# Analog & Digital Electronics

Course No: PH-218

## Lec-8: BJT Small Signal Analysis

Course Instructors:

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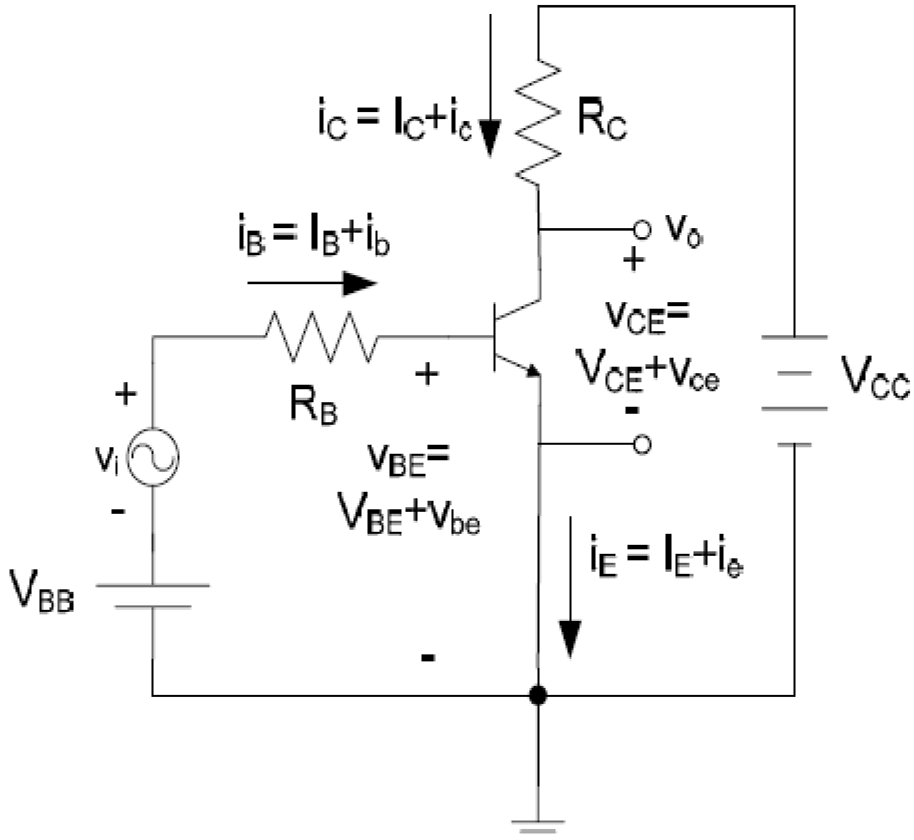
# BJT with input ac signal




$i_B = I_B + I_b$

$i_E = I_E + i_e$

$i_C = I_C + i_c$

$I_B$, $I_C$, $I_E$ - D.C. currents

$I_b$, $i_c$, $i_e$ – A.C. currents

$i_B$, $i_C$, $i_E$ – D.C + A.C. currents

Similarly,

$V_{BE}$, $V_{CE}$ – D.C. Voltages

$v_{be}$, $v_{ce}$ – A.C.Voltages

$v_{BE}$, $v_{CE}$ – D.C+ A.C Voltages

# BJT with small ac input signal

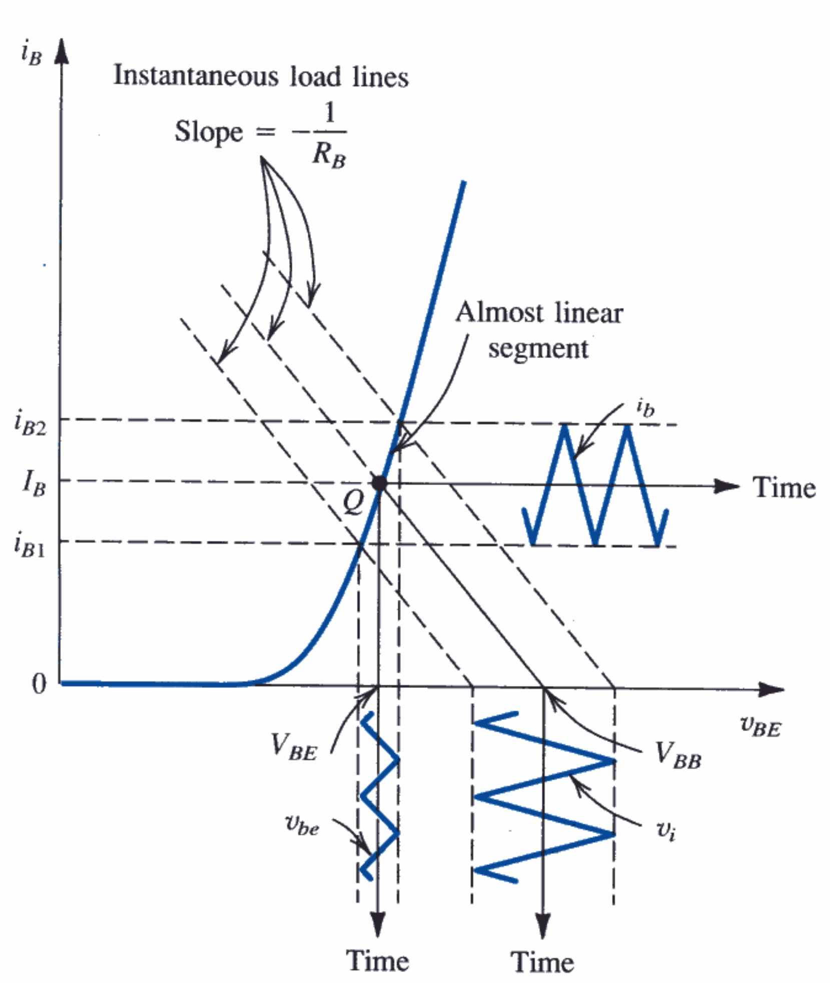
**Small ac signal refers to the input signal ($v_{be}$) whose magnitude is much small than thermal voltage ($V_T$) i.e. $v_{be} << V_T$**

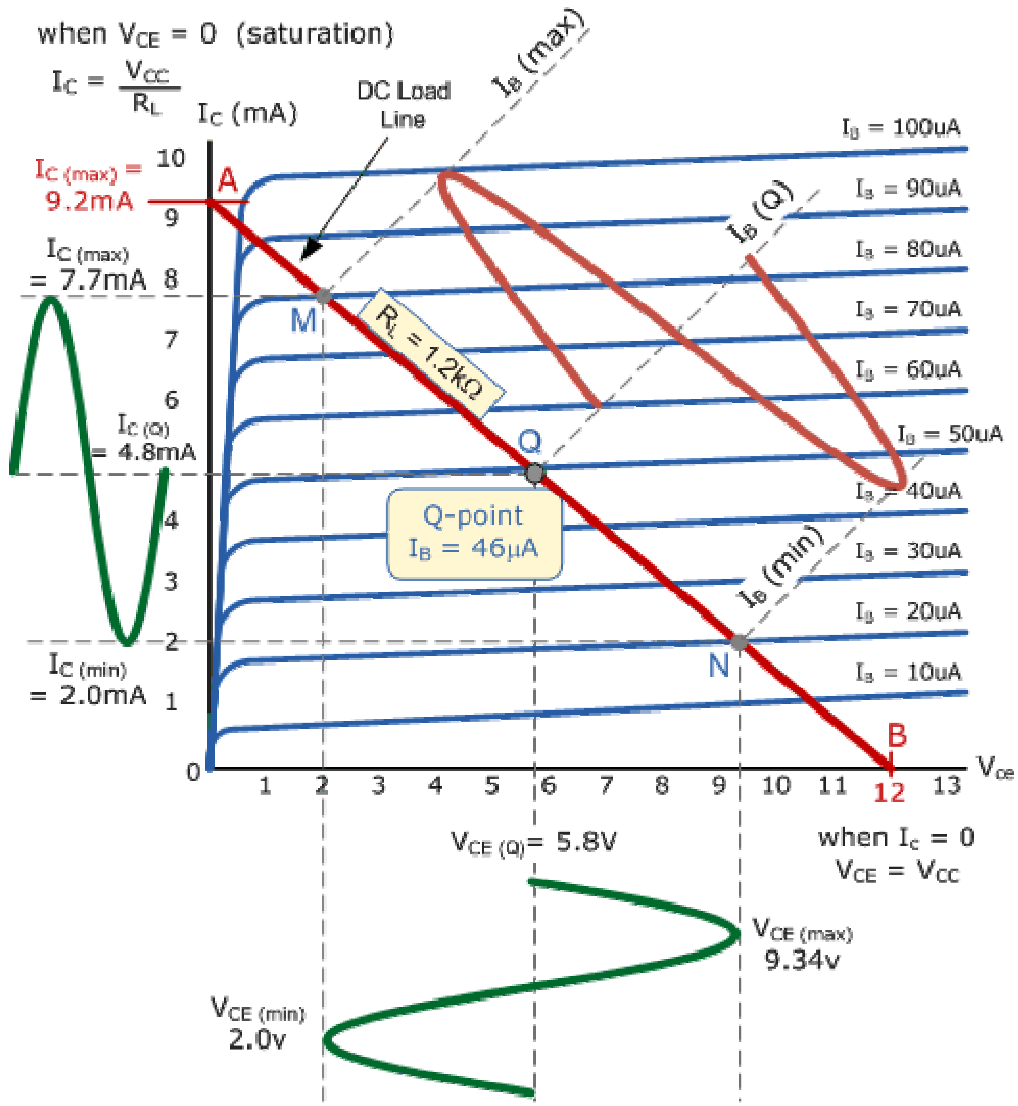
Magnitude of the ac signal applied for amplification must be small so that

- the transistor operates in the linear region for the whole cycle of input (called as a linear amplifier)
- the transistor is never driven into saturation or cut-off region

- On the other hand, if the input signal is too large. The fluctuations along the load line will drive the transistor into either saturation or cut off. This clips the peaks of the input and the amplifier is no longer linear.

# Graphical Analysis: Load Line

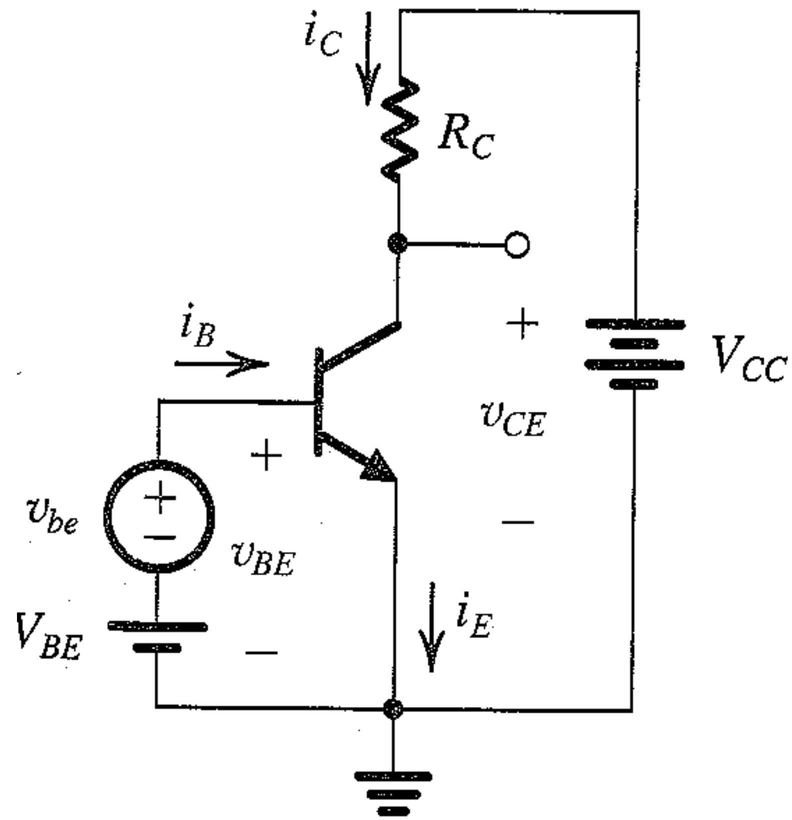


# Small Signal Analysis

If an ac+dc input signal the total $v_{BE}$ becomes

$$v_{BE} = v_{be} + V_{BE}$$




**The collector current becomes**

$$i_C = I_S \exp(v_{be} + V_{BE})/V_T$$

$$i_C = I_S \exp(V_{BE}/V_T)\exp(v_{be}/V_T)$$

$$i_C = I_C \exp(v_{be}/V_T)$$

For small signal $v_{be} \ll V_T$ (10mV) hence

$$i_C = I_C (1 + v_{be}/V_T)$$

$$i_C = I_C + I_C v_{be}/V_T$$

The ac component of the collector current is: $i_c = I_C v_{be}/V_T$

$$g_m = \frac{i_c}{v_{be}} = \frac{I_C}{V_T}$$

$g_m$ is called the small signal ***transconductance***

It represents the slope of $i_C$-$v_{BE}$ curve at the Q point.

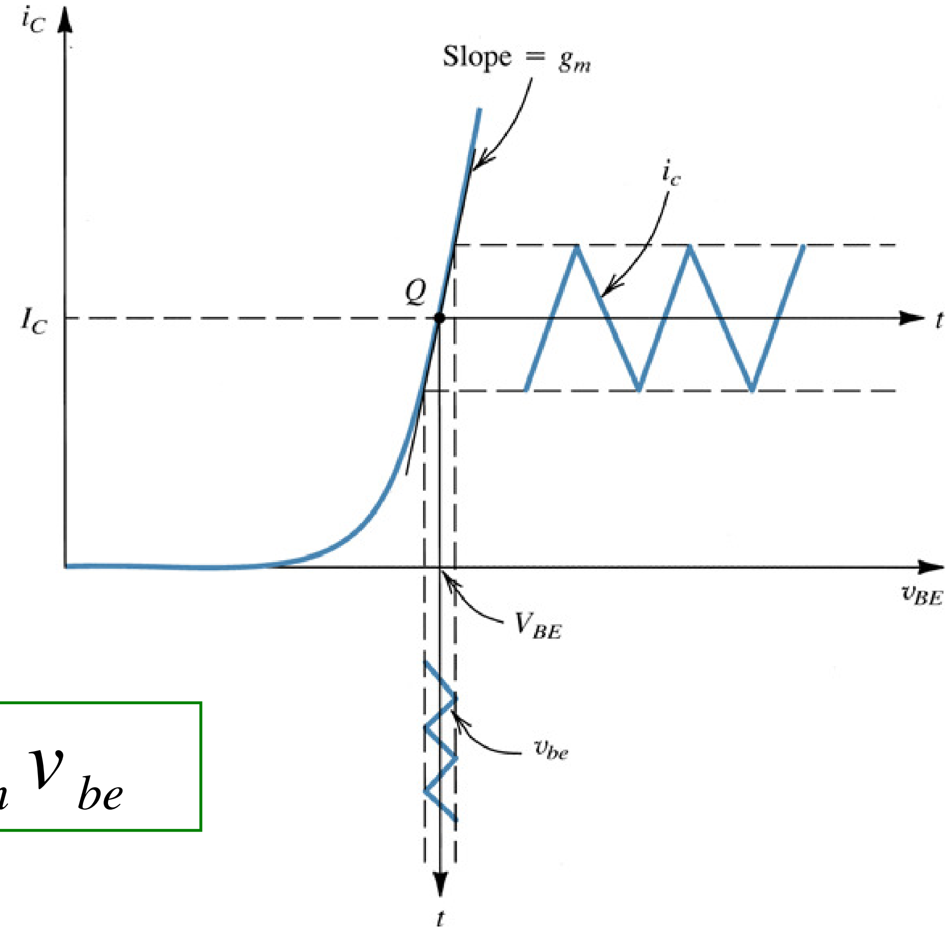
# Small-signal Transconductance

The small signal approximation implies that signal is so small that operation is restricted to an almost linear segment of the $i_C$-$v_{BE}$ exponential curve.

$$g_m = \left(\frac{i_c}{v_{be}}\right)_{v_{be}=0} = \left(\frac{\partial i_C}{\partial v_{BE}}\right)_{i_C = I_C}$$

$$g_m = \frac{i_c}{v_{be}} = \frac{I_C}{V_T}$$

$$i_c = g_m v_{be}$$




The small signal analysis suggests that for a small signal, transistor behaves as a voltage controlled current source. The input port of the controlled current source is between base and emitter and output port is in between collector and emitter.

# Small-signal Analysis: Current and input resistance

**The total base current:** $i_B = I_B + i_b$

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta}\frac{I_C}{V_T}v_{be}$$

Signal component of base current: $i_b = \frac{1}{\beta}\frac{I_C}{V_T}v_{be} = \frac{g_m}{\beta}v_{be}$

$$r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

**$r_\pi$ is the small-signal input resistance between base and emitter, *looking into the base.***

**The total emitter current:** $i_E = I_E + i_e$

$$i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha}$$

$$i_e = \frac{i_c}{\alpha} = \frac{I_C}{\alpha V_T}v_{be} = \frac{I_E}{V_T}v_{be}$$

$$r_e = \frac{v_{be}}{i_e} = \frac{\alpha}{g_m} = \frac{V_T}{I_E}$$

**$r_e$ is the small-signal input resistance between base and emitter, *looking into the emitter.***

$$v_{be} = i_b r_\pi = i_e r_e$$

$$r_\pi = (\beta + 1) r_e$$

# Hybrid-π small signal model of BJT

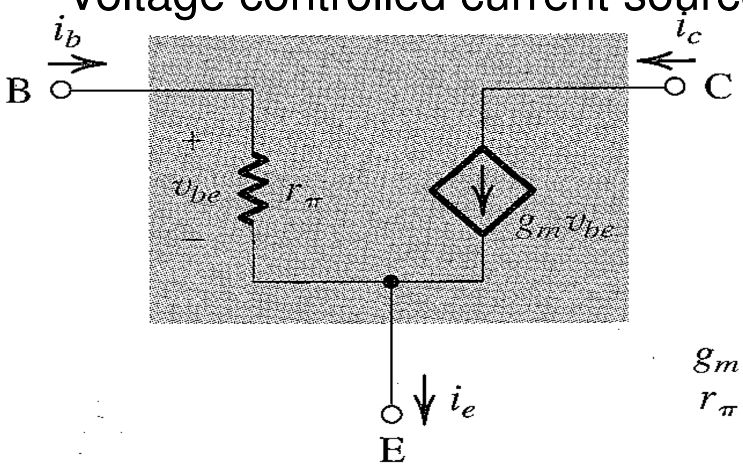
This model represents that transistor as a voltage controlled current source with control voltage $v_{be}$ and include the input resistance looking into the base.

$$r_\pi = \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B}$$

$$r_e = \frac{v_{be}}{i_e} = \frac{\alpha}{g_m} = \frac{V_T}{I_E}$$

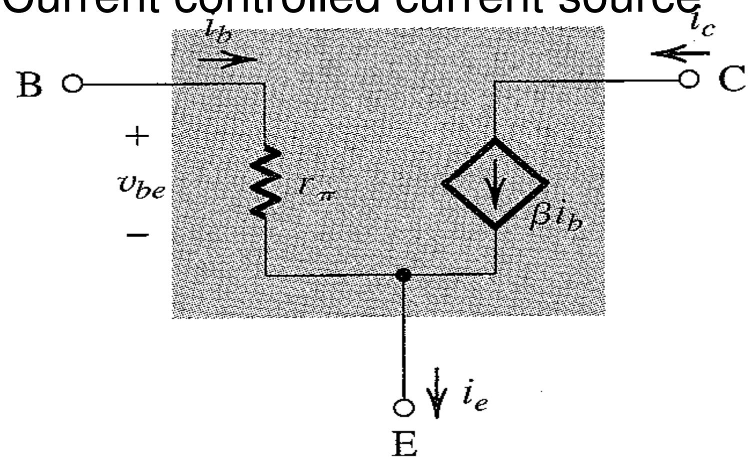
$$g_m = \frac{i_c}{v_{be}} = \frac{I_C}{V_T}$$

Voltage controlled current source




$$g_m = I_C/V_T$$
$$r_\pi = \beta/g_m$$

Current controlled current source




$$\begin{aligned} i_e &= \frac{v_{be}}{r_\pi} + g_m v_{be} = \frac{v_{be}}{r_\pi}(1 + g_m r_\pi) \\ &= \frac{v_{be}}{r_\pi}(1+\beta) = v_{be} \Big/ \left(\frac{r_\pi}{1+\beta}\right) \\ &= v_{be}/r_e \end{aligned}$$

$$\begin{aligned} g_m v_{be} &= g_m (i_b r_\pi) \\ &= (g_m r_\pi) i_b = \beta i_b \end{aligned}$$

# Hybrid-π model including Early effect

$$I_C = I_s\, e^{\left(\frac{eV_{BE}}{kT}\right)}\left[1+\frac{V_{CE}}{V_A}\right]$$

*$V_A$ is called the Early voltage and ranges from about 50 V* to 100 V.

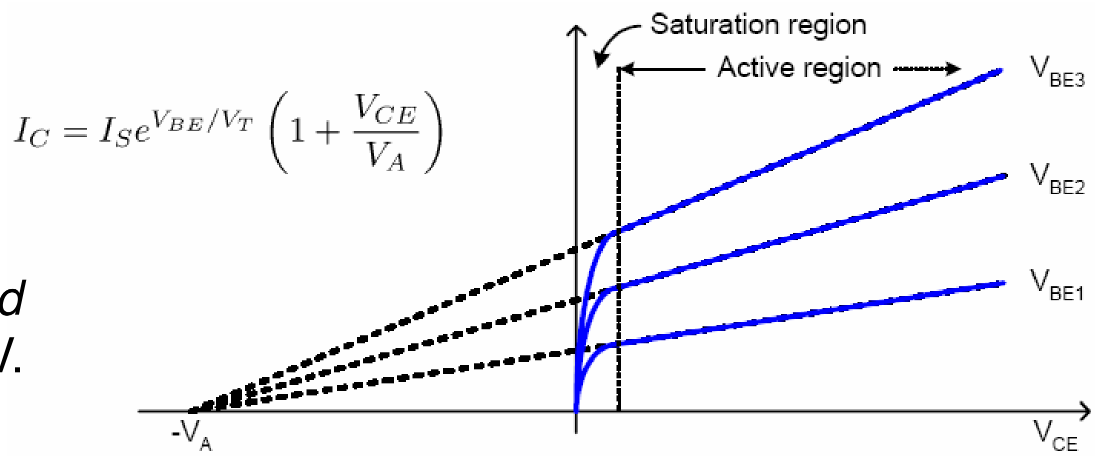
*Early effect can be modeled as*

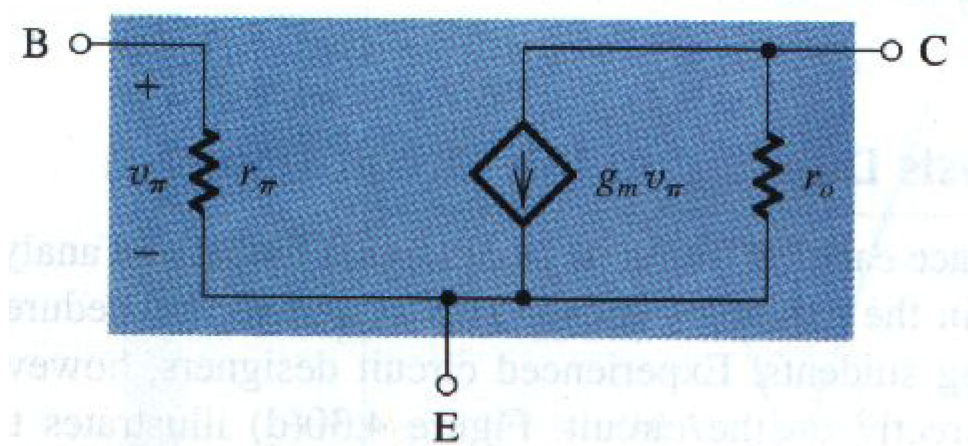
$$I_C = I_c^{'} + \frac{V_{CE}}{r_o}$$

$$I_c^{'} = \mathrm{I_s}\exp\left(\frac{eV_{BE}}{kT}\right)$$

$$r_o \equiv \left[\left.\frac{\partial i_C}{\partial v_{CE}}\right|_{v_{BE}=\text{constant}}\right]^{-1}$$

$$r_o = \frac{V_A}{I'_C}$$



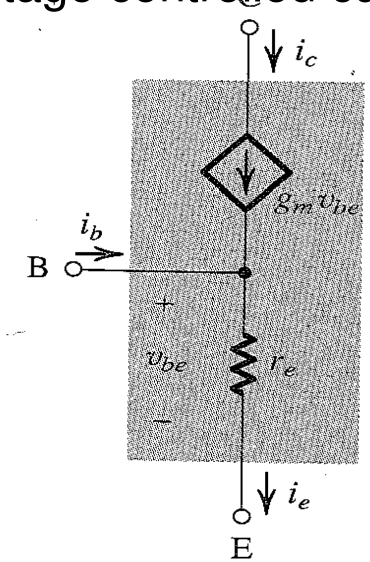




Include Early Effect in the model:

# *Small signal T model of BJT*

This model represents that transistor as a voltage controlled current source with control voltage $v_{be}$ and include the input **resistance looking into the emitter**.

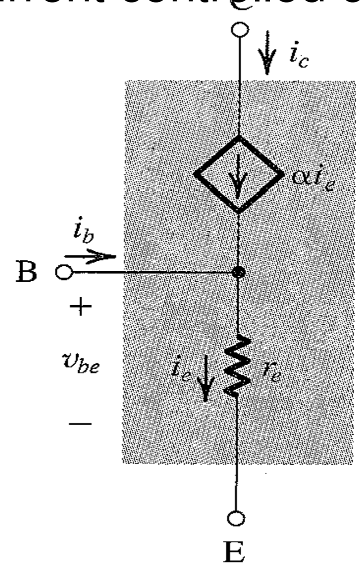
Voltage controlled current source




$$g_m = I_C / V_T$$

$$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$$

Current controlled current source




$$i_b = \frac{v_{be}}{r_e} - g_m v_{be} = \frac{v_{be}}{r_e}(1 - g_m r_e)$$

$$= \frac{v_{be}}{r_e}(1-\alpha) = \frac{v_{be}}{r_e}\left(1 - \frac{\beta}{\beta+1}\right)$$

$$= \frac{v_{be}}{(\beta+1)r_e} = \frac{v_{be}}{r_\pi}$$

$$g_m v_{be} = g_m(i_e r_e)$$

$$= (g_m r_e) i_e = \alpha i_e$$

# BJT Circuit Analysis using Small Signal Model:

1. **Determine the DC operating point of the BJT and in particular, the collector current $I_C$**

2. **Calculate small-signal model parameters $g_m$, $r_\pi$, & $r_e$ for this DC operating point**

3. **Eliminate DC sources**

❖ **Replace DC voltage sources with short circuits**

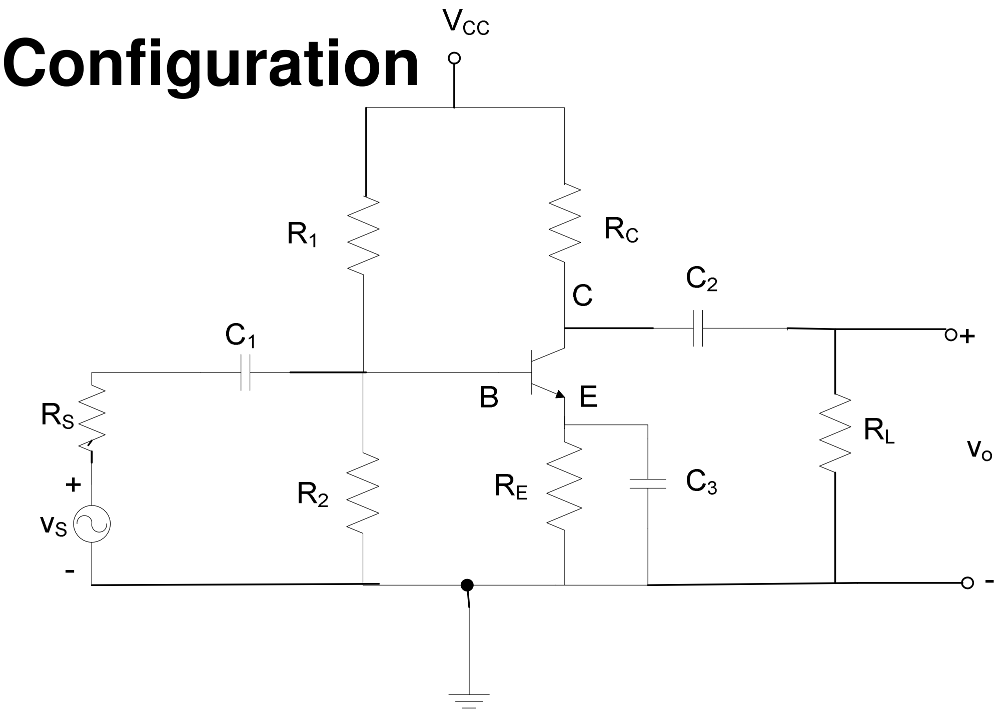
❖ **Replace DC current sources with open circuits**

4. **Replacing all capacitors by a short circuit equivalent and remove all elements bypassed by the short circuit equivalents.**

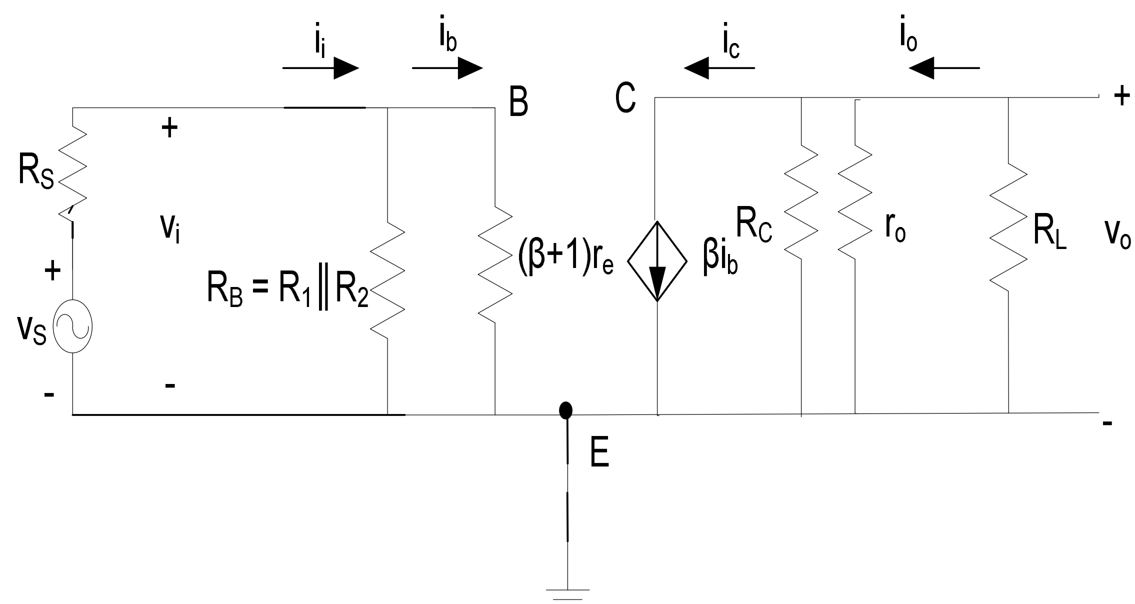
5. **Replace BJT with an equivalent small-signal model**

6. **Analyze the resulting circuit to determine the required quantities e.g. voltage gain, input resistance…etc.**

# CE Voltage- Divider Bias Configuration




# A.C. Equivalent Circuit



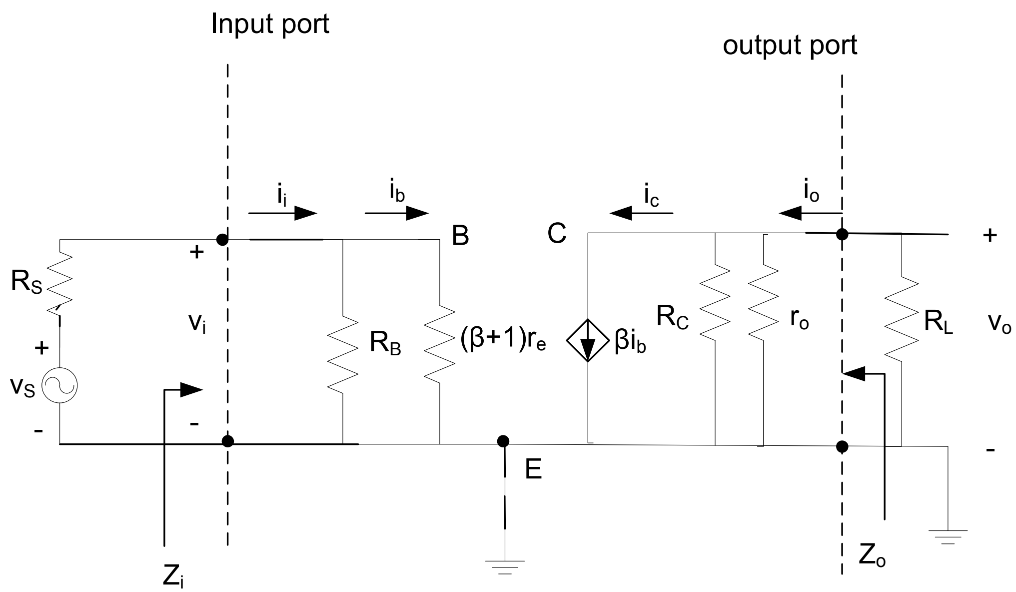
# Small Signal Performance Parameters

**In AC analysis, we are interested to find**

- ❖ Input resistance or impedance
- ❖ Output resistance or impedance
- ❖ Voltage gain

## Input resistance

The total resistance looking into the amplifier at coupling capacitor $C_1$ represents total resistance of the amplifier presented to signal source




$$Z_i = \frac{v_i}{i}; i = i_1 + i_b$$

$$v_i = i[R_B \backslash\backslash (\beta+1) r_e]$$

$$Z_i = [R_B \backslash\backslash (\beta+1) r_e]$$

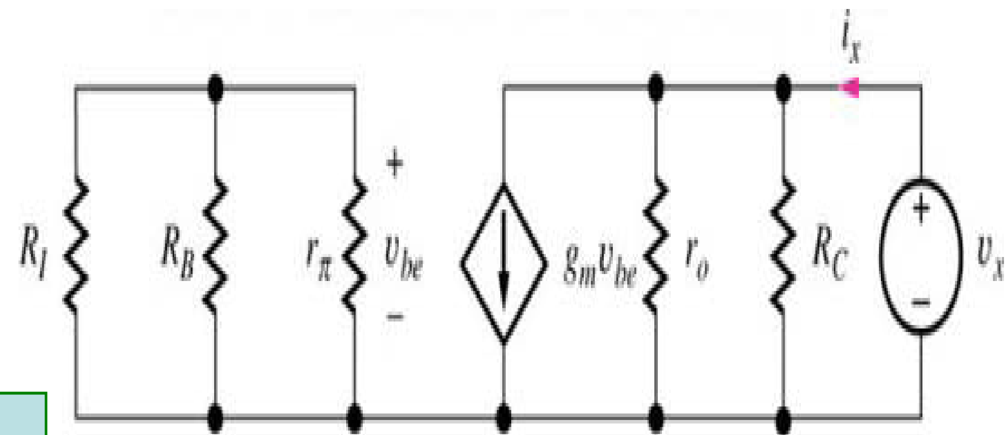
$$Z_i = [R_1 \backslash\backslash R_2 \backslash\backslash (\beta+1) r_e]$$

## Output resistance

The total resistance looking into the output of the amplifier at coupling capacitor $C_2$ represents output resistance of the amplifier.

To find $Z_{out}$, input source is set to 0 and test source is applied at output

$$i_x = \frac{v_x}{R_c} + \frac{v_x}{r_o} + g_m v_{be}$$




But $v_{be}$ is zero . Therefore

$$Z_{out} = \frac{V_x}{i_x} = (R_C \,//\, r_o)$$

$$Z_{out} = R_C$$ Because $r_o >> R_C$

# Voltage Gain

$$A_v = \frac{v_o}{v_i} = \frac{-i_o R_L}{i_b(\beta+1)r_e} = \frac{-\beta i_b\left(\frac{R_C \| r_o}{R_C \| r_o + R_L}\right)R_L}{i_b(\beta+1)r_e}$$

$$\text{or,}\quad A_v = \frac{-\beta R_C \| r_o \| R_L}{(\beta+1)r_e} \approx \frac{-R_C \| r_o \| R_L}{r_e}$$