A SIMPLE ANALYTICAL MODEL OF LASER BENDING

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Abstract

Laser bending is a process of bending a sheet by the irradiation of laser beam on the surface of the sheet. A number of analytical and numerical methods have been proposed for the estimation of bend angle. A brief review of these methods is presented. A finite element analysis to simulate the laser bending process is carried out with ABAQUS package for the purpose of understanding the physics of this process. Afterwards, a simple analytical model is developed to evaluate the bending angle in laser bending of metal sheet. The model is based on bending of sheet using the elastic-plastic theory. It is ascertained from the experimental results available in literature that the results from the proposed model provide reasonably good prediction of bend angle. It is also shown that the model can be used for the quick estimation of yield stress of the material during laser bending process.

Keyword: *Laser bending,Finite element method, Temperature distribution*

1Introduction

Laser forming is a thermo-mechanical process used to deform sheets by means of thermal heating by a laser beam. The process is suitable for rapid prototyping and deforming the low ductility materials. Since last two decades, several papers have been published on laser forming process, amongst them straight line bending by laser beam has been researched extensively. The researchers tried to understand the physics of the process and produced various models for the prediction of bend angle. Some of these models are analytical and some are based on numerical methods such as finite element method (FEM).

Shen and Vollertsen (2009) presented a literature review for the modelling of laser forming. There are three prominent mechanisms of laser bending. These are temperature gradient mechanism (TGM), buckling mechanism (BM) and upsetting mechanism (UM). The temperature gradient mechanism activates when laser beam diameter is in the order of sheet metal thickness and scan speed is high. In buckling mechanism, the beam diameter is relatively larger and scan speed is lower. In the upsetting mechanism, the beam diameter is much smaller than the thickness of the sheet. Vollertsen (1994) derived an expression for the bending angle for TGM. His model does not include the effect of yield stress of material. Yau*et al*. (1997) included this effect in their model. A mathematical model was developed by Kyrsanidi *et al*. (2000) who considered non-uniform temperature distribution throughout the thickness of the plate due to the developed plastic strains. This model, although computationally efficient, requires programming and includes iterative steps. Cheng *et al.* (2006) proposed analytical model for plate with varying thickness. Shen *et al.* (2006) proposed analytical model based on the assumption that the plastic deformation is generated only during heating, while during cooling the plate undergoes only elastic deformation. The model is valid for TGM as well as BM.Recently, Lambiase (2012) proposed expression for the bending angle based on assumption of elastic-bending theory without taking into account plastic deformation during heating and cooling phases.

Several other analytical models have been proposed. Shi *et al*. (2007) provided a model for estimating the bend angle in an in-plane axis perpendicular to the scan direction. Vollertsen (1994) provided an expression for

estimating the bend angle during BM mechanism. Kraus (1997) provided a closed−form expression for estimating the bend angle during upsetting mechanism.
Several FEM methods have been proposed (Chen *et al.*, 1999; Kyrsanidi *et al.*, 1999; Chen and Xu, 2001; Hu *et al.*, 2001). However, FEM simulation takes several hours and is not suitable for online optimization and control (Kyrsanidi *et al.*, 2000). Finite difference models have also been proposed to model the process, however, they also require a large computational time. In the present work a simple analytical model based on the elastoplastic bending of beams is proposed. This model takes into account the effect of the yield stress of the material. The temperature is estimated by a series solution. The method is very efficient and matches well with FEM simulations.

2 Modelling by ABAQUS

In the present work, FEM simulations were carried out using ABAQUS package. 8-noded brick elements (DCC3D8) were used with uniform mesh. The material properties of D36 shipbuilding steel were chosen based the table provided in the literature (Dixit *et al.*, 2012). However, these properties were considered as temperature independent. A plate of size 80 mm × 40 mm × 6 mm, laser power P= 1000 W, η *(absorptivity)* =0.3, scan velocity= 8 mm/s and laser beam radius r= 8 mm were chosen. The temperature distribution is assumed to be Gaussian and the heat flux is given by

$$q = \frac{2\eta P}{\pi r^2} \exp\left(-2\frac{x^2 + y^2}{r^2} \right), \quad (1)$$

The co-efficient of thermal expansion is taken as constant for the range or temperatures considered here, and is = $12\times10^{-6}/^{\circ}$C and the room temperature is taken as 20 °C. In this model, the laser beam is assumed to traverse along the width direction and it is sufficiently far away from the edges in the length direction. A schematic diagram of this arrangement is shown in Figure 1. The sheet is clamped at the right edge (surface) parallel to scan direction, but the clamping boundary conditions are not shown in the figure.

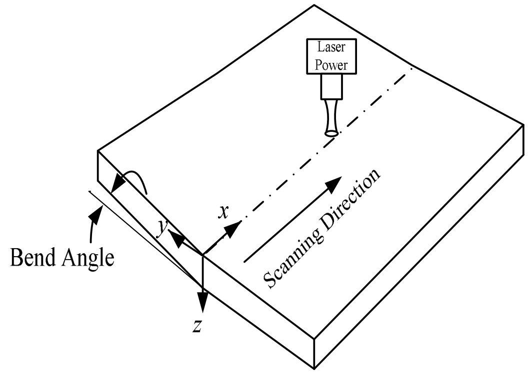

Figure 1 Schematic diagram of straight line bending process clamped at the right edge surface

No external loads are applied on the sheet. The right edge of the sheet is fixed to avoid the rigid body movements. The boundary condition of zero displacement at the fully constrained right edge of metal plate is imposed. The plate is divided into 80 elements along the length side (y-direction) of the plate, 40 elements along the width side (x-direction) of the plate and 6 elements along the thickness direction of the plate. The total elements are 19200. Three FEM simulations were carried out with different values of yield stress to show the effect of yield stress on the bend angle. The calculation time required for the finite element analyses presented in this work is 6-7 hours on a PC. Table 1 shows the bend angle for three different values of yield stress. It is observed that the yield stress has significant effect on the bend angle and neglecting the effect of yield stress in an analytical model may not be appropriate. The maximum temperature in the sheet was about 165 °C, which gives the maximum increase in the temperature of about 145 °C. If the local heated material is sufficiently constrained, the maximum thermal stress is $\Delta T\alpha E=145\times12\times10^{-6}\times200\times10^{9}=358\times10^{6}$ Pa. If the yield stress is more than 358 MPa, no plastic deformation takes and the bend angle should be zero. The results of Table 1 confirm that at a yield stress of 495 MPa, the bend angle is zero.

Table 1 Variation of bend angle with yield stress

Yield stress (MPa)	Bend angle(degree)
50	0.79
335	0.51
495	0.0

Figure 2 shows the variation of top and bottom surface temperatures with time for a material with yield stress of 335 MPa. Initially, the temperature of both the surfaces increases, which causes counter bending due to the temperature difference in the temperatures of top and bottom surfaces. As the material underneath the laser beam is surrounded by the cold material and the yield stress drops due to temperature, there is some localized plastic deformation. After that, cooling starts and both top and bottom surfaces contract. The relative contraction on the top surface is more than that at bottom surface, causing the upward bending (towards laser beam side). After a certain time, the temperature at bottom and top surfaces becomes equal. There is a small decrease in the final bend angle, when the cooling beneath the surface is going on, whilst the top surface has already cooled.

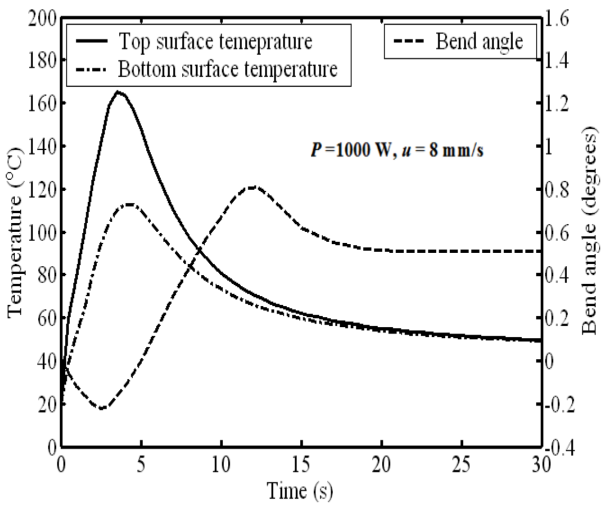

Figure 2 Variation of temperature with time on the top and bottom surface of the sheet and variation of Bend angle during heating and cooling

3 Model for the prediction of bending angle

The model for the prediction of bending angle is based on the following assumptions:

(1) During heating, the material beneath the laser beam undergoes plastic deformation. No counter bending is assumed.
(2) During the cooling phase, the top surface contracts and the bottom surface expands as both surfaces attain the average temperature.
(3) The strains during cooling are caused due to equivalent mechanical stresses. If a surface cools by ΔT amount, it amounts to applying $\alpha_{th} E \Delta T$ compressive stress to surface. A similar analogy can be given when the surface heats.
(4) The equivalent mechanical stresses cause elastoplastic bending and the bend angle can be obtained by usual elastoplastic theory of bending described in standard textbooks, e.g., Chakrabarty (2006).

The temperature is estimated by a model provided in reference by Mishra and Dixit (2013). The heat flux due to laser beam is assumed as Gaussian of the following form in Mishra and Dixit's work (2013):

$$q(x, y) = \frac{\eta P}{\pi r^2} \exp\left\{ -\frac{\left(x^2 + y^2\right)}{r^2} \right\}, \quad (2)$$

However, in the present work the heat flux is assumed to follow Eq. (1) given earlier. Accordingly the expressions are modified. For a heat source moving with velocity u parallel to x-axis, the time to scan the sheet is denoted by t_1. Temperature at a point (x, y, z) at time $t > t_1$ is given as

$$T - T_\infty = \frac{\eta P}{\pi c_p h} \int_0^{t_1} \left[\begin{array}{l} \dfrac{2}{8\alpha(t-t') + r^2} \exp\left(-\dfrac{2(\{x - u(t-t')\}^2 + y^2)}{8\alpha(t-t') + r^2} \right) \\ \times \left\{ 1 + 2\sum_{n=1}^{n=\infty} \exp\left(-\dfrac{\alpha n^2 \pi^2 (t-t')}{h^2} \right) \cos\dfrac{n\pi z}{h} \right\} \end{array} \right] \mathrm{d}t', \quad (3)$$

where T is the temperature at time t at point (x, y, z), P is the laser power, η is the absorptivity of the material, r is the laser beam radius at the surface of the material, α is the thermal diffusivity and h is the thickness of the sheet. Temperature at time $t < t_1$ at a point (x, y, z) is given as

$$T - T_\infty = \frac{\eta P}{\pi c_p h} \int_0^{t} \left[\begin{array}{l} \dfrac{2}{8\alpha(t-t') + r^2} \exp\left(-\dfrac{2(\{x - u(t-t')\}^2 + y^2)}{8\alpha(t-t') + r^2} \right) \\ \times \left\{ 1 + 2\sum_{n=1}^{n=\infty} \exp\left(-\dfrac{\alpha n^2 \pi^2 (t-t')}{h^2} \right) \cos\dfrac{n\pi z}{h} \right\} \end{array} \right] \mathrm{d}t'. \quad (4)$$

Figure 3 shows the typical variation of temperature along thickness direction of the sheet. The average temperature can be calculated as

$$T_{av} = \frac{\int_0^h T \,\mathrm{d}z}{h}, \quad (5)$$

Then, the equivalent longitudinal thermal stresses at any location along the thickness is given as

$$\sigma_x = \alpha_{th} E (T_{av} - T), \quad (6)$$

where α_{th} is coefficient of thermal expansion and E is Young's modulus of elasticity. The moment resulting from thermal stresses is given as

$$M = \int_0^h \sigma_x z \, \mathrm{d}z \, . \quad (7)$$

Figure 4 shows schematic diagram of plain strain bending of the sheet of thickness h, where line corresponding to T_{av} is a neutral axis, c is the distance from neutral axis to elastic plastic boundary, s is the distance from neutral axis to the top surface of the sheet, $(h-s)$ is the distance between neutral axis and elastic plastic boundary.

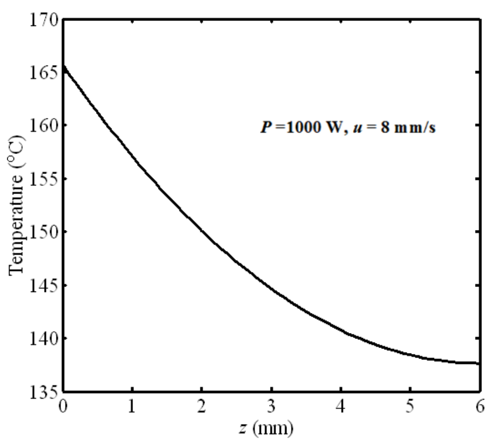

Figure 3 Variation of temperature along thickness direction

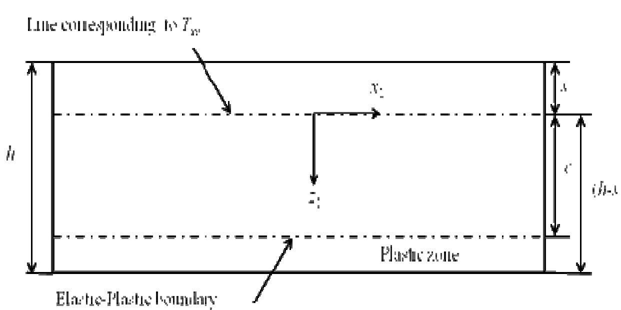

Figure 4 Schematic diagram of plain strain bending of the sheet

when $c>s$, the moment during elastic plastic bending is given as

$$M = \int_0^s \frac{\sigma_y z_1^2}{c\sqrt{1-\nu+\nu^2}} \mathrm{d}z_1 + \int_0^c \frac{\sigma_y z_1^2}{c\sqrt{1-\nu+\nu^2}} \mathrm{d}z_1 + \int_c^{(h-s)} \frac{2\sigma_y z_1}{\sqrt{3}} \mathrm{d}z_1, \quad (8)$$

where ν is the Poisson's ratio. After simplification,

$$M = \frac{2\sigma_y}{\sqrt{3}} \left\{ \frac{1}{3c}(s^3 + c^3) + \frac{1}{2}\left[(h-s)^2 - c^2 \right] \right\}, \quad (9)$$

when $c<s$, then in both sides of the neutral axis, there are elastic as well as plastic zones and the moment is given as

$$M = 2\int_0^c \frac{\sigma_y z_1^2}{c\sqrt{1-\nu+\nu^2}} \mathrm{d}z_1 + \int_c^{(h-s)} \frac{2\sigma_y z_1}{\sqrt{3}} \mathrm{d}z_1 + \int_c^s \frac{2\sigma_y z_1}{\sqrt{3}} \mathrm{d}z_1, \quad (10)$$

After simplification,

$$M = \frac{2\sigma_y}{\sqrt{3}} \left[\frac{1}{2}\left\{ (h-s)^2 + s^2 \right\} - \frac{c^2}{3} \right], \quad (11)$$

The value of M can be calculated from Eq. (7). Thereafter, c can be obtained from Eq. (9) or Eq. (11) using bisection method. To begin with, it is assumed that $c>s$ and Eq. (9) is used to find out c. If c comes out to be less than s, then Eq. (11) is used.

For the plane strain elastic bending, the longitudinal stress is given as (Chakrabarty, 2006),

$$\sigma_{x_1} = \frac{E z_1}{(1-\nu^2) R}, \quad (12)$$

where R is the radius of curvature. Applying this equation at the elastoplastic boundary and using von-Mises yield criterion

$$\frac{2\sigma_y}{\sqrt{3}} = \frac{E c}{(1-\nu^2) R}. \quad (13)$$

when the yielding just begins on the bottom surface

$$\frac{2\sigma_y}{\sqrt{3}} = \frac{E (h-s)}{(1-\nu^2) R_e}, \quad (14)$$

where R_e is the radius of curvature when the yielding just commences. From Eq. (13) and Eq. (14), the radius of curvature during elastic plastic bending is given as

$$R = \frac{c}{(h-s)} R_e, \quad (15)$$

where R_e is given as

$$R_e = \frac{\sqrt{3} E (h-s)}{2\sigma_y (1-\nu^2)} \quad (16)$$

Hence, the final radius of the curvature during the elastic plastic bending is given as

$$R = \frac{\sqrt{3}\,E\,c}{2\sigma_y\left(1-\nu^2\right)} \quad (17)$$

Referring to Figure 5, the bend angle is calculated as

$$\theta = \frac{d}{R} \quad (18)$$

where*d* is the diameter of laser beam. Here, it is assumed that *R* is large compared to the thickness of the sheet.

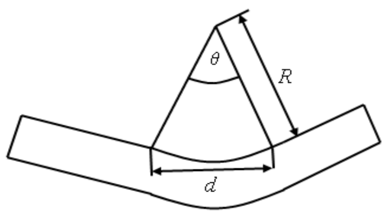

Figure 5 Schematic representation of bend angle

4 Validation of model

For validating the model, the analytical results have been compared with experimental results reported by Kyrsanidi *et al*. (2000). In all cases, the dimension of the sheet is equal to 300 mm × 150 mm × 6 mm, and it is made of 1.0584 (D36) shipbuilding steel. The scanning velocity ranges between 0.03 and 1.4 m/min and the power between 1 and 3 kW. The laser beam diameter is equal to 16 mm. The material properties are as follows: specific heat= 427J/(kg.°C), thermal conductivity= 35.1W/(m.°C), density= 7860 kg/m^3, absorption coefficient of the material= 0.3, coefficient of thermal expansion 12×10^{-6} 1/°C , Young's modulus= 200 GPa, Poisson ratio= 0.3. The values of yield stress are taken from Dixit *et al*. (2012) based on the average temperature. Average temperature is taken as the mean of maximum temperature and room temperature. Table 2 shows that at 1 kW laser power, the bend angle prediction matches well with the experimental results except at the highest and the lowest bend angle cases.The large percentage deviation in many cases may be due to uncertainty in the coefficient of absorption.

Table 2 Comparisons between present model and experimental data Kyrsanidi et al. (2000) for 1 kW laser power.

Scanning velocity (m/min)	Experimental bend angle(degrees)	Predicted bend angle(degree)	% Deviation
0.05	1.067	0.93	12.84
0.1	1.153	0.81	29.75
0.15	0.7	0.63	10
0.25	0.367	0.485	11.8
0.3	0.268	0.3	11.94
0.45	0.048	0.11	129.17

To further validate the proposed model, the present analytical solution is compared with the experimental results from Shen *et al*. (2006). Here, the line energy (*LE*) which is defined as *LE* =*P* /*v* applied to heated surface is constant. The material properties are as follows: Young's modulus= 210 GPa, yield stress= 275 MPa,mass density = 7860 kg /m^{3,} specific heat= 427 J/(kg.°C), thermal conductivity= 35.1 W/(m°C). The absorption coefficient= 0.8. The sheet size is 80 mm ×80 mm × 2.3 mm. The diameter of laser beam is 4 mm and *LE* = 20 J/mm. The scan velocity is varied between 40 and 65 mm/s. Table 3 shows comparisons between proposed model and experimental results for three different values of laser power and scan velocity. It shows that the results predicted by the proposed model are in good agreement with the experimental values reported by Shen *et al*. (2006).

Table 3 Comparisons between presented model and Shen *et al.* (2006) model

Power (W)	Scan velocity (mm /s)	Experiment bend angle (degrees)	Predicted bend angle (degrees)	% Deviati on
800	40	0.41	0.463	12.93
1000	50	0.47	0.542	15.32
1300	65	0.49	0.58	18.37

5 Estimation of yield stress by inverse analysis and model updating

With the model presented in this work, the yield stress can be estimated in an inverse manner to match the experimental results. The bisection method can be used to find out the proper value of σ_y, that produces very small error between the model predicted and experimental bend angle. Table 4 uses the data of Kyrsanidi *et al*. (2000). It is observed that inverse estimated yield stress is slightly different from the yield stress values used in direct model. This difference is not more than ±5%. In fact this much uncertainty is always present in yield stress of any typical material. Thus, the proposed model can be used for the quick estimation of yield stress of the sheet material.

Table 4 Inverse estimation of yield stress

Power (kW)	Yield stress (MPa)	
	Used in direct model	Predicted by inverse analysis
3	287.35	296.25
3	287.35	296.25
3	287.35	296.25
3	319.9	326.25
3	344.7	336.25
3	344.7	336.25
2	359.54	374.75
2	359.54	374.75
2	359.54	374.75

6 Conclusions

In this work a simple analytical model has been developed for estimating bend angle during laser forming of the sheet based on elastic- plastic theory. This model has been compared with some experimental results. A good agreement is obtained between experimental and predicted results. Although in some cases, the error between predicted and experimental results seems to be high, but as per the review paper of Shen and Vollertsen (2009) the average model prediction error by the best available analytical model is about 40% and generally the error is always more than 20%. Considering this, the present model provides a very good estimate and is fairly simple. The model can also be used for the quick estimate of yield stress.

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