AN EFFICIENT INVERSE METHOD FOR DETERMINING THE MATERIAL PARAMETERS AND COEFFICIENT OF FRICTION IN WARM ROLLING PROCESS

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Abstract

In this work, the material parameters for power law and coefficient of friction are obtained using inverse analysis by measuring exit strip temperature and slip. The procedure makes use of finite element model for deformation and an analytical method for the estimation of temperature. A heuristic optimization algorithm is used for this purpose that minimizes the error between the measured and estimated flow stresses. The method is verified by conducting some numerical experiments. Less than 1% error is observed.

Keywords: Flow stresses, Temperature distribution, Finite element method, Inverse analysis

1 Introduction

Rolling is one of the oldest and most important metal forming processes. The rolling process can be classified into three categories *viz.*, cold, warm and hot rolling. The focus of the present work is to develop an inverse procedure for the estimation of material properties and friction coefficient in a warm rolling process. A few researchers have applied inverse analysis to estimate parameters in rolling process. Lenard and Zhang (1997) estimated the friction through an inverse technique by matching the measured and computed roll force, roll torque and forward slip. Lenard and Nad (2002) estimated the coefficient of friction as a function of rolling process and material parameters by inverse analysis. The coefficient of friction was chosen to allow the matching of the measured and calculated roll force and roll torque. Cho and Ngaile (2003) proposed an inverse method to determine the flow stresses and friction coefficient. The inverse method was based on the minimization of the difference between the experimental loads and the corresponding finite element method (FEM) predictions. In their work, the rigid visco-plastic finite element formulation was used to obtain the flow stresses while optimization algorithm adjusts the parameters used in the simulation until the calculated response matches the experimental measurements within a specified tolerance. Han (2005) applied the modified two-specimen method (MSTM) for the online determination of flow stresses and coefficient of friction in rolling. In their method, the strip is rolled twice with two different sets of roll radii. Instead of real experiments, ABAQUS FEM simulations were used for validating the methodology. Cho and Altan (2005) proposed the inverse method to determine flow stress and friction factor of the bulk material under isothermal condition. Byon *et al.* (2008) proposed inverse method for the prediction of flow stress-strain curve and coefficient of friction using actual mill data. The roll force and forward slip is taken as the basis for inverse estimation. In their work, forward slip is decided to find the coefficient of friction and the material parameters are subsequently obtained based on roll force matching. Chen and Yang (2010) used conjugate gradient method to estimate the heat generation at the roll-strip interface by temperature measurement at some locations in the roll. For validation of the proposed method, simulated temperatures were used instead of the real experiments. Weisz-Patrault *et al.* (2011) proposed an inverse method to determine the temperature field at the surface of the roll by measuring the temperature using thermocouple at only one point of the roll.

In the present work, an inverse analysis is carried out to predict the material parameters and coefficient of friction during warm rolling. The heuristic based optimization is used to identify the unknown material parameter and friction. The main contributions of this paper are as follows. Firstly, a simple computationally and experimentally efficient algorithm has been used for minimization of the objective function. Secondly, a method is proposed for estimating process parameter dependent coefficient of friction as well as material parameters based on the measurement of slip and temperature of strip at exit.

2 Model of Warm Rolling

A two-dimensional thermo-mechanical analysis based on Eulerian formulation has been employed to simulate the strip rolling process. The analysis is divided into two modules– deformation module and thermal module. Deformation module is based on the flow formulation of FEM. Thermal module uses an approximate analytical method. Initially the deformation module carries out the analysis at inlet temperature of the strip. The output of deformation module is used in thermal module to estimate the average temperature of the deformation zone. Now, deformation module is activated again to carry out the analysis at this average temperature. This iterative procedure continues till convergence. Figure 1 shows the flow chart of the procedure.

2.1 Deformation module

Eulerian flow formulation based FEM is employed to analyze stresses, roll torque, roll force and forward slip considering plane strain condition. This formulation is based on steady-state deformation of the strip. Due to symmetry, only half domain is considered. The FEM meshed domain of strip along with boundary conditions is shown in Fig. 2. In this work, elasticity is neglected. Hence, the plastic strain is the same as the total strain.

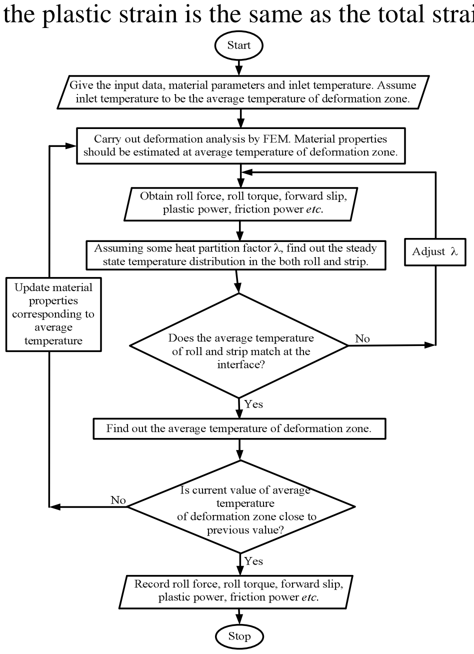

Figure 1 Flow chart illustrating the methodology of direct model

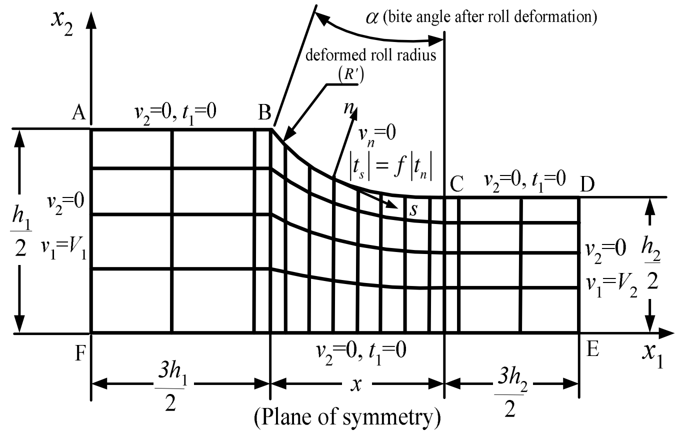

Figure 2 A meshed domain of strip with boundary conditions

Two models of flow stress are used in this work. One is the Johnson-Cook (J-C) model [Boisse *et al.* (2007)] given by

$$\sigma_{eq} = \left(A + B\varepsilon_{eq}^{n}\right)\left[1 + C\ln\frac{\dot{\varepsilon}_{eq}}{\dot{\varepsilon}_0}\right]\left[1 - \left(\frac{T - T_{amb}}{T_m - T_{amb}}\right)^m\right] , \qquad (1)$$

where $\dot{\varepsilon}_0$, T_{amb} and T_m are the reference strain rate, the ambient temperature and the melting temperature, respectively. *A*, *B*, *C*, *m* and *n* are the material parameters. The other is a power law given by

$$\sigma_{eq} = \sigma_0 \varepsilon_{eq}^{n}\left(\frac{\dot{\varepsilon}_{eq}}{\dot{\varepsilon}_0}\right)^{\beta}\left(\frac{T}{T_m}\right)^{-\gamma}, \qquad (2)$$

where β and γ are the material parameters. The J-C model has a fairly wide range of applicability. Hence, it is used for carrying out master simulations of warm rolling and these simulation results are taken in lieu of real shop floor experiments. The power law model is used for inverse estimation of material parameters in somewhat restricted range of parameters.

In this formulation, two dimensional continuity and momentum equation for the steady state process are solved. The FEM procedure is similar to cold rolling model of Dixit and Dixit (1996) except that material models contain the effect of strain-rate and temperature. In the notations of this paper *f* is the ratio of the magnitudes of normal and tangential tractions at the roll-work interface and μ is the equivalent Coulomb coefficient of friction.

2.2 Thermal module

A methodology of Yadav *et al*. (2013) has been employed to obtain the average temperature of both roll and strip at the interface. In this work, two sub-modules are used for obtaining the temperature distribution. One sub-module finds the temperature distribution in the strip when the heat generation due to plastic deformation and due to friction is known. The other sub-module estimates the temperature distribution in rolls, when the heat transfer through the roll-strip interface is known.

Analytical approach developed by Yadav *et al*. (2013) extending the work of Fischer *et al*. (2004) has been employed for the temperature distribution in rolls. The plastic deformation is assumed to occur in the domain BCHG as shown in Fig. 3. An approach similar to Yadav *et al*. (2011) which is based on the work of Kim *et al*. (2009) has been adopted for the estimation of temperature distribution in strip after exit from the rolls.

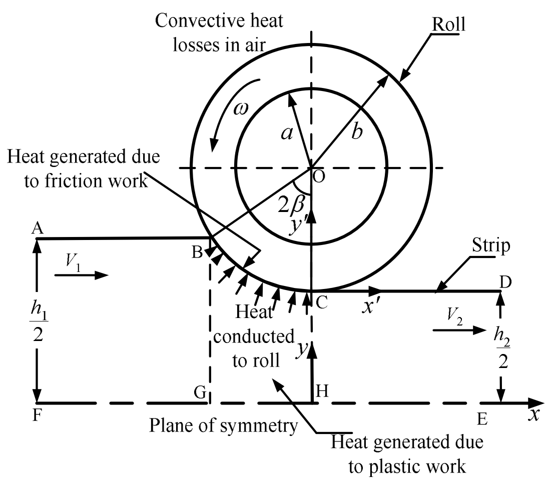

Figure 3 Geometric arrangement of the roll and strip

2.3 Validation

Yadav *et al*. (2013) did not present the validation results for warm rolling. Here, the validation with warm rolling experimental data available in literature is done. Serajzadeh and Mohammadzadeh (2007) performed the warm rolling experiments of low carbon steel with steel rolls. They conducted the experiments in the range 500–750 °C of inlet strip temperature for different rolling speed and percentage of reduction. The dimension of the strip was $(100\times5\times3)$ mm^3 and the work-roll diameter was 150 mm. The inner diameter of the roll is not mentioned by authors. Here, it is assumed as 130 mm, considering typical practical value. (Later on, the inner diameter of 120 mm was also chosen and it was found that results differ by an amount less than 1%. Thus, the inner diameter has an insignificant effect on roll torque and roll force in this case.) The convective heat losses at the outer and inner periphery roll are assumed as 260 W/m^2-°C and 2.6 W/m^2-°C respectively. The thermal properties of strip and roll are taken from Khalili *et al*. (2012). The flow stress of the strip material is governed by [Serajzadeh and Mohammadzadeh (2007)]

$$\sigma_{eq} = B\dot{\varepsilon}^m \varepsilon^n \tag{3}$$

where B, m and n are the temperature dependent material constants. The average temperature at the deformation zone are compared with the experimental results of Serajzadeh and Mohammadzadeh (2007). The inlet temperature of strip, inlet thickness of strip, percentage of reduction and angular velocity of roll are given in Table 1. It is observed that error in the prediction of temperature is less 9%. Thus, the model is validated.

3 Inverse estimation of parameter

It is assumed that the material behaves as per the power law material model given in Eq. (2). This model may not be valid over wide ranges of strain, strain-rate and temperature. However, it is always possible to select smaller domains for fitting this model. A number of such models in various smaller domains collectively can model the behavior in a wider zone.

The root mean squared (RMS) fractional error, E of the estimated and the measured quantity (exit temperature, roll force or roll torque) is taken as the objective function, which is minimized with respect to the decision variables σ_0, n, β, γ and μ. Mathematically,

$$E = \sqrt{\frac{1}{n}\times\sum_{i=1}^{n}\left(\frac{\text{measured parameter} - \text{estimated parameter}}{\text{measured parameter}}\right)^2}, \tag{4}$$

where n is the number of experimental observations. It was found that any one parameter among exit temperature, roll force and roll torque can be basis for calculating E, although the exit temperature measurement is the most convenient from a practical point of view. Alternatively, one can measure as many parameters as convenient for better reliability. Minimization of the objective function is carried out by a heuristic method inspired by the similar method used by the Chandrasekeran *et al*. (2012) and Eideh and Dixit (2013).

Table 1 Comparison of present model with experimental results [Serajzadeh and Mohammadzadeh (2007)]

Sample No.	Initial temperature of strip T_0(°C)	Inlet thickness of strip h_1(mm)	Percentage reduction (% *r*)	Rolling speed *V* (r.p.m)	Average temperature at deformation zone (°C)	Expt. [Serajzadeh and Mohammadzadeh (2007)] Temperature (°C)	% error
1	500	3	30	50	426.95	441	–3.19
2	650	3	15	50	599.62	595	0.78
3	650	3	30	50	577.16	530	8.89
4	750	3	15	65	692.52	700	3.28

3.1. Details of the algorithm

The methodology for finding out the material parameter and friction during rolling is as follows:

Step 1: Decide the ranges of strain, strain-rate and temperature in which the power law model has to be fitted. The total domain can be represented as a rectangular parallelepiped in a three-dimensional space with strain, strain-rate and temperature as axes. Carry out 8 experiments approximately corresponding to eight corners of rectangular parallelepiped. It is not possible to carry out the experiments at exact desired values of strain, strain-rate and temperature. Actual controllable parameters are percentage reduction, rolling speed and initial temperature. The computational model of the rolling guides in the proper selection of controllable parameters.

Step 2: Choose suitable ranges for material parameters, σ_0, n, β, and γ of power law and friction coefficient μ.

Step 3: For each parameter the range is divided into three linguistic zones *viz*., low (L), medium (M) and high (H) for two parameters. Thus, the entire domain gets divided into $3^5 = 243$ cells.

Step 4: Select the middle (M) values of all the parameters as initial guess parameters. Estimate E (Eq. 4) between measured and estimated temperature.(In place of temperature, roll force and roll torque can also be used.) The E is first calculated only on the basis of two measurements– one corresponding to high reduction, low temperature and high strain rate and other corresponding to low reduction, high temperature and low strain rate. If $E \geq 0.1$, the error is considered very high and there is no need to calculate errors for other experimental points. If $E < 0.1$, then gradually other experimental points are included in the error calculation for finding out E subject to maximum of 8 experimental points. This strategy helps in reducing the computational time involved in function evaluation.

Step 5: Keeping 4 other parameters (n, β, γ and μ) constant, carry out one dimensional search for optimum σ_0 in the following manner:

- If the estimated temperature at the current point is greater than the measured temperature, then decrease the value of σ_0 by jumping to the center of adjacent cell. If reduction in E is significant based on test of significance, then the center of adjacent cell becomes the current point. If the reduction in E is insignificant, both points are taken as current point and further exploration is carried out in a parallel manner from both points. If E increases, then σ_0 is not changed.
- If the estimated temperature at the current point is lesser than the measured temperature, then increase the value of σ_0 by jumping to the center of adjacent cell. If reduction in E is significant based on test of significance, then the center of adjacent cell becomes the current point. If the reduction in E is insignificant, both points are taken as current point and further exploration is carried out in a parallel manner from both points. If E increases, then σ_0 is not changed.
- Else do not change the value of σ_0.

Step 6: The similar methodology as discussed in Step 5 is repeated for optimizing other parameters *i.e*., change the one parameter at a time keeping other four parameters to be constant. After completing an iteration consisting of five one-dimensional searches, the search domain gets reduced to one cell.

Step 7: For the further refinement, the optimum cell is further divided as in Step 3. Repeat the procedure of Step 4 to Step 6.

After carrying out this procedure, if the E could not be reduced significantly, then the ranges of strain rate, temperature and strain need to be reduced.

3.2 Modification of the algorithm for variable friction case

In many cases, the friction is a function of process parameters and it cannot be considered constant. Most predominantly, it depends on the temperature. The friction can be measured by measuring the slip at each experimental point. For each experimental point, the coefficient of friction may be different. The algorithm developed in Section 3.1 can be used for this case also.

Now, the number of decision variables reduces from 5 to 4. However, in each cell, while estimating the observable parameter (temperature, roll force and roll torque), the proper value of coefficient of friction as estimated based on slip measurement needs to be considered.

4 Results and Discussion

4.1 Inverse estimation of material parameters and coefficient of friction

In the present work, actual shop floor experiments have not been carried out. For validating the proposed procedure, numerical experiments have been carried out. It is assumed that the actual flow stress is governed by the well known J-C relation. The constants of J-C model are A=598 MPa, B=768 MPa, n=0.2092, C=0.0137, m=0.807, $\dot{\varepsilon}_0 = 0.001$ given in Boisse *et al*. (2007). The actual friction coefficient is assumed to be 0.20. The roll radius (R) is 65 mm and inlet thickness of the strip (h_1) is 1 mm. With these data, virtual simulations are carried out using the FEM analysis for different initial temperature of strip, percentage of reduction and exit velocity of strip. Forward slip is recorded. The work-roll material is considered as steel only for the entire analysis. The thermal properties of work roll are given in Khalili *et al*. (2012).

Inverse analysis provides the optimum value of following unknown parameters σ_0, n, m, γ and μ of AISI steel. The maximum number of function evaluations is 20 which is sufficient to obtain the optimum value of material and process parameters. The optimum material parameters for power are obtained by inverse analysis as: for temperature range 450–650K σ_0= 461.1 MPa, n =0.085, β=0.013, γ=0.565 and μ=0.2005, for temperature range 650–850K σ_0= 408 MPa, n= 0.097, β=0.012, γ=0.83 and μ=0.1995. Figures 4 and 5 show the variation of flow stresses with strain at the estimated and the actual material properties for work material (steel) of strip. It is assumed that the power law gives the better results for smaller range of process parameter. Hence, temperature is defined in the two ranges *i.e*. 450–650 K and 650–850 K. Later on the larger range is also considered. Figs. 4 (a) and (b) show the variation of flow stresses with strain at temperatures 520 K and 600 K corresponding to temperature range (450–650 K) at different strain rates. Similar graph has been plotted at 720 K and 800 K corresponding to temperature (650–850 K) in Figs. 5 (a) and (b). It is observed that the estimated flow stresses are in good agreement with actual material flow stresses, the maximum error is found to be less than 1%.

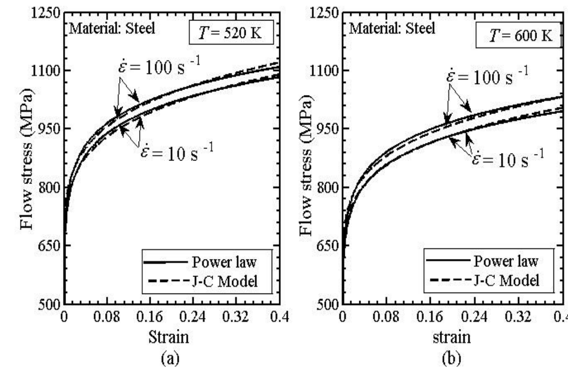

Figure 4 Variation of flow stresses with strain for steel at actual and estimated material properties at temperatures (a) 520 K and (b) 600K

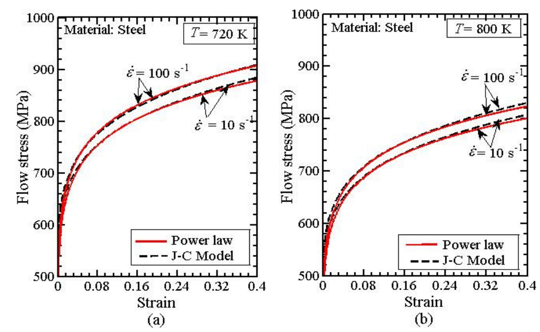

Figure 5 Variation of flow stresses with strain for steel at actual and estimated material properties at temperatures (a) 720 K and (b) 800K

Figure 6 show the variation of flow stress with strain at different strain rates at 520 K and 800 K corresponding to temperature range (450–850 K). The inverse estimation of material parameters is carried for larger range of temperature. The same inverse methodology is used to obtain the following material parameters: σ_0 = 443 MPa, n = 0.084, β = 0.0105, γ = 0.701 and friction coefficient μ=0.1995. It is observed that in this case, there is larger deviation with J-C model.

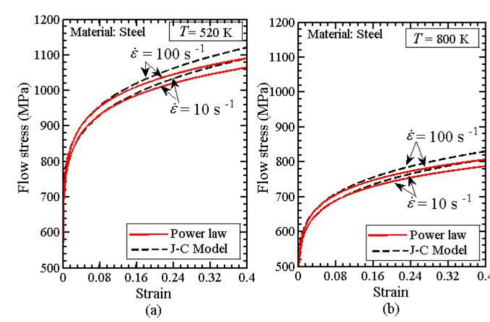

Figure 6 Variation of flow stresses with strain for steel at actual and estimated material properties at temperatures (a) 520 K and (b) 800K

4.2 Inverse estimation of material parameters and coefficient of friction for variable friction case

In this section the effect of variation of friction coefficient with temperature is studied. The friction coefficient is decided on the basis of forward slip measurement. The forward slip is calculated by deformation FEM module. The friction coefficient is assumed to follow following relation:

$$\mu = 0.41 - 0.00025T - 0.028V, \qquad (12)$$

where T is the average temperature at the deformation zone in °C and V is the roll speed in m/sec. Table 2 shows the estimated friction coefficients for different cases.

Table 2 Optimum friction coefficient

Case	Inlet temperature T_0(K)	Exit velocity V_2 (m/sec)	% reduction r	Optimum friction coefficient μ
1.	600	0.5	4	0.243
2.	600	0.5	28	0.232
3.	600	3	4	0.235
4.	600	3	28	0.227
5.	520	0.5	4	0.252
6.	520	0.5	28	0.263
7.	520	3	4	0.241
8.	520	3	28	0.234

The optimum material parameters for power law are obtained as σ_0 = 456 MPa, n = 0.077, β = 0.0035, γ = 0.735. Figure 7 shows the variation of flow stresses with strain for actual and estimated material parameters. The difference between the actual and estimated flow stresses is observed to be less than $\pm 3\%$.

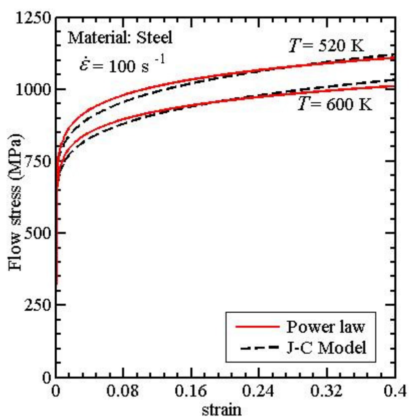

Figure 7 Variation of flow stresses with strain at actual and estimated material properties of steel at temperatures (a) 520 K and (b) 600 K

5 Conclusions

In the present work, an inverse method is proposed for the estimation of material parameters of power law and coefficient of friction. The methodology requires the estimation of the exit temperature of the strip and slip. The slip measurement gives the proper estimation of coefficient of friction. If the coefficient of friction is assumed constant for different process conditions, then only the measurement of temperature can be carried out. In place of the exit temperature, the roll torque or roll force can also be measured. A heuristic method is used for the minimization of the error between actual and estimated temperature of exit strip.

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