Prediction of Forming Limit Curves for Extra Deep Drawn (EDD) steel using Marciniak and Kuczynski (MK) model

Sashank Srinivasan[1]*, Geetha Krishna A[1], Shyam Krishna Shenoy A[1], Swadesh Kumar Singh[2], Amit Kumar Gupta[1]

[1]Dept. of Mechanical Engineering, BITS – Pilani, Hyderabad Campus, AP – India – 500078,

[2]Dept. of Mechanical Engineering, GRIET, Hyderabad, AP – India – 500072,

*Corresponding author email ID: sashank1371993@gmail.com

Abstract

Forming Limit Curves (FLCs) are an important tool for predicting the forming behaviour of sheet metals. Experimental measurements of FLCs are often time consuming and costly, and therefore, empirical prediction methods carry significant practical importance. In this paper, an attempt is made to predict the FLCs for Extra Deep Drawn (EDD) steel using Marciniak and Kuczynski (MK) model. In developing MK model, three different yield criteria are used based on Hill (1948) and Barlat (1989) which are modelled based on the true stress-true strain data obtained from the uniaxial tensile tests. The theoretical FLCs from MK model have been validated with the experimental results obtained by the hemispherical dome tests with specimens of different widths. The theoretical and experimental results are found to be in good agreement.

Keywords: Forming limit curves, Yield functions, EDD steel, MK model

1. Introduction

In industrial sheet metal forming operations involving thin sheets, formability is limited by the onset of localized necking (Kuroda and Tvergaard, 2000). Forming limit curves (FLC) has proved to be a useful tool to represent conditions for the onset of necking and evaluate formability of sheet metals. Experimentally constructing the FLC is expensive and time consuming. This has generated a great need for precisely predicting the FLC numerically.

The Marciniak–Kuczynski (MK) analysis has been one of the most commonly used approaches for numerical determination of FLCs. In MK analysis, a thickness imperfection is introduced far away from the sheet metal boundary to simulate pre-existing defects in the sheet material (Gologanu *et al*. (2013)). It has been shown that the presence of even slight intrinsic inhomogeneity in load bearing capacity throughout a deforming sheet can lead to unstable growth of strain in weaker regions, causing localized necking and failure. Necking is considered to occur when the ratio of the effective total strain in the groove region to that in the nominal region of the sheet is above a critical value.

The MK analysis has been used extensively in numerical analyses based on constitutive models at two different length scales; micro scale and macro scale. The micro scale models incorporate crystal plasticity theories into the MK model and thus account for the microstructure of the material. The macro scale models are based on phenomenological yield functions to predict the material response.

This paper deals with numerical simulation of FLC's for Extra Deep Drawn (EDD) Steel sheets using the MK analysis. There has been a continuing trend towards development of materials with improved formability, which led to development of deep drawing quality and extra-deep drawing quality steel sheets and several nonferrous alloys. Extra deep drawing (EDD) steels are the most widely used steel material today for automotive applications involving simple and complex components, which require very high formability. Exterior components such as starter end covers, petrol tanks, are made up of deep drawing grade steels. The low carbon steel sheets are also used extensively in enamelling applications such as baths, sink units, kitchen ware, cookers and refrigerator panel (Singh *et al*. (2010)). Thus, prediction of forming limit curves is very crucial in determining the limits of formability of EDD steel.

The plastic behaviour of the EDD steel has been modelled using different yield functions which then have been integrated with the MK analysis to predict FLC. First, the various yield functions have been evaluated based on comparison of the predicted yield stress and then the performance of theoretical FLC's with the experimental data is done. The parameters for the yield functions were determined using yield stresses and anisotropy values obtained from uniaxial tensile tests along different orientations with respect to rolling direction of the sheet metal.

2. Experimentation and Data Development

The chemical compositions of as-received EDD sheets based on spectrometry is presented in Table 1 (Singh *et al*. (2010)).

Table 1: Chemical composition of EDD steel sheets (in weight percent).

Element	C	Si	Mn	S	P
Wt. (%)	0.048	0.83	0.39	0.024	0.019
Element	Sn	Cu	Ni	Mo	Cr
Wt. (%)	0.004	0.019	0.054	0.028	0.027

* Rest of the composition is Iron

2.1. Procedure for uniaxial tensile tests

The mechanical properties of EDD steel were obtained from uniaxial tensile tests conducted using universal tensile testing machine with a maximum tensile force capacity of 5 kN.

From Table 2, it can be seen that EDD steels have high values of strength coefficient (*K*) and work hardening exponent (*n*) (which indicates the ability of the metal to undergo plastic deformation prior to necking/fracture) and thus these sheets are expected to have good formability. Higher strain hardening exponent (*n*) increases the ability of the metal to undergo uniform plastic deformation before localized necking/excessive thinning occurs and hence enhances the deformability of material.

Table 2: Material Data obtained from UTM uniaxial tensile testing

Orientation (degrees)	Yield Stress (MPa)	K(MPa)	N	r
0	204.967	678.422	0.3059	1.1549
90	195.442	664.966	0.3019	1.4850

2.2. Test procedure for experimental FLC curve

The experimental FLC curves were plotted after conducting hemispherical dome tests on Nakazuma specimens made of EDD steel.The experiments were carried out on the test rig which is shown in Figure 1. The test rig comprises of a 20 ton hydraulic press which was used to conduct the hemispherical dome tests. Circular blanks were machined by using wire-cut electro-discharge machining process for high accuracy and finish. Grids were made on the blanks using chemical etching. After the hemispherical dome tests, the deformed grids were measured using a travelling microscope.

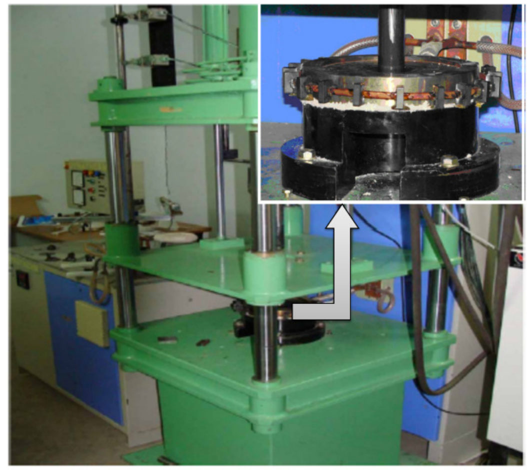

Figure 1: Hemispherical dome test setup

3. Formulation of Yield Criteria

In order to model the inherent anisotropy in the EDD sheet metal, i.e., the variation in the yield stress and *r*-values with orientation (with respect to rolling direction), two anisotropic yield functions have been considered. A brief description of the yield functions used, along with their parameter determination is presented below.

3.1. Hill's 1948 yield criterion

Hill (1948) proposed an anisotropic yield criterion as a generalization of the Huber-Mises-Hencky criterion with anisotropy in three orthogonal symmetry planes (Banabic (2010), Hill .R (1948)). The yield criterion is expressed by a quadratic function of the following type:

$$2f(\sigma_{ij}) \equiv F(\sigma_{22}-\sigma_{33})^2 + G(\sigma_{33}-\sigma_{11})^2 + H(\sigma_{11}-\sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 = 0 \quad (1)$$

where *f* is the yield function; F, G, H, L, M and N are constants specific to the anisotropy state of the material, and *x, y, z* are the principal anisotropic axes. Axis 1 is usually parallel to the rolling direction, 2 is parallel to the transverse direction and 3 is collinear with the normal direction. The relations between the anisotropy coefficients, the yield stresses and the coefficients may be easily obtained from the flow rule associated to the yield function:

$$r_0 = \frac{H}{G};\ r_{90} = \frac{H}{F};\ r_{45} = \frac{N}{F+G} - \frac{1}{2}; \quad (2)$$

$$\frac{\sigma_0}{\sigma_{90}} = \sqrt{\frac{r_0(1+r_{90})}{r_{90}(1+r_0)}} \quad (3)$$

In case, for plane stress condition, principal directions of the stress tensor are coincident with the anisotropic axes ($\sigma_{11} = \sigma_1; \sigma_{22} = \sigma_2;\ \sigma_{12} = 0$), the Hill 1948 yield criterion can be written as a dependence of the principal stress in the form. (Using the anisotropy coefficient and σ_0)

$$\sigma_1^2 - \frac{2r_0}{1+r_0}\sigma_1\sigma_2 + \frac{r_0(1+r_{90})}{r_{90}(1+r_0)}\sigma_2^2 = \sigma_0^2 \quad (4)$$

3.2. Another method of formulation of Hill '48 yield criterion

Hill'48 yield criterion can also be expressed in a different way. This is done by using r_0, σ_0 and σ_{90} as:

$$\frac{{\sigma_1}^2}{{\sigma_0}^2} - \frac{2r_0}{(1+r_0){\sigma_0}^2}\sigma_1\sigma_2 + \frac{{\sigma_2}^2}{{\sigma_{90}}^2} = 1 \quad (5)$$

In this paper Eq (4) is referred to as Hill-1 and Eq (5) as Hill-2.

3.3. Barlat 1989 yield criterion

Barlat and Lian (1989) published a generalisation of Hosford's criterion (Hosford WF. (1979)) for materials exhibiting normal anisotropy by introducing the following yield function (Barlat and Lian (1989) and Banabic (2010)):

$$f \equiv a|k_1 + k_2|^M + a|k_1 - k_2|^M + c|2k_2|^M = 2\sigma^a \quad (6)$$

where k_1 and k_2 are invariants of the stress tensor while M is an integer exponent (= 6 as EDD steel has BCC crystal structure (Banerjee (2012))); k_1 and k_2 are obtained from

$$k_1 = \frac{\sigma_{11} + h\sigma_{22}}{2}; \quad (7)$$

$$k_2 = \sqrt{\left[\left(\frac{\sigma_{11} - h\sigma_{22}}{2}\right)^2 + p^2\sigma_{12}^2\right]} \quad (8)$$

and *a, c* and *h* are material parameters determined by

$$a = 2 - c = 2 - 2\sqrt{\frac{r_0 r_{90}}{(1+r_{90})(1+r_0)}} \quad (9)$$

$$h = \sqrt{\frac{r_0(1+r_{90})}{r_{90}(1+r_0)}} \quad (10)$$

p is to be calculated using numerical procedures. However, since $\sigma_{12} = 0$, calculation of *p* is not required.

4. Marciniak Kuczynski Model Description

To predict the theoretical forming limit curves, the MK model assumes an initial thickness imperfection in the geometry of the sheet in the form of a groove across the width of the sheet.

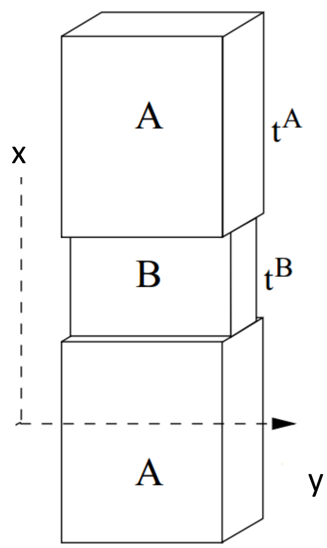

Figure 2: Initial inhomogeneity assumed by MK model

The zone outside the groove is referred to as zone A and the groove is referred to as zone B, as shown in Figure 2. A Cartesian coordinate system is aligned with the symmetry axes: the x-axis is along the rolling direction (RD), and the y-axis is along the transverse direction (TD). This initial imperfection can be defined by a thickness ratio:

$$f_0 = \frac{h_0^B}{h_0^A} < 1 \quad (11)$$

where h_0^A, h_0^B are the initial thicknesses of zone A and zone B, respectively. f_0 is a parameter of the MK model. The boundary of the sheet (assumed to be far away from the groove) is subjected to monotonic proportional straining parallel with the symmetry axes.

$$\rho = \frac{\epsilon_y^A}{\epsilon_x^A} \qquad (12)$$

where ϵ_x, ϵ_y are components of strain along the coordinate axes. The ϵ_x component of the strain is usually referred to as the major strain, whereas ϵ_y is called the minor strain (the case when the major strain is along the transverse direction is treated similarly).

The value of f_0 is varied until the theoretically predicted FLC curve agrees best with the experimental curve at the plane strain point, i.e., for $\rho = 0$. For the present analysis, the value of f_0 chosen is 0.99.

As the straining at the boundary increases, the thickness of zone B reduces continuously and faster than that of region A. Hence it has to bear increasingly higher stresses than those in zone A. There will be a point when the region B has deformed substantially more than region A, signalling the start of necking. The failure criterion is thus:

$$\frac{d\overline{\epsilon_A}}{d\overline{\epsilon_B}} < N \qquad (13)$$

$d\overline{\epsilon_A}$, $d\overline{\epsilon_B}$ denote the equivalent plastic strains in the respective regions. From a computational point of view, the constant N should be a small number so as to ensure that region B has deformed sufficiently more than region A. Then it can be said with certainty that necking would have occurred. N= 0.15 was used for this analysis.

4.1 Formulation of MK model

As presented by Xiaoqiang *et al.* (2013) and Xu and Weinmann (1998), the general stress state of the material is described by the power law equation:

$$\bar{\sigma} = K\overline{\epsilon^n}\dot{\epsilon^m} \qquad (14)$$

where *n* is the strain hardening coefficient, *m* is the strain rate sensitivity coefficient.

The ratio of principal stresses and strains is defined as:

$$\alpha = \frac{\sigma_y}{\sigma_x}, \rho = \frac{\epsilon_y}{\epsilon_x} = \frac{d\epsilon_y}{d\epsilon_x} \qquad (15)$$

The effective stress and strain are defined as:

$$\bar{\sigma}\bar{\epsilon} = \sigma_x\epsilon_x + \sigma_y\epsilon_y$$

$$= \sigma_x\epsilon_x(1 + \alpha\rho) \qquad (16)$$

The associative flow rule is given by:

$$d\epsilon_{ij} = d\lambda\frac{\partial\bar{\sigma}}{\partial\sigma_{ij}} \qquad (17)$$

From the associative flow rule and the constant volume condition $d\epsilon_x + d\epsilon_y + d\epsilon_z = 0$, expressions for $d\epsilon_x, d\epsilon_y, d\epsilon_z$ are obtained.

The MK model incorporates a compatibility condition

$$d\epsilon_y^A = d\epsilon_y^B \qquad (18)$$

Furthermore, the sheet metal being deformed will always be in equilibrium. This is represented by the force balance equation:

$$\varphi_A(\overline{\epsilon^A} + \overline{d\epsilon^A})\epsilon^{\dot{m}}{}_A = f\varphi_B(\overline{\epsilon^B} + \overline{d\epsilon^B})\epsilon^{\dot{m}}{}_B \qquad (19)$$

where $\varphi = \frac{\sigma_x}{\bar{\sigma}}$ and $f = \frac{t_A}{t_B}$. t_A, t_B denote the instantaneous thicknesses of regions A and B. This ratio can be found by using the equation:

$$f = f_0 exp\,(\epsilon_z^A - \epsilon_z^B) \qquad (20)$$

Initially values of f_0 and ρ are assumed. Small strain increments of $d\epsilon_x^B$ are imposed in the groove region. The values of $d\epsilon_y^B, d\epsilon_x^b$ are found using the corresponding equations described above. Assuming a value for $d\epsilon_x^A$, the values of $d\epsilon_y^A, d\epsilon_x^A$ are computed. The equality of the force balance equation is checked. If the equality is satisfied, then the necking criterion is checked. If the necking criterion is also satisfied, then that particular strain state of region A corresponds to a point on the FLC. If the assumed value of $d\epsilon_x^A$ does not correspond to equal values of left and right hand sides of the force balance equation, the assumed value is changed and the process is repeated. This procedure is done for different values of f_0 and ρ to plot the full FLC.

5. Results and Discussion

5.1. Yield curves

The yield criteria obtained using Eq. (4) has coefficients, obtained using anisotropy coefficients and not the yield stresses along different directions, Eq. (4) gives a poor prediction of yielding along different directions. However, this equation gives a good estimate of anisotropy.

Eq. (5) which has been formulated with σ_0 and σ_{90} gives better prediction of variation in yield stress than Eq. (4). This can be seen in Figure 3, where σ_{90} was found to be 195.442 MPa (equal to the experimental result for σ_{90}) for Hill-2; whereas Hill-1 predicts it to be 216.43 MPa. Thus, Hill-2 captures the variation in yield stress more accurately than Hill-1. It should also be noted that Hill-1 captures anisotropy better than Hill -2. Note that, yield curve of Hill-2 does not pass through $\sigma_2/\sigma_0 = 1$ when σ_1=0 because the yield functions are normalised with σ_0.

5.2. Comparison of numerical FLC with experimental results

The experimental as well as the theoretical FLC's are plotted in Figure 4. The point on the FLC corresponding to the plain strain condition approximately has the value of n, the strain hardening coefficient. The plotted numerical FLC's intersect the major strain axis at almost the same point. This confirms that the value of f_0 considered for the three numerical FLC's are correct.

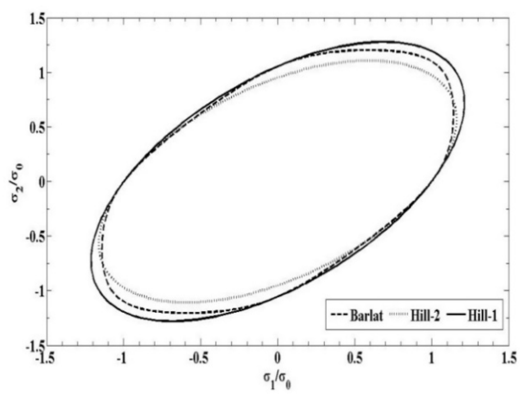

Figure 3: Yield curves of the formulated yield functions for EDD steel

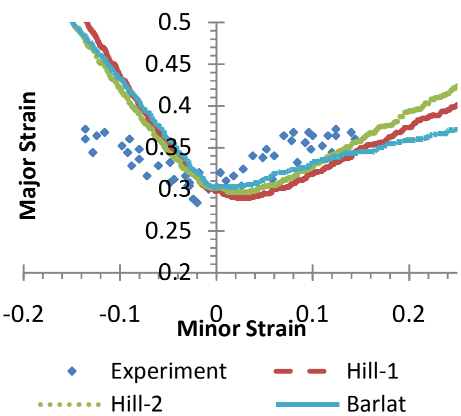

Figure 4: Comparison of FLC's with the experimental results

It is observed from Figure 4, that the Barlat criterion is best able to follow the trend of the experimentally obtained FLC. This is especially true for the tension-tension region. Even in the tension-compression region of the FLC, the Barlat criterion has a similar trend to that of the experimental FLC. However, FLC modelled using Barlat wield criteria over predicts the safe region in second quadrant. In fact, all three yield criteria over-predict the safe region in the FLC for negative minor strains. Since all three numerically plotted FLC's over-predict the safe region in the tension-compression region, this can be as an attributed drawback of the MK analysis.

To ascertain the accuracy of the three numerical FLC plots, a correlation factor R is found by comparing the experimental data with the numerical prediction. Since it has already been ascertained that the numerical plots have sufficient accuracy only in the tension-tension region, the correlation factors are found only for the prediction in this region. The general expression for correlation factor is:

$$R = \frac{n(\sum xy) - (\sum x)(\sum y)}{(\sqrt{n(\sum x^{2)} - (\sum x)^2})(\sqrt{n(\sum y^{2)} - (\sum y)^2})} \quad (21)$$

The R values obtained for the three numerical FLC's along with the FLC plots are shown in Figures 5-7.

The R value for the Barlat prediction (0.7780) is the highest among the three models. This confirms that Barlat model is the best among the yield models considered in this work to predict FLC.

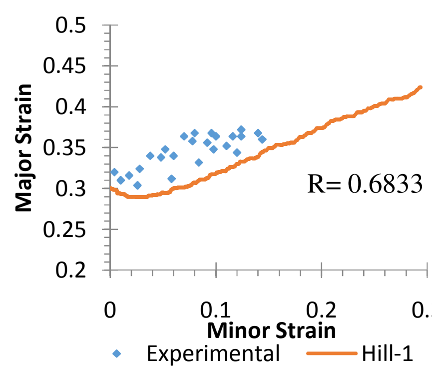

Figure 5: Comparison of FLD based on Hill-1 with experimental FLD results and the corresponding R value

Also, from R-values obtained, Hill-2 predicts FLC more accurately than Hill-1. So it can be said that the yield criteria which captured variation in yield stress better than other gave satisfactory results. Therefore variation in yield stress is a factor which affects FLC and it should be taken into consideration.

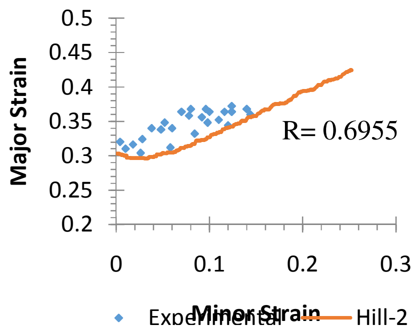

Figure 6: Comparison of FLD based on Hill-2 with experimental FLD results and the corresponding R value

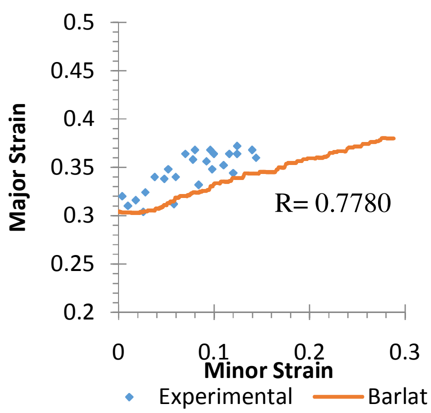

Figure 7: Comparison of FLD based on Barlat with experimental FLD results and the corresponding R value

6. Conclusion

A study has been done to check the ability of the MK analysis in predicting FLC's of EDD steel.

On comparison of the experimental and numerical FLC's, it is observed all the numerical FLC were able to predict the FLC with sufficient accuracy only in the tension-tension region. Since all the three yield models fail to predict the tension-compression region of the FLC accurately, this can be attributed as a drawback of the MK analysis.

Correlation factors for numerical FLC based on the experimental data were found for the tension-tension region. The FLC plotted using Barlat yield criterion had the highest correlation factor and hence it is concluded that among the three yield models considered, the Barlat model is the most suitable to predict FLC's accurately.

Future work involves prediction of FLC's at higher temperatures for warm forming using MK analysis. Further, other yield criteria like Hill 1994 and Barlat 2000 can be used to predict better yield curves and consequently better forming limit curves.

7. References

Barlat F, Lian K., (1989) Plastic behavior and stretchability of sheet metals. Part I: a yield function for orthotropic sheets under plane stress conditions. Int J Plast 1989; 5(1):pp. 51–66.

Dorel Banabic,(2010) Sheet Metal Forming Processes- Constitutive mocelling and numerical simulation, Elsivier

Hill R. A theory of the yielding and plastic flow of anisotropic metals. Proc R Soc Lond 1948; 193(1033):p. 281–97.

Hosford WF, (1979) On yield loci of anisotropic cubic metals. In: Proceedings of the 7th North American metalworking research conference. Michigan, USA: Ann Arbor; 1979. pp. 191–7.

Kumkum Banerjee (2012), Physical Metallurgy and Drawability of Extra Deep Drawing and Interstitial Free Steels. Recrystallization, Edited by Prof. Krzysztof Sztwiertnia

Li Xiaoqiang, *et al.* (2013) Prediction of forming limit curve (FLC) for Al–Li alloy 2198-T3 sheet using different yield functions, Chinese Journal of Aeronautics

Mihai Gologanu *et al.* (2013), Theoretical model for forming limit diagram predictions without initial inhomogeneity, AIP conference proceedings, volume 1532, pp. 245-253

Siguang Xu, Klaus J. Weinmann (1998) Prediction of forming limit curves of sheet metals using Hill's 1993 user-friendly yield criterion of anisotropic materials, Int. J. Mech. Sci. Vol. 40, No. 9, pp. 913-925

Swadesh Kumar Singh *et al.* (2010) A study on the extent of ironing of EDD steel at elevated temperature, CIRP Journal of Manufacturing Science and Technology 3, pp. 73–79.

Swadesh Kumar Singh, *et al.* (2010) Understanding formability of extra-deep drawing steel at elevated temperature using finite element simulation, Materials and Design 31, pp. 4478–4484