and 0.4. The different magnitudes of the cross-correlation coefficient between the shear strength parameters of soil (c and φ) are considered as -0.5, 0 and +0.5, which indicates that the chosen parameters are negatively correlated, uncorrelated or positively correlated, respectively. All these forms of correlations can possibly exist between the shear strength parameters over a region [56]. A negative correlation coefficient indicates that as c draws a higher magnitude from its distribution, φ would draw a lower value; whereas a positive correlation coefficient indicates that both c and φ would simultaneously draw higher or lower values from the distribution. If the parameters are uncorrelated, they can draw any values from their corresponding distribution. The significance of the different correlation coefficients is well explained in standard literature [34, 47, 56]. The choice of these three $\rho_{c\varphi}$ values, as adopted in the present study, covers the broad spectrum of correlation coefficients that can possibly exist between the shear strength parameters, and is helpful to develop a notional understanding of their effects on the probability of failure.

For a particular value of dimensionless correlation length ($\Theta = 1$), the results are presented in Fig. 5. Considering a cross-correlation coefficient of 0.5, the probabilities of failure (P_f) of the cut slope with b_t = 10 m are estimated as zero, 0.73% and 5.83% for CoV values of 0.2, 0.3 and 0.4, respectively. Hence, it is seen that the probability of failure augments with the increase in CoV value. A higher CoV indicates that the c and φ values become more dispersed from their mean, thereby resulting in the increased P_f values. For the same cut slope section, considering a CoV of 0.4, Fig. 5 shows that the probabilities of failure (P_f) are estimated as 0.23%, 2.1% and 5.83% for cross-correlation coefficient values of -0.5, 0 and +0.5, respectively. It is seen that the probability of failure of the cut slope decreases when the shear strength parameters are either not correlated or becomes negatively correlated. The observation is in consent with the findings from the LEM-based probabilistic analysis available in literature [14]. Therefore, a slope with negative or no correlation between the shear strength parameters is likely to be more stable as compared to a slope having a positive correlation between c and φ. Therefore, the RFEM analysis conducted on the cut slope section shows that the probability of failure is highly sensitive to the selection of CoV and cross-correlation coefficient value of the random variables (in this case the shear strength parameters). Hence, it can be realised that a proper selection of CoV and cross-correlation coefficient value is required for estimation of probability of failure of cut slope using random finite element method.

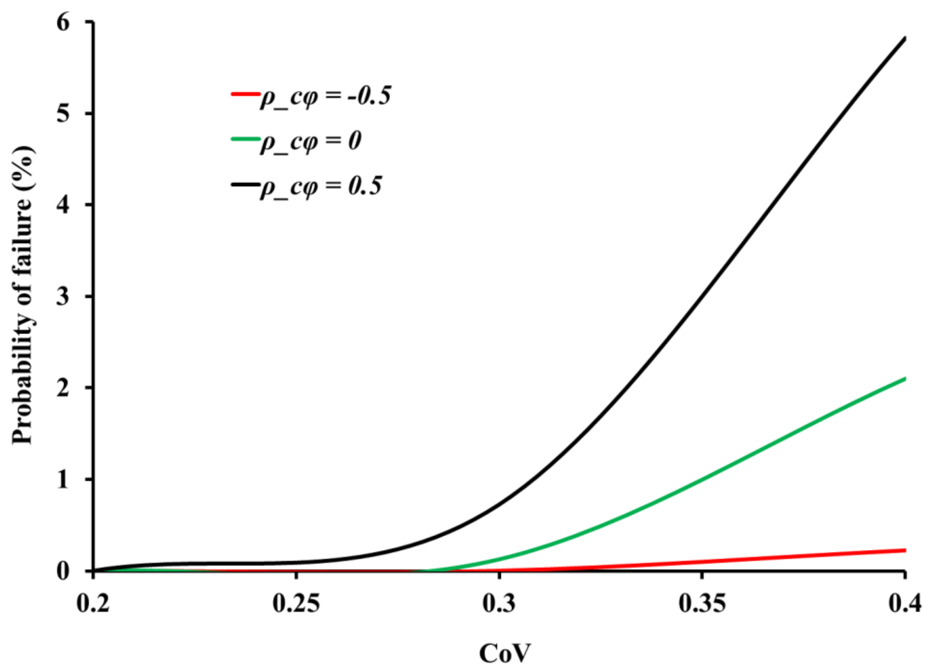


Fig. 5 Variation of probability of failure with CoV for various cross-correlation coefficients

8. Conclusions

This paper reports the contrast between the random finite element method (RFEM) and limit equilibrium method (LEM) based probabilistic stability analysis of a cut slope. The study shows that the estimated failure probability is highly governed by the spatial correlation length. The reported LEM-based probabilistic ignores the spatial variation of shear strength parameters in vertical direction and also presumes the critical failure surface. On the other hand, RFEM is capable of simulating spatial variation in both the directions and also allows the slip surface to generate along the weakest path. It is concluded that in comparison to 2-D spatial variation, considering 1-D spatial variation of shear strength parameters underestimates the failure probability of the cut slope in the intermediate ranges of correlation length. Thus, consideration of 2-D spatial variability in the soil shear strength parameters using RFEM approach proves to be more realistic. It is noted that for LEM and RFEM-based probabilistic study, a slope section is adjudged safe having a low value of probability of failure (defined by a performance level 'above average') for a dimensionless correlation length (Θ) up to 0.7 and 0.5, respectively. Any higher value of correlation length would result in an unsafe cut slope with higher probabilities of failure.

The RFEM-based analysis highlighted that the selection of CoV and cross-correlation coefficient have significant influence on computed failure probability. It is highlighted that increase in CoV results in an increment of failure probability. Furthermore, positively correlated shear strength parameters increases the probability of failure of a cut slope. For a cross-correlation coefficient of +0.5, P_f of the cut slope varies from zero to 5.83% for CoV ranging from 0.2 to 0.4. For a CoV of 0.4, the P_f of the cut slope varies 0.23% to 5.83% for the cross-correlation coefficient values ranging from -0.5 to +0.5, respectively. In a nutshell, the study reveals the importance of conducting a detailed site specific investigation to assess the spatial variability of the soil shear strength parameter. However, in case a 2-D spatial variability is not readily available based on limited site investigation, standard literature may be referred for generating synthetic yet reasonable spatial variabilities, and the same can be used for various probabilistic analysis for unsupported or retained slopes.

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