Turning Time Calculation for Airborne Surveys

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Abstract

This KanGAL report shows the formulation for various turning mechanisms which are mostly adopted for airborne surveys for data collection of LiDAR, bathymetry etc wherein the area of interest on ground is covered by traveling a aerial platform (aircraft or helicopter) on parallel flight lines of same altitude. Three turning mechanisms, namely, consecutive turning, non-consecutive turning, and hybrid turning are demonstrated. Consecutive turning is performed by an aerial platform on two adjacent or consecutive flight lines. However, when an aerial platform performs a turn from a flight line to non-consecutive flight line, turning is called non-consecutive turning. Hybrid turning is combination of the consecutive and non-consecutive turning. A flight planning algorithm for selection of optimal turning mechanism, which should result in minimum time, is presented. Pseudo codes for implementation of these mechanisms are also presented.

1. Consecutive Turning

An aircraft can perform a turn from a flight line to the consecutive flight line or non-consecutive flight line. Generally both types of turnings are represented by the order of strips flown. Information of the turn can be conveyed by augmenting the direction of flying with ordering of the strips. Generally, the order of flight lines being covered is given using their direction of flight and the flight strip number. A flight direction can be forward (F) or reverse (R). The flight direction is forward if the flight is being carried out in the same direction as that of the direction of flight on the first flight line. A flight in the opposite direction to the forward direction is called reverse. Similarly, a trun can be also called clockwise or anti-clockwise depending how an aircraft is taking a turn when seen from zenith. A combination of flight lines and flight directions can be shown as 1F-2R-3F-4R, which represents the order of flight strips being covered and their direction of coverage. This further represents that between two flight strips a turn is being taken, which is consecutive turn in this case as successive flight lines are being covered as flight progresses. In case of a non-consecutive turn the flight numbers will not occur in monotonically increasing order but will change, e.g., 1F-6R-2F-7R-3F-8R-4F-9R-5F-10R.

It is evident that in either of the turning types, the aircraft alters the direction of flight from one particular flight line to the next flight line till survey operations are completed. The time of a turn is modeled for consecutive and non-consecutive turning in the following sections.

1.1 Modelling of consecutive turn

The spacing between the consecutive flight lines is equal to the effective swath (B). According to the effective swath, there are three possibilities: swath is more than, less than or equal to the width (h_t) of the 180º level turn (also called horizontal course reversal). Read & Graham (2002; pp 285-290) and Graham & Koh (2002; pp 133-136) mention the simple geometric arrangement of direct turn (DT), U turn (UT) and S turn (ST). Read & Graham (2002; pp 285-290) further suggest 30/30 procedure turn, which is a modification over S turn, for consecutive turning in GPS assisted aerial surveys. The characteristic design of all these turns utilize the available space or create an adequate space for 180º level turn with a given speed of aircraft and bank angle. When the effective swath is less than the width of half circular turn ($B < h_t$), it requires change in the direction of flight (or heading) such that it creates a space for 180º level turn (figure 3). In this case, consecutive turns are formulated appropriately by modifying 30/30 turn and S turn for varying locations of the ends of the consecutive strips. Conversely, an effective swath value more than or equal to the diameter (or width h_t) allows an aircraft to switch to consecutive flight line by U turn which is formed by two quarter turns separated by straight horizontal flight line segment. Depending on the value of the swath, the length of the horizontal line segment varies from zero to a finite value.

The mathematical formulations for all three cases, which combine and modify the 30/30 procedure turn, direct turn and U turn as per the aviation practices commonly adopted by pilots in field, are presented below to calculate total turn time (T_T). Following symbols are used in formulations and figures:

$i = i^{th}$ flight line

$i+1 = (i+1)^{th}$ flight line

$X_i^L = X$ coordinate of left edge of the i^{th} flight line

$X_{i+1}^L = X$ coordinate of left edge of the $(i+1)^{th}$ flight line

$X_i^R = X$ coordinate of right edge of the i^{th} flight line

$X_{i+1}^R = X$ coordinate of right edge of the $(i+1)^{th}$ flight line

ψ = Change in the heading angle of aircraft at the end of i^{th} flight line

ψ_{Max} = Maximum allowable change in the heading angle of aircraft (user input)

Case-1 ($B < h_t$): Figure (1) depicts the consecutive turning by 30/30 turn which consists of inclined linear path and circular path. The turning process starts at the end of the strip (point A). From point A to C, the aircraft travels along an inclined line. At point C, the aircraft starts to follow a circular trajectory and performs a horizontal course reversal (or 180º level turn) and reaches the subsequent flight line at point D. During turning, the change in the heading angle is realized by following a combination of transition curves and an arc of a circle. Consequently, in addition to the straight line and circular curve, the length of the flying path (travel length) is also made of the transition curves that provide a smooth change in the bank angle and the direction of flight. In addition to that, the relative positions (X coordinates) of the two adjacent flight lines or strips also dictate the length of the travel.

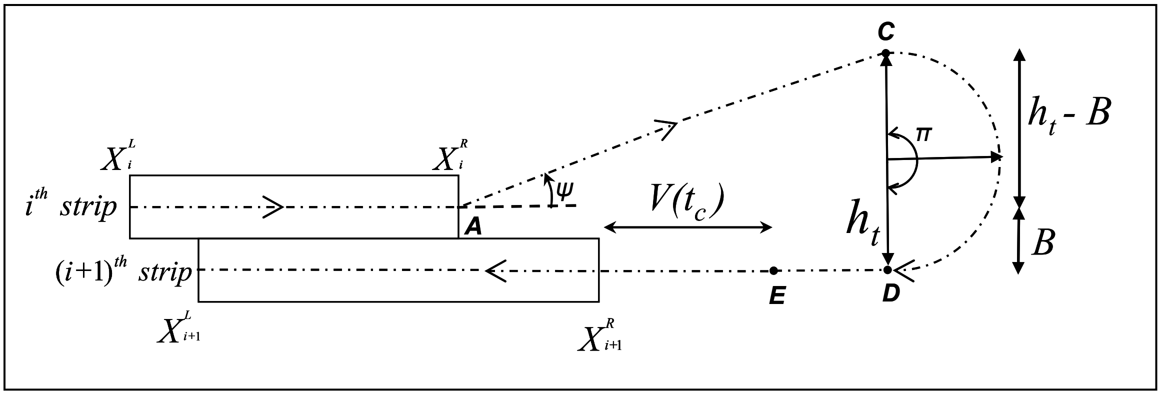

Fig. 1: Schematic view of two consecutive flight lines and flight path with cushion period in a consecutive turn

The complete geometric arrangement including the transition curves from point A to point C can be modeled by different geometric methods and curves. Conventionally, transition curve in level turn is modeled by Cornu spiral or Euler spiral which is a clothoid curve (Dai & Cochran Jr., 2009; Techy et al., 2010). The solutions proposed by Dai & Cochran Jr. (2009), Techy et al. (2010) and others (e.g. McCrae, 2008) are complicated and iterative due to the involvement of Fresnal integral in mathematical expressions.

On the other hand, a pilot performs a turn according to his/her personal comfort. A naturally comfortable agreement is to adhere to flat turns. A flat turn ensures better control in traveling over a predefined path and avoids the fatigue that is apparent in repetitive operations like turnings (Capt. K.B. Singh & Capt. Amit Dahiya, personal discussion). Moreover, the heading change in a turn performed with standard rate turn (SRT) is reliable and more predictable (AVA, 2001). As a consequence, between two given points in air that requires a heading change for a level flight, instead of a shorter curvilinear path with higher turn rate, a longer path of small curvature with turning rate less than that of the SRT is generally practiced (AVA, 2001; Capt. Amit Dahiya, personal discussion). According to AVA (2001), the rule of thumb is to perform a SRT with bank angle less than one half of the change in the heading. For a 30/30 procedure turn, Read & Graham (2002; pp 285-290) mentions a maximum allowable change in heading angle by 30º. Therefore, in order to change the heading by 30º, maximum bank angle should be limited to 15º. Further, if the bank angle reaches its maximum value of 15º, it should be maintained at 15º to follow a circular level turn. Finally, the aircraft should start coming back to the level flight when there remains a heading difference equal to the maximum bank angle (15º in this case). This arrangement ensures the turn rate less than SRT. Similarly, 1/2 SRT turn is formulated with thumb rules by aviation authorities (IP, 2012).

Considering the practical experience and preference of flying crew and the complexity of iterative solution for solving the spiral based equations, an approximate model is developed in this thesis for calculation of the length of the trajectory from point A to C and from C to D by simplifying the possible geometric arrangement. The length of travel over the modeled trajectory of aircraft in curvilinear path from point A to C and in 180º level turn from point C to D are connected at point C with zero banking angle. Maintaining the continuity of curve with zero bank angles at point C, these curves can be formulated separately. The mathematical expressions for transition curves and circular curves are based on the same calculation steps. First, the calculation steps are shown for the 180º level turn from point C to point D and later similar steps are adopted for approximating the curvilinear path from point A to C.

Zdanovich (1967; pp 18-27) describes a heuristic method and formulation for modelling a turn described by circular arc that is adjoined by transition curves at the ends and change the heading of aircraft by a specific angle in a level turn. The illustrated method by Zdanovich (1967; pp 18-27) adjusts the travel length of the circular arc for the period of transition curves by appropriate logically derived mathematical approximations.

Figure (2) shows the turning geometry of 180º level turn on consecutive flight lines. An aircraft enters at point C on a transition curve with zero bank angle, joins the circular curve at D_1 and travel over it to D_2 with maximum banking angle (β_m) and finally exits with transition curve at D. Ideally, point D should match with the right end coordinates of $(i+1)^{th}$ strip. Therefore, modeling the geometry of circular curve should be initiated with point C. Coordinates of point C can be obtained by projecting the geometry of lower half of curves utilizing the property of symmetry. Angle ψ can be calculated using the coordinates of points A and C. If angle ψ turns out to be more than maximum allowable change in the heading angle ($\psi_{\max}$), the curve for turn from point C to D should be shifted rightward by required distance so as to limit ψ up to $\psi_{\max}$.

Zdanovich (1967; pp 18-27) assumes a constant rate of banking angle ($\dot{\beta}$) and consequently evaluates the transition period (T_m) to attain maximum bank angle (β_m) as linear and nonlinear function of time, respectively. During the transition period (T_m), an aircraft is banked (roll in) to the maximum bank angle (β_m). According to Zdanovich (1967; pp 18-27), during the period of transition (T_m), the maximum value of heading change (ψ_T) and the change along X and Y coordinate axes to reach the end point of a transition curve where circular curve commences, are given by:

$$T_m = \left(\frac{\beta_m}{\dot{\beta}}\right) \quad \text{… (5a)}$$

$$\psi_T = \left(\frac{V\,T_m}{2\,r}\right) \approx \left(\frac{g\,T_m\,\beta_m}{2\,V}\right) \quad \text{… (5b)}$$

$$\Delta X = V\,T_m \quad \text{… (5c)}$$

$$\Delta Y = \frac{g\,T_m^{\,2}\,\beta_m}{6} \quad \text{… (5d)}$$

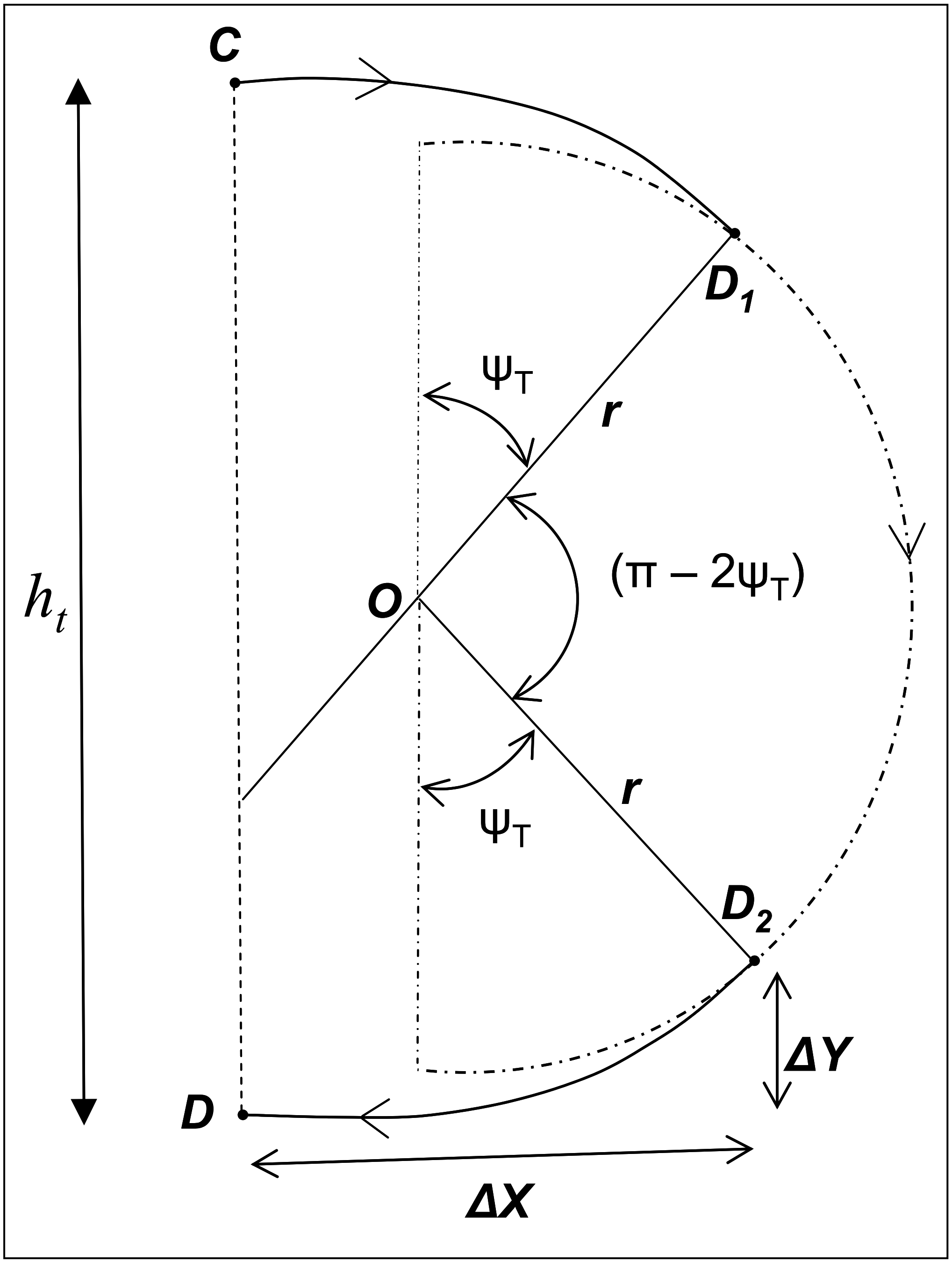

Fig. 2: Schematic view of circular trajectory and transition curves of flight path

Radius of a circular and level turn with given aircraft speed and banking angle (β_m) is provided by (Wolper, 1994):

$$r = \left(\frac{V^2}{g \tan \beta_m} \right) \qquad \dots (6)$$

Coordinates of point C (X_C, Y_C), point D (X_D, Y_D), center point (X_0, Y_0) of the circular portion and horizontal distance (h_t) between point C and D are given by:

$$X_C = X_{i+1}^R + V\,t_c$$
$$Y_C = Y_{i+1} + 2(\Delta Y) + 2(r\cos\psi_T) \qquad \dots (7a)$$

$$X_D = X_{i+1}^R + V\,t_c$$
$$Y_D = Y_{i+1} \qquad \dots (7b)$$

$$X_0 = X_D + \Delta X - r\sin\psi_T$$
$$Y_0 = Y_D + \Delta Y + r\cos\psi_T \qquad \dots (7c)$$

$$h_t = 2(\Delta Y + r\cos\psi_T) \qquad \dots (7d)$$

The length of traveling path along the curve from point C to D can be written as:

$$L_c = 2V\,T_m + \left(\pi - 2\psi_T\right) r \qquad \dots (8)$$

where angle ψ_T should be expressed in radians.

Figure (3) shows a possible mechanism of maneuvering of the aircraft in the trajectory from point A to C for a consecutive turn. This figure illustrates clearly the different components of travel, i.e., an arrangement of possible curvilinear paths and straights.

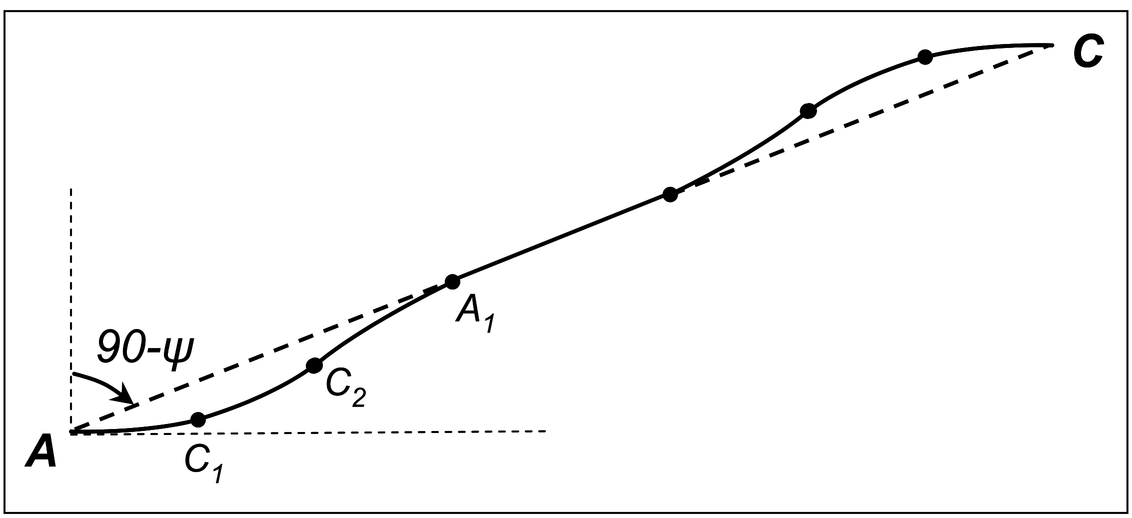

Fig. 3: Schematic view of conceptualization of flight path from point A to C

In order to change the heading of an aircraft by an angle ψ over the length L_1 of the straight line from point A to point C, the travel length from point A to point C is illustrated as curvilinear path formed by two curves and a straight line connecting these curves. Curve $AC_1C_2A_1$ is shown originating at point A and another curve, which is similar to $AC_1C_2A_1$, is shown ending at point C. The aircraft from zero banking angle at point A, by travelling along the transition curve AC_1 gradually reaches to bank angle (β_1) in time period T_1 and changes it's heading by angle ψ_1. At the end of the transition curve at point C_1, the aircraft moves along circular arc C_1C_2 with a constant bank angle (β_1). At point C_2, the aircraft starts travelling over transition curve C_2A_1 and reaches to point A_1 with zero banking angle in time T_1. After desired change in heading (ψ), at point A_1, the aircraft starts travelling along the straight line. After travelling over the straight line, the aircraft again follows the curve, which is similar to $AC_1C_2A_1$, and reaches to point C with zero value of banking angle. In the process of travelling over the transition curves, circular arcs, and straight line from point A to point C, the aircraft change its heading angle by ψ. In some cases, it is possible that the length of the straight line in the curvilinear path may be zero and curvilinear path consists of only two similar and connected curves.

The change in heading angle ψ from point A to C is measured positive in the direction away from the next strip. Moreover, as stated earlier, the change in the heading angle (ψ) is decided by calculation subjected to its maximum allowable value (ψ_{Max}). Hereafter, as explained earlier with calculations, the aircraft enters through a transition curve into a circular curve from point C and exits too through a transition curve and reaches at point D.

The travel length from point A to point C comprises of four transition curves, two circular curves and one straight line segment. However, the calculation of the length of travel for six curves and a straight line is complicated and cumbersome. The complicated calculations are avoided by adopting a simplified model suggested by Walker (1964). As shown in figure (4), the model approximates the length of travel from point A to point C by an S curve or two inverted but connected circular curves. The S curve is appended with two transition curves at the ends (i.e. at point A and C) for better estimate of the turning time. The transition curves are assumed to occur only at point A and point C and all other transition curves and straight line, which may occur intermittently, are assumed to be included in the circular arcs. The proposed model of two circular curves with transition curves at the ends provides a slight overestimate of travel length. However, it avoids the cumbersome calculations by avoiding the connected intermediate transition curves and straight line segment.

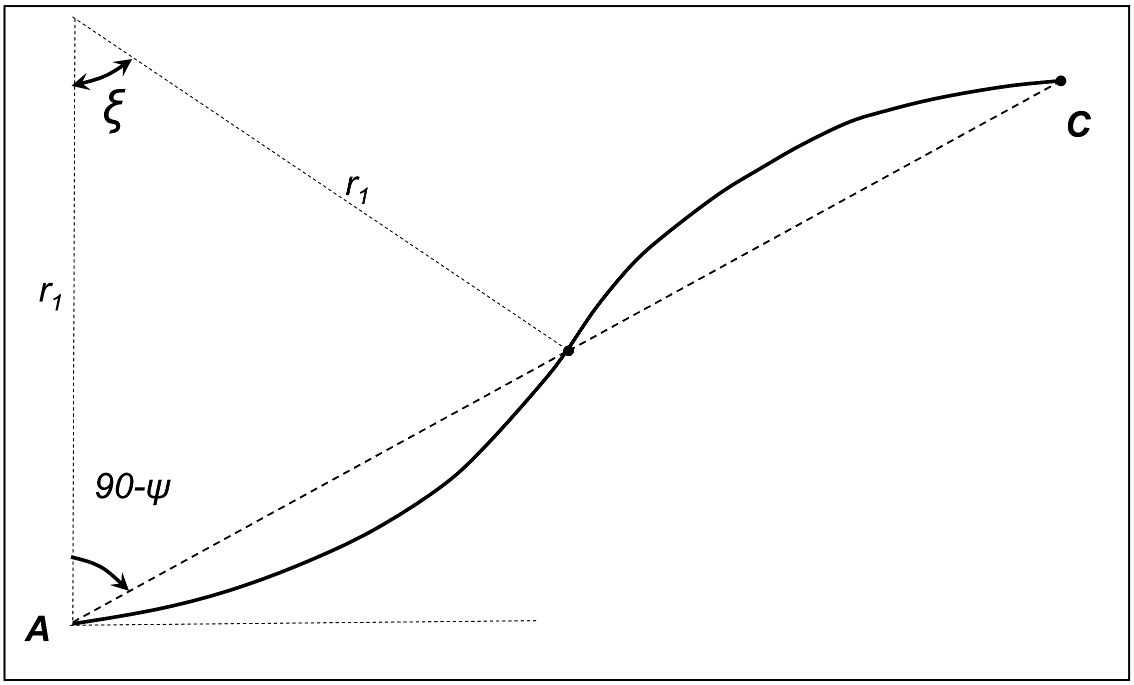

Fig. 4: Schematic view of modelled flight path (after Walker, 1964)

Based on this simple and approximate model which consists of two circular arcs, the maximum allowable change in heading angle at point A ($\psi_{\max}$) can be estimated. The angle $\psi_{\max}$ confirms the geometric arrangement of consecutive turn by ensuring the compatibility and connectivity at point C between the curvilinear path from point A to C and 180º level turn from point C to D. In order to change the angle formed by S turn by a value equal to two times of ξ, the minimum length of straight line from point A to point C should be equal to:

$$L_0 = 4\,r_1 \sin\left(\frac{\xi}{2}\right) \qquad \ldots (9)$$

Walker (1964) provides the mathematical expression to estimate the angle ξ. The modified expression according to the width of the 180º level turn (h_t) and effective swath (B) is written as:

$$\cos\xi = \left(\frac{2\,r_1 - (h_t - B)}{2\,r_1}\right) \qquad \ldots (10)$$

where

$r_1 = \dfrac{V}{\dot{\psi}_{Max}}$ = Radius of a circular arc of curvilinear path from point A to point C

$\dot{\psi}_{Max}$ = Maximum allowable rate of turn on a circular path, i.e., maximum angular speed

Substituting the value of ξ from (10) to (9) gives:

$$L_0 = \sqrt{\frac{8\,V\left(h_t - B\right)}{\dot{\psi}_{Max}}} \qquad \ldots (11)$$

The available length (L_1) of straight line between point A and C is calculated by considering the width of 180º level turn (h_t) and effective swath (B) as:

$$L_1 = \left(\frac{h_t - B}{\sin\psi}\right) \qquad \ldots (12)$$

Coordinates of the ends of the consecutive flight lines are not considered as these may calculate less value of angle ψ. As explained earlier, a pilot always prefers to travel by standard rate of turning (SRT) or less, the available length (L_1) should be at least equal to or more than the minimum length (i.e. $L_1 \geq L_0$). Therefore, the following inequality restricts the value of angle ψ as:

$$\sin\psi \leq \sqrt{\left(\frac{h_t - B}{8\,V}\right)\dot{\psi}_{Max}} \qquad \ldots (13)$$

The above expression (13), which limits the value of angle ψ, is a function of speed of aircraft, width of 180º level turn, effective swath (or spacing between adjacent flight lines), and the rate of turning. As shown in formulations earlier, the width of a 180º level turn itself is a function of speed. As maximum rate of turning is used, the expression (13) provides the maximum allowable value of angle ψ. Therefore, maximum allowable value of angle ψ can be written as:

$$\psi_{Max} \leq \sin^{-1}\left(\sqrt{\left(\frac{h_t - B}{8V}\right)\dot{\psi}_{Max}}\right) \qquad \ldots (14)$$

Taking SRT (rate of turn ≤ 3º per second) as the maximum rate of turning, curves showing the values of maximum allowable change in heading angle (ψ_{Max}) against the speed (V) are drawn for different values of effective swath (B) in figure (5). The speed is selected in the range of 40 to 77 m/s (approximately 80 to 150 knots). The values of effective swath are 30, 130, 230, 330, 430, and 530 meters that covers a wide range of the possible values from minimum to maximum.

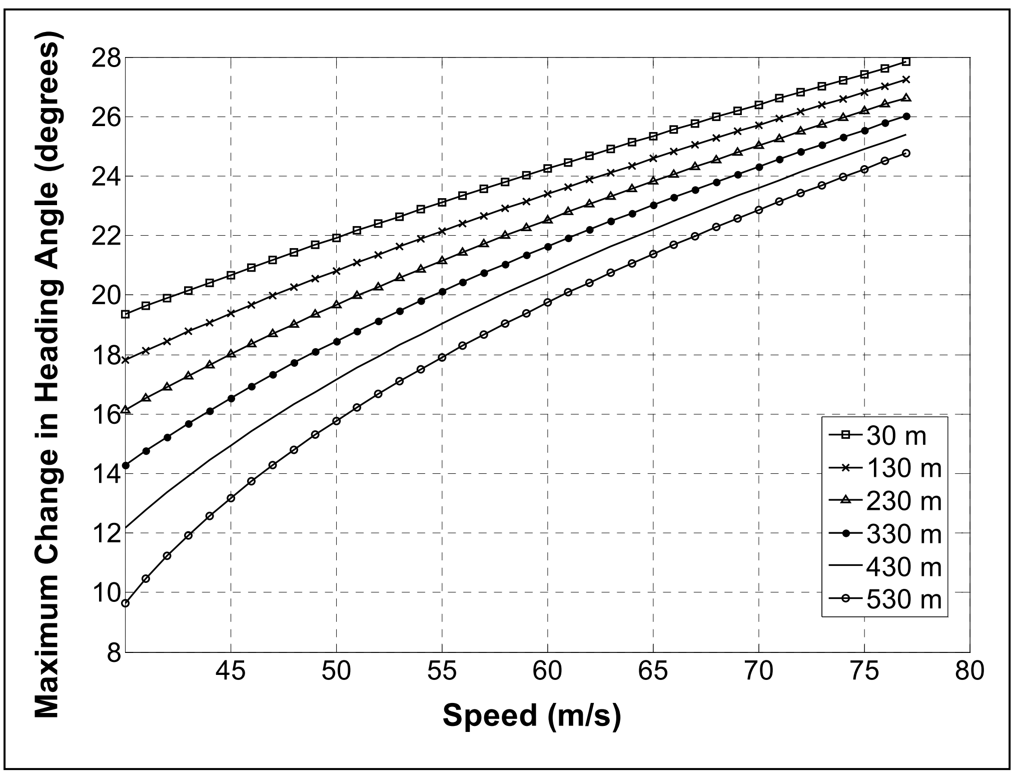

Fig. 5: Maximum allowable change in heading angle at point A

In the calculation process for ψ_{Max}, the value of radius of turn of circular arc (r_1) is calculated according to the maximum rate of turning ($\dot{\psi}_{Max}$). However, for another value of angle ψ, which is less than ψ_{Max}, the rate of turning ($\dot{\psi}$) will also be less than the SRT as shown by equation (15a) below. Consequently, the radius of circular arc (r_1), bank angle at the end of the transition curve (β_1), time required for the transition (T_1 or transition period), and the change in the heading during transition period (ψ_1) can be calculated as (Zdanovich, 1967; pp 18-27):

$$\dot{\psi} = \left(\frac{8V \sin^2 \psi}{h_t - B} \right) \qquad \text{… (15a)}$$

$$r_1 = \left(\frac{V}{\dot{\psi}} \right) \qquad \text{… (15b)}$$

$$\beta_1 = \tan^{-1}\left(\frac{V\dot{\psi}}{g} \right) \qquad \text{… (15c)}$$

$$T_1 = \left(\frac{\beta_1}{\dot{\beta}} \right) \qquad \text{… (15d)}$$

$$\psi_1 = \left(\frac{V\,T_1}{2\,r_1} \right) \approx \left(\frac{g\,T_1\,\beta_1}{2V} \right) \qquad \text{… (15e)}$$

In the above equations, $\dot{\psi}$ is the rate of turn and $\dot{\beta}$ is the rate of banking angle during transition period. The value of rate of turn for SRT is 3 degrees per second. Zdanovich (1967; pp 18-27) considers the banking of 30º in 3 seconds duration with constant rate. Therefore, rate of banking angle can be calculated as 10º per second. The change in heading angle during the transition period is expressed by equation (15e). If this heading change (ψ_1) is more than that of the straight line (i.e. ψ), it conveys that transition curve is not required as circular curve is extremely flat. Extremely flat curve can be considered as single circular arc of small curvature (or straight line). Owing to the small discrepancy raised due to the mismatch between the circular arc length and actual length of the trajectory, transition period (T_1) is added. Therefore, the length of the curvilinear path from point A to point C is calculated as:

$$L' = L_1\,;\;\; if\ (\psi_0 > \psi)$$
$$L' = 2VT_1 + 4\, r_1 \sin^{-1}\left(\frac{L_1}{4r_1}\right);\;\; if\ (\psi_0 \le \psi) \qquad \ldots (16)$$

where $\psi_0 = \tan^{-1}\left(\frac{g\,T\,\beta_1}{6V}\right)$

Case-2 ($B \ge h_t$): This is a U turn i.e. two 90º turns joined by a straight line. These two turns, each performing the change in direction by 90º, require two pairs of transition curves. Each pair of transition curve provides time to achieve the maximum banking at the beginning of circular curve and zero banking after completing the circular curve. In this case, the minimum horizontal distance between point C and point D and travel length should be:

$$h_t' = 2\,(\Delta Y + \Delta X + r\,(\cos\psi_T - \sin\psi_T)) \qquad \ldots (17a)$$

$$L_c = 4V\,T_m + 2\,r\left(\frac{\pi}{2} - 2\psi_T\right) \qquad \ldots (17b)$$

As shown in figure (2), for maintaining a constant bank angle on circular path from point D_1 to point D_2, the desired horizontal distance between point C and point D and travel length are given by equations (17a) and (17b), respectively. There may be a conflicting situation where the available distance between point C and point D is higher than h_t and smaller than h_t'. In such intermediate situations, though pilot can skillfully maneuver using navigational aids for an ignored short distance and reach destination point successfully, the estimation of correct time is critical. Transition curves at the end of first circular turn and the start of second circular turn between point C and point D, cover the angular distance of $2\psi_T$. It can be shown by a numerical example again that assuming circular path for the angle of $2\psi_T$, a distance equal to $2(\Delta X)$ in a turn will underestimate the time duration approximately by an amount equal to transition period (T_m). Performing the banking by maximum angle of 30º at the rate of 10º per second demands the transition period (T_m) and the change in X and Y coordinates (ΔX, ΔY) as 3 seconds, 150 meters, and 7.7 meters, respectively. Therefore, though it is possible to estimate the correct time by applying the correction to transition period, it is not advised as aircraft is not guaranteed to reach the consecutive flight line. The horizontal offset ($=2\Delta X + 2\Delta Y$) is equals to 315 meters for the numerical example shown above. Another possible solution is to reduce the bank angle. It will reduce both angle ψ_T and the transition period (T_m), while increasing the radius of circular turn (r). Increased radius of the turn for circular portion will certainly increase the travel length and travel time as well but ensure the destination correctly. Equating the radius of turn to half of the swath is a good approximate criterion and will obtain the new value of bank angle (β'_m). Recalculations for transition period (T_m), angle ψ_T, radius of turn (r), and the travel length (L_c) using equations (5a), (5b), (6) and (8) are imperative. New angle of bank (β'_m) is given by:

$$\beta'_m = \tan^{-1}\left(\frac{2V^2}{B\,g}\right) \qquad \ldots (17c)$$

Algorithmic steps in the form of pseudo codes are presented in Appendix A.

2. Non-consecutive Turn

2.1 Modelling of non-consecutive turning

When an aircraft, after negotiating a flight line, turns and reaches a flight line which is not juxtaposed, the turn is called non-consecutive. In this case, the aircraft turns naturally according to banking angle and turning radius in a constant direction (clockwise or anti-clockwise). A clockwise turn at the end of a forward direction flight will take an aircraft from a lower number flight line to a higher number flight line. This type of turn, in view of the increase in the number of flight line is called a forward turn. Similarly, a clockwise turn at the end of a reverse direction flight line will take an aircarft from a higher number flight line to a lower number flight line. As this turn is taking an aircraft from a higher flight number to a lower flight number, it is also calleda reverse turn. The same, as in the above, will be true if the term 'clockwise' is replaced by 'anti-clockwise' and 'forward' by 'reverse'. In a forward turn, two flight lines, which are covered by an aircraft in sequence, are separated by a specific integer number of lines that creates an interval of lines or the line interval. Similarly, another value of line interval is required for performing a reverse turn. As a result, the line intervals for reverse and forward turns are different. The spacing for reverse direction turn is less by one line than that of the forward direction. Figure (6) shows the non-consecutive turns (NCT) over a rectangular AOI. As turning is performed in a systematically smooth and natural way along a constant direction (say clockwise and counterclockwise or right handed and left handed), the non-consecutive turns are also called constant direction turns or CDT (Read & Graham, 2002; pp 285-290).

As shown in figure (8), there are 19 flight lines. The aerial survey commences from point S and ends at point E. The flight lines on AOI are indicated by dashed lines with arrows which show the flight direction. The flying operation starts at point S. However, the turning starts from the upper end of the extreme left strip (first strip) to the next strip in clockwise direction with respect to nose of the aircraft or viewing direction of pilot. As a result, after the end of the first strip, the aircraft takes a long forward turn to the sixth strip. Further, at the end of the sixth flight line, the aircraft performs a short reverse turn to the second numbered flight line. Therefore, a long clockwise forward turn and a short clockwise reverse turn are formed by forward and reverse turnings, respectively. Accordingly, the aircraft covers first ten flight lines of AOI in the similar manner of forward-long-turn and reverse-short-turn in a clockwise direction. After travelling over the tenth flight line, i.e. once all preceding flight strips are covered, the aircraft starts travelling in a counter-clock wise direction (or left handed direction) to fly over the remaining flight lines in a similar fashion till the end of the survey operation at point E. As a result, ordering of flying operations over the flight lines is 1F-6R-2F-7R-3F-8R-4F-9R-5F-10R-15F-11R-16F-12R-17F-13R-18F-14R-19F. Upper and lower ends of vertical flight strips are similar to the right and left ends of horizontal flight strips.

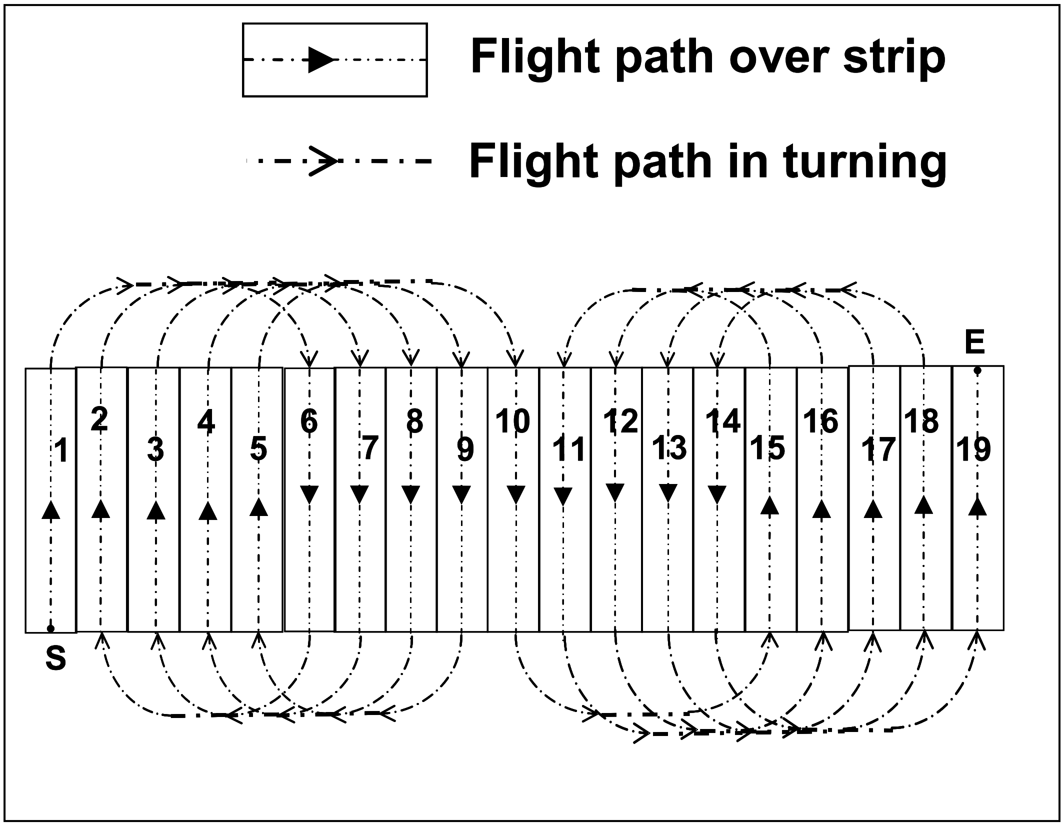

Fig. 6: Schematic view of non-consecutive turns over rectangular AOI

A non-consecutive turning mechanism, as can be referred in figure (6), is a set of forward long turns and reverse short turns in a constant direction between the flight lines or flight strips. The line interval, i.e., the separation between flight lines between which a short or long-turn is taken, can have different values. However, minimizing the line interval, i.e. by taking the turn at the earliest opportunity, will ensure minimum time taken for these turns.

The line interval is the number of flight lines within which a turn is realized, which is two in the case of consecutive turns. The distance for minimum line interval can be decided by minimum travel length (h_t') which is given by equation (17a). The line interval for a long-turn requires one additional line. For a given spacing (effective swath) of flight strips, the travel length of a long-turn or short-turn is equivalent to the travel length in case of a consecutive turn with swath width being more than the minimum distance (h_t') required for consecutive turning. Each long and short-turn has a constant travel length for turning except for the length caused by the difference in the coordinates of the end of strips that are joined by the turns. Therefore, the line interval for a forward turn (l_f) and a reverse turn (l_r) can be calculated as:

$$l_f = ceiling\left(\frac{h'_t}{B}\right) + 1 \qquad \dots (18a)$$

$$l_r = l_f - 1 \qquad \dots (18b)$$

Turning by a long-turn from i^{th} strip to j^{th} strip requires a line interval of l_f number of lines. The line number of j^{th} strip can be written as:

$$j = i + l_f \qquad \dots (19)$$

As indicated earlier, the coordinates of the right end and left end of a flight strip are increased and decreased, respectively, by a distance that is necessary for creating the cushion time (i.e. $V\,t_C$). A short-turn will be created by turning from j^{th} strip to $(i+1)^{th}$ strip. The turning lengths for a long-turn (L_f) and a short-turn (L_r) can be determined as:

$$L_f = 4V\,T_m + 2r\left(\frac{\pi}{2} - 2\psi_T\right) + B\,(l_f) - h'_t + V\,t_C \qquad \dots (20a)$$

$$L_r = 4V\,T_m + 2r\left(\frac{\pi}{2} - 2\psi_T\right) + B\,(l_f - 1) - h'_t + V\,t_C \qquad \dots (20b)$$

After deciding the line interval, turn length, source line and destination line, the mechanism of non-consecutive turning can be modeled using an algorithm. According to the algorithm, an aircraft travelling over any flight strip preferably takes a reverse turn with a short-turn provided the destination flight line exists and yet not covered. However, if the short reverse turn is not possible to be performed, a long forward turn is formed. This process is repeated till all the flight strips are covered. This algorithm can be explained by figure (6). In figure (6), a total of 19 flight lines are shown. After covering the first flight line, the aircraft takes a forward turn to the sixth flight line by a long-turn as it is not possible to form a short reverse turn. At the end of the sixth flight line, the aircraft turns to second flight line by a short reverse turn because the second flight line exists and is not covered yet. This continues till first to tenth flight lines are travelled by a series of long and short-turns in clockwise direction. After reaching the tenth flight line from the fifth flight line by a long forward turn, it is imperative that the aircraft should turn by a short-turn in reverse direction. However, as all preceding flight lines have already been covered, the aircraft rather takes a long forward turn to the fifteenth flight line in counterclockwise direction. In the above process of non-consecutive turning, the line interval for long-turn and short-turn are 5 and 4, respectively. At this stage the aircraft repeats the same steps by a series of short reverse turns and long forward turns to cover the rest of the AOI. This continues till all the flight lines are exhausted.

In the process of turning as shown above, a situation may arise, as shown in figure (7), when turning in forward direction or in reverse direction is not possible because the destination flight lines, both for long and short-turns, are already covered or unavailable. However, there are some more flight lines which are still not covered. There are two methods of covering the remaining flight lines: natural non-consecutive turning (NNCT) and hybrid non-consecutive turning (HNCT).

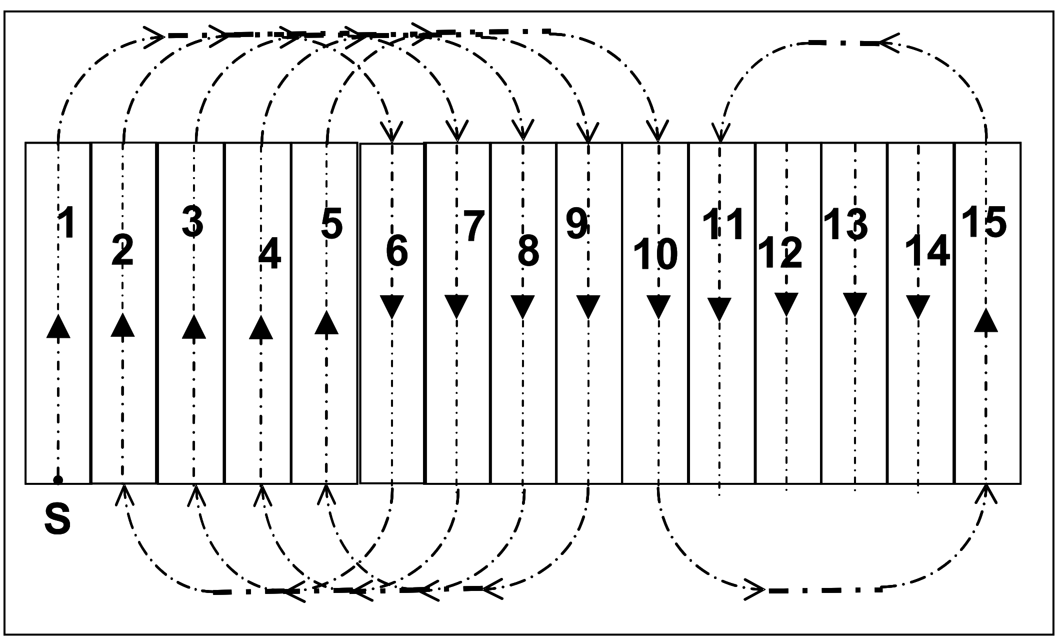

Fig. 7: Schematic view of non-consecutive turns over rectangular AOI

The complete arrangement of turning, since the beginning of the survey from point S to the end point E in figure (8) is termed as natural non-consecutive turnings (NNCT). More specifically, this turning mechanism is named here as natural non-consecutive turning of second type (or NNCT-2). The natural non-consecutive turning of first type (NNCT-1) will be explained later. Turning by NNCT-2 mechanism comprises of a series of long and short-turns (as shown in figure 8). As is evident from figure 8, each of the remaining flight lines is covered by performing a set of long-turn and short-turn outside the AOI. The long-turn, short-turn, and the extra flight lines together are addressed as extra turn. It can be observed that unlike a set of long-turn and short-turn that covers two flight lines in the AOI, an extra turn can only cover one flight line as the extra turn lies outside the AOI.

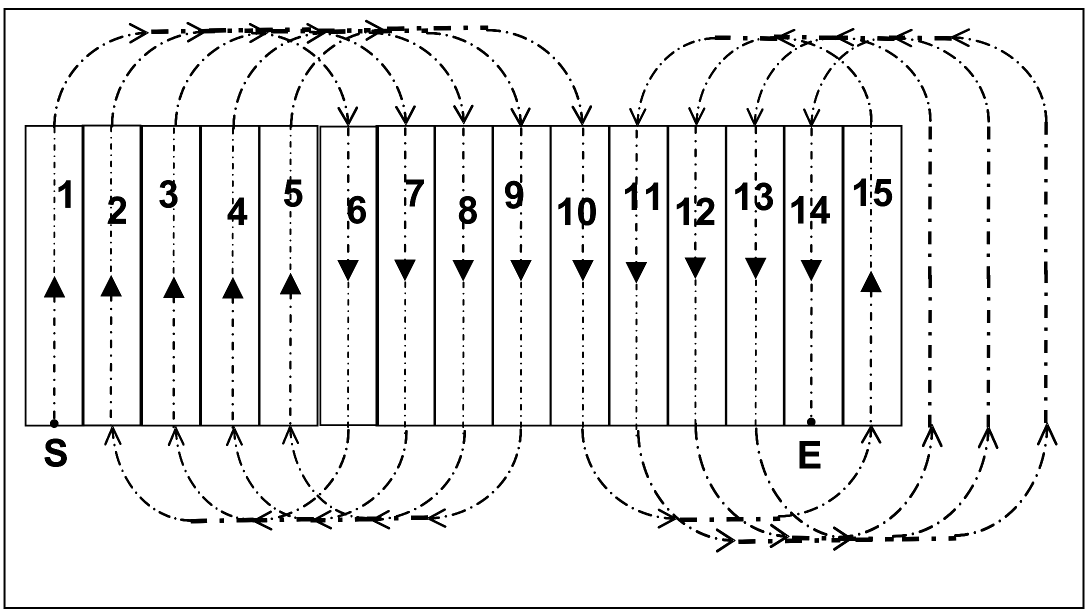

Fig. 8: Schematic view of natural non-consecutive turning with extra turns (type-2 or NNCT-2)

An extra turn essentially consists of a pair of long-turn and short-turn with cushion times and an additional flight line outside the AOI. The length of additional flight line, which lies outside the AOI, varies according to values of X coordinates of opposite ends of two consecutive flight lines. However, in each extra turn, travel length for a pair of long-turn and short-turn with a distance equivalent to the cushion time is a constant. The constant travel length in an extra turn is given by:

$$L_e = 8 V T_m + 4 r \left(\frac{\pi}{2} - 2 \psi_T \right) + B (2 l_f - 1) - 2 h_t' + V t_C \qquad \ldots (20c)$$

While performing a NNCT-2 turn, a special case may occur where there is a further possibility of minimizing the length of travel. In other words, in order to achieve the economy in flight duration, it is imperative to search a possibility of forming a short forward turn. Such a case is illustrated by figure (9). Instead of performing an extra turn (by NNCT-2), i.e. covering one flight line lying inside and other outside the AOI, a better solution is to cover both flight lines inside the AOI. This turning mechanism is termed as natural non-consecutive turning of first type (or NNCT-1). In this turning mechanism, long and short-turns are performed in such a way that while one of the remaining flight lines of AOI is covered by one flight path, the other flight path lies over the already covered flight line on the AOI. As shown in figure (8), instead of initiating the extra turn by a long-turn for the remaining flight lines, a short-turn is performed. The short-turn repeats the flight line as the aircraft covers a flight line which is already covered. After this short-turn, a set of long-turn and short-turns are travelled to cover the remaining flight lines in successive manner. NNCT-1 mechanism results in a few strips being covered twice as shown in figure (9). Turning with NNCT-1, which is applicable under certain circumstances, certainly reduces the time duration compared to NNCT-2. In figure (9), for better explanation of NNCT-1 mechanism, some of the long and short-turns are drawn with different lengths and shapes.

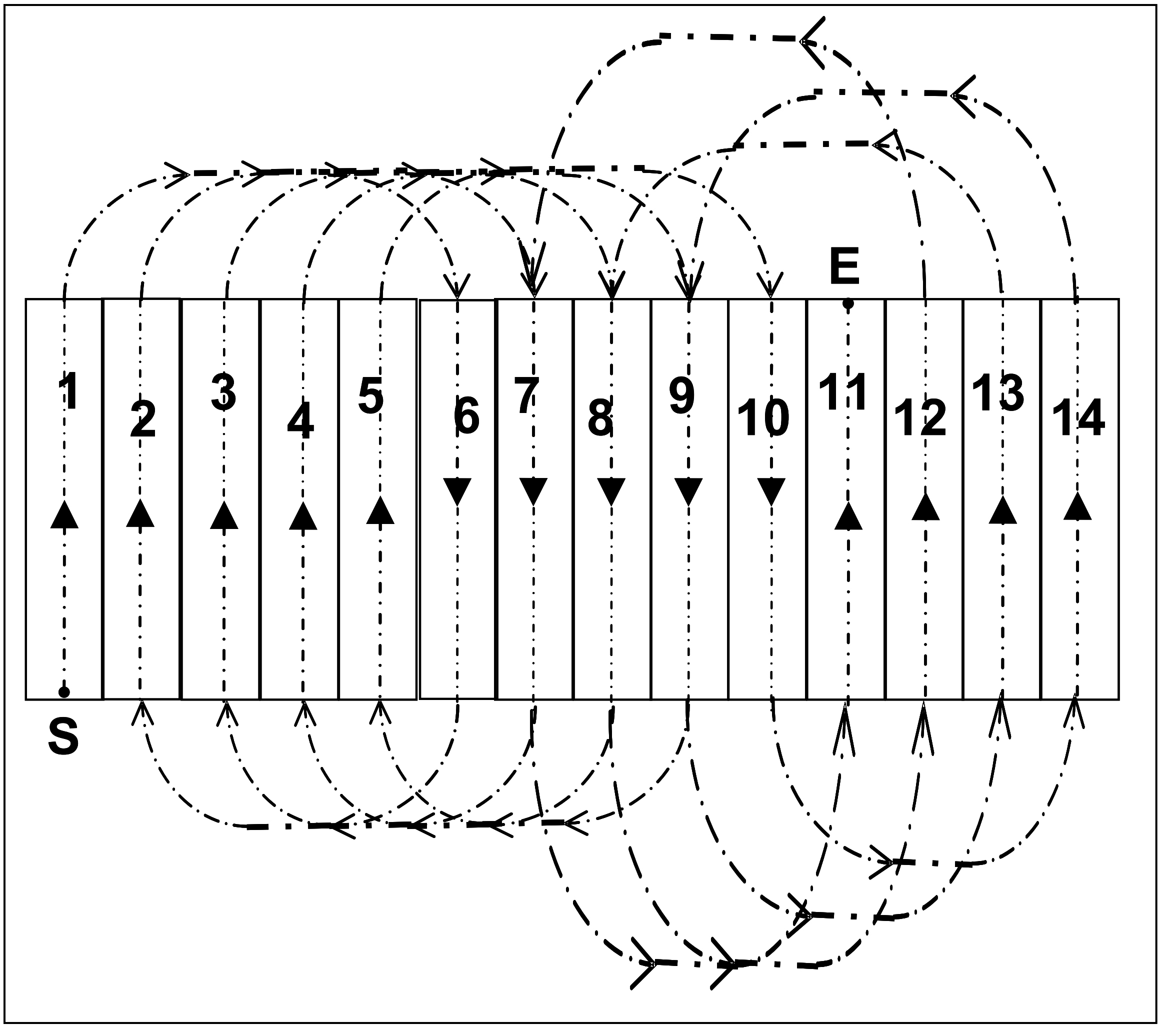

Fig. 9: Schematic view of natural non-consecutive turning of type-1 (NNCT-1)

3. Hybrid Turning

The NNCT mechanism also suggests that instead of turning by NNCT-1 or NNCT-2 mechanisms, consecutive turning can be performed for the remaining flight strips. However, as the non-consecutive turning is desired only when consecutive turning is difficult to perform, the possibility of hybridizing the non-consecutive and consecutive turnings in one mission is preferably not adopted in field practices (Adriaan Combrink, private communication). Considering the possible economy in the flight operations and depending upon the choice of a pilot, there is a slight possibility to perform hybrid non-consecutive turnings (HNCT). Like NNCT, HNCT can also be imagined of two types:

HNCT-1 and HNCT-2. Turning by NNCT-2 and HNCT-2 mechanisms are shown by figures (10) and (11).

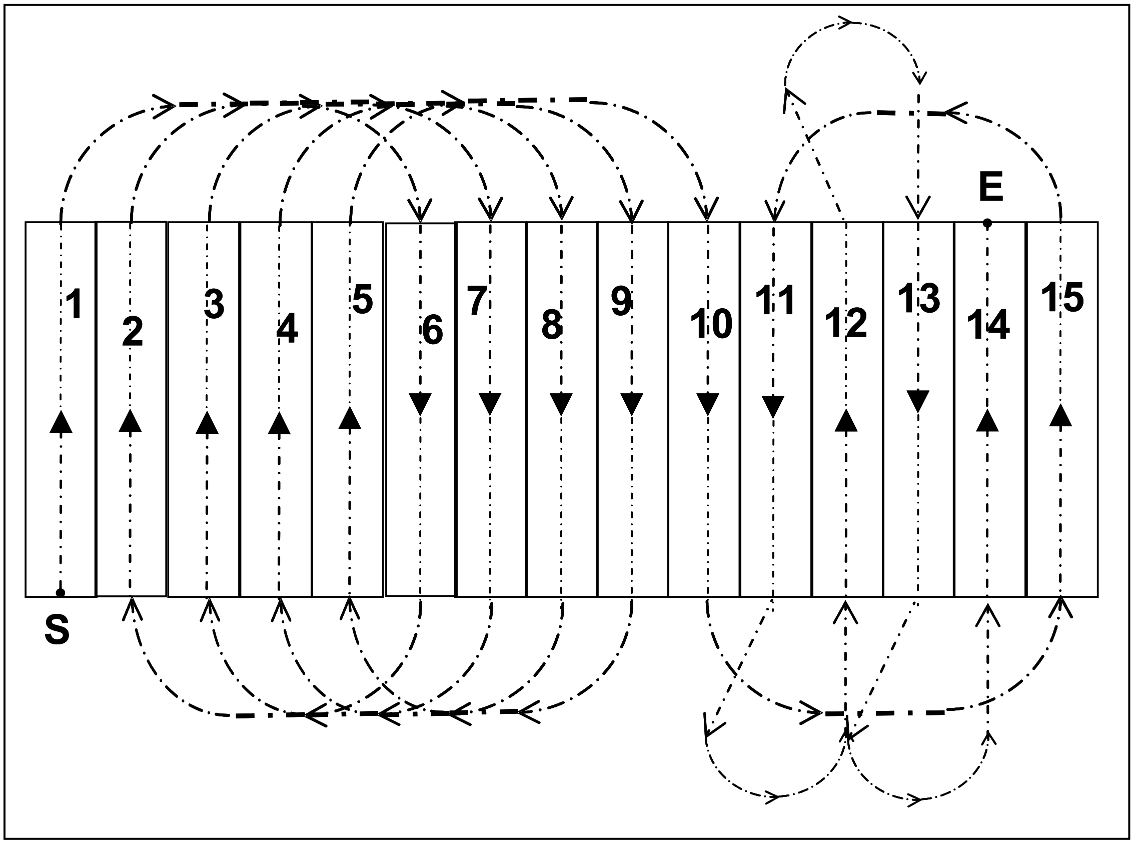

Fig. 10: Schematic view of hybrid non-consecutive turning of type-1 (HNCT-2)

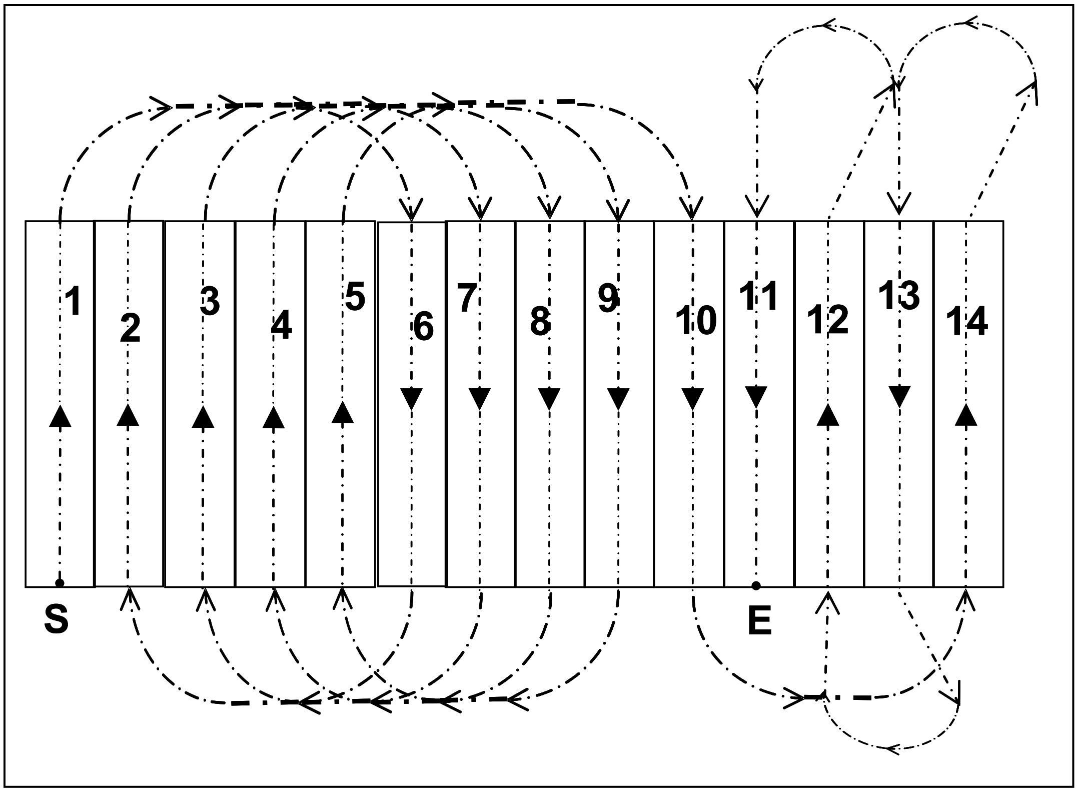

Fig. 11: Schematic view of hybrid non-consecutive turning of type-1 (HNCT-1)

It should be noted that NNCT-1 and HNCT-1 turning mechanisms are specific cases of NNCT-2 and HNCT-2, respectively. It means that, when a situation appears where NNCT-1 or HNCT-1 can be performed, NNCT-2 or HNCT-2, respectively, can also be performed. However, in such cases, NNCT-2 or HNCT-2 will take more time. In other words, NNCT-1 or HNCT-1 mechanisms are more economical than NNCT-2 or HNCT-2 mechanisms, respectively. With 14 flight lines, NNCT-2 and HNCT-2 turning mechanisms are pictorially presented by figures (12) and (13).

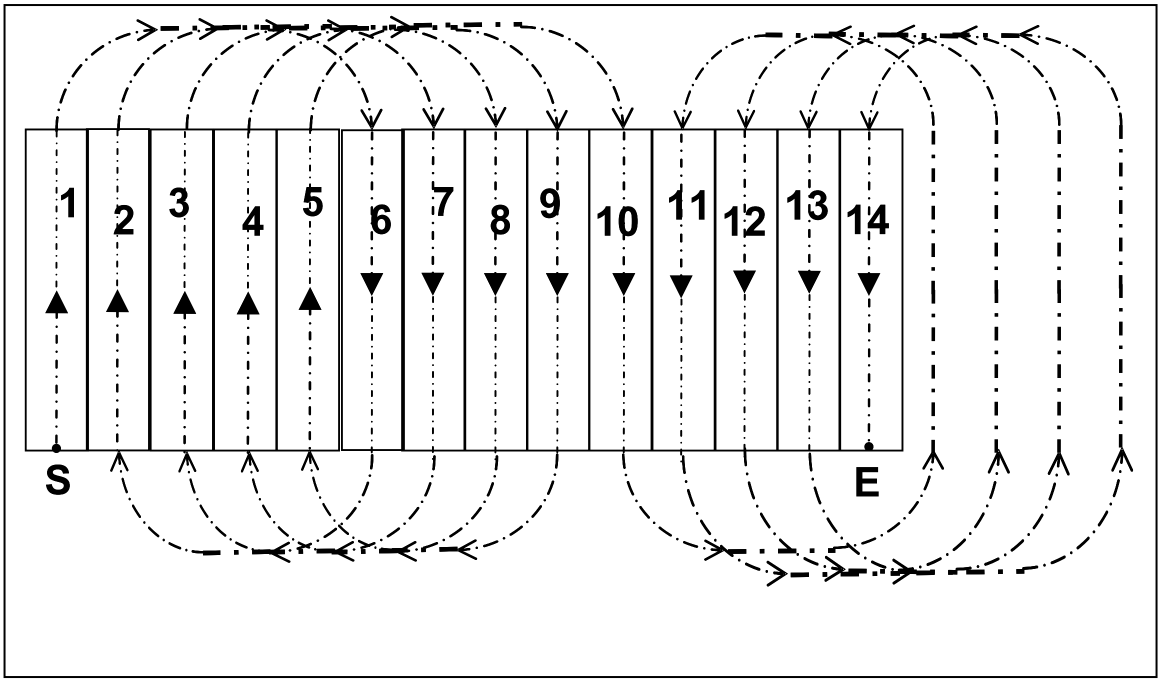

Fig. 12: Schematic view of natural non-consecutive turning of type-2 (NNCT-2) with 14 flight lines

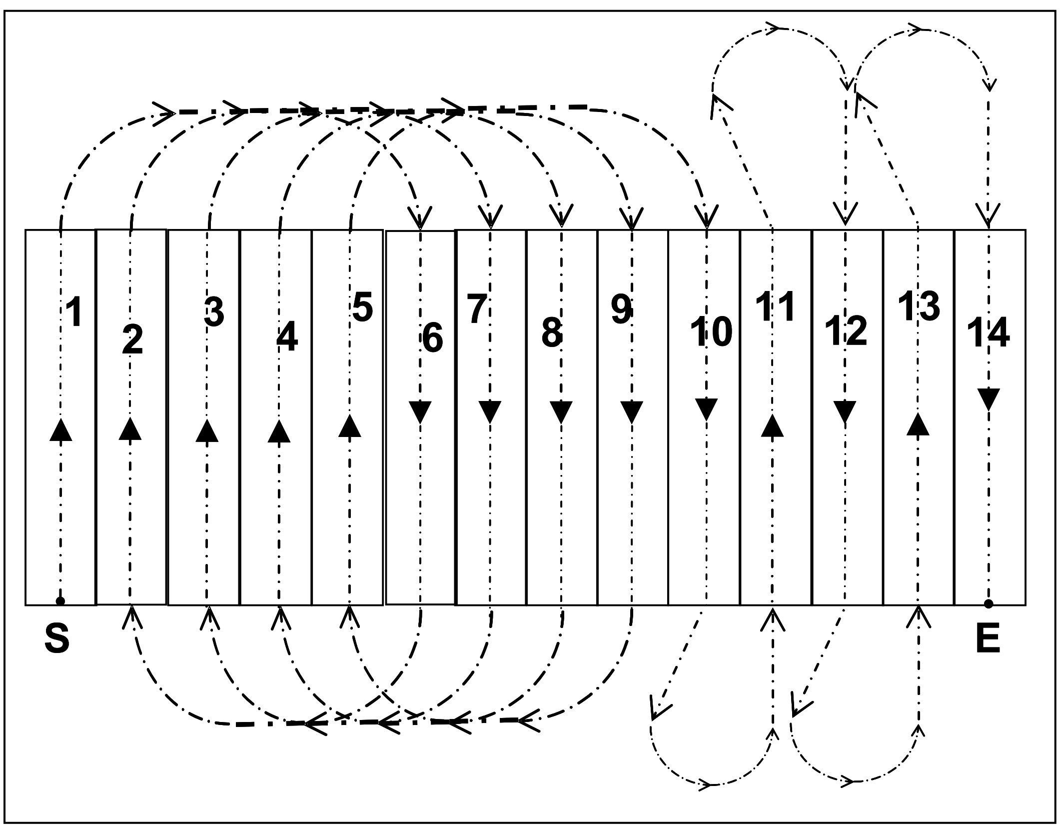

Fig. 13: Schematic view of hybrid non-consecutive turning of type-2 (HNCT-2) with 14 flight lines

Next section details the flight planning algorithm, which considers various turning mechanisms, for calculation of total turning time.

4. Flight planning algorithm for turning time

The algorithmic steps for consecutive, non-consecutive and hybrid turnings in the form of pseudo codes are presented in Appendix A. However, due to various turning mechanisms, namely consecutive, non-consecutive and hybrid, a decision making process is required for selection of the turning mechanism which corresponds to the minimum turning time. The decision making process is complicated due to the variants of the above mentioned non-consecutive turning mechanisms (e.g. NNCT-1, NNCT-2, HNCT-1 and HNCT-2 mechanisms). The following flight planning algorithm summarizes the decision making process for the selection of right turning mechanism.

Calculate:

  Consecutive Turning Time:

    Calculate: Tuning time

  Non-Consecutive Turning:

    Check: Possibility for NNCT-1

    Calculate: Turning time for NNCT-2 and NNCT-1 (if exists)

  Hybrid Turning:

    Check: Possibility for HNCT-1

    Calculate: Turning time for HNCT-2 and HNCT-1 (if exists)

Decision Making:

  Calculate: Minimum turning time among the times contributed by all mechanisms (consecutive, NNCT-1, NNCT-2, HNCT-1, and HNCT-2)

  Select: Turning mechanism corresponding to minimum turning time

The flight planning algorithm written above calculates the minimum turning time and the corresponding turning mechanism. The next section discusses the relationship between data density, overlap and terrain relief, as besides deciding the turning type it is important to see how the desired data can be obtained optimally under the available terrain conditions. Next section presents the pseudo codes for various turning mechanisms to calculate the turning time.

5. Pseudo codes for calculation of turning time for various turning mechanisms

5.1 Pseudo code (algorithm steps) for consecutive turn

Case-1 ($B<h_t$): In this case, aircraft will first approach the point C by appropriately changing the yaw angle ψ (maximum 30º or less; $\psi \le \psi_{Max}$, $\psi_{Max}=30°$) at A. It starts the half circular turn (180º turn) at point C(X_C, Y_C). Angle ψ is measured positive in the direction away from the next strip. Algorithm in the form of pseudo code for time calculation is following:

Table 1: Algorithm Steps for Consecutive Turning (Case-1)

Estimation (or Calculation) for	**If** $(i = odd = 1,3,5,7,...)$	**If** $(i = even = 2,4,6,8,...)$
Maximum Allowable Change in Hading Angle	$\psi_{Max} = \sin^{-1}\left(\sqrt{\left(\frac{h_t - B}{8V}\right)\dot{\psi}_{Max}}\right)$	
X Coordinates of 'point C' $(\psi = \psi_{Max})$	$\psi = \psi_{Max}$ $X_\psi = X_i^R + \frac{(h_t - B)}{\tan\psi_{Max}}$	$\psi = \psi_{Max}$ $X_\psi = X_i^L - \frac{(h_t - B)}{\tan\psi_{Max}}$
Direct distance from 'point A' to 'point C'	If $(X_\psi < X_{i+1}^R + V\,t_C)$ $L_1 = \sqrt{(X_{i+1}^R + V\,t_C - X_i^R)^2 + (h_t - B)^2}$ $\psi = \tan^{-1}\left(\frac{h_t - B}{X_{i+1}^R + V\,t_C - X_i^R}\right)$	If $(X_\psi > X_{i+1}^L - V\,t_C)$ $L_1 = \sqrt{(X_{i+1}^L - V\,t_C - X_i^L)^2 + (h_t - B)^2}$ $\psi = \tan^{-1}\left(\frac{h_t - B}{X_i^L - X_{i+1}^L + V\,t_C}\right)$

	Else if $(X_{\psi} \geq X_{i+1}^{R} + V\,t_C)$ $L_1 = \dfrac{h_t - B}{\sin \psi_{Max}}$ $\psi = \psi_{Max}$	Else if $(X_{\psi} \leq X_{i+1}^{L} - V\,t_C)$ $L_1 = \dfrac{h_t - B}{\sin \psi_{Max}}$ $\psi = \psi_{Max}$
Length of travel from 'point A' to 'point C'	Calculate r_1, β_1, T_1 by equations (15); Calculate L_i' by equation (16);	
	$L_i' = L_i' + \left\| X_{\psi} - X_{i+1}^{R} - V\,t_C \right\|$	$L_i' = L_i' + \left\| X_{i+1}^{L} - V\,t_C - X_{\psi} \right\|$
Total turn time (T_T)	Calculate L_c by equation (8); $T_T = \dfrac{1}{V}((n-1)(L_c + V\,t_C) + \sum_{i=1}^{n-1} L_i')$	

Case-2 ($B \geq h_t$): In this case, since space desired for aircraft turning is available, the difference between x coordinates of the consecutive flight strips will also decide the turning time. Algorithm in the form of pseudo code for time calculation is following:

Table 2: Algorithm Steps for Consecutive Turning (Case-2)

Estimation (or Calculation) for	If $(i = odd = 1,3,5,7,...)$	If $(i = even = 2,4,6,8,...)$
Difference between two flight lines	$L_i' = \left\| X_i^R - X_{i+1}^R - V\,t_C \right\|$	$L_i' = \left\| X_{i+1}^L - V\,t_C - X_i^L \right\|$
Length of travel	If $(h_t = B)$ Calculate T_m, ψ_T, r, L_c by equations (5a), (5b), (6), and (8) Else if $(h_t < B \leq h_t')$ Recalculate bank angle (β_m') by equation (17c)	

	Recalculate T_m, ψ_T, r, L_c by equations (5a), (5b), (6) and (17b) Else if $(B > h_t')$ $$L_c = 4V\,T_m + 2\,r\left(\frac{\pi}{2} - 2\psi_T\right) + B - h_t'$$
Total turn time	$$T_T = \frac{1}{V}((n-1)(L_c + V\,t_C) + \sum_{i=1}^{n-1} L_i')$$

5.2 Pseudo code (algorithm steps) for non-consecutive turn

Computational steps for non-consecutive turnings start with counting the number of short and long-turns along with the differences of ends of strip which are joined by a particular turn. Direction or mode of turning (reverse or forward) is changed regularly after every turn. Strip number of a covered strip is recorded. For extra turns, travel length from the end of the covered flight strip to starting point of the uncovered flight strip is calculated without considering the long or short-turn separately. Pseudo code for the non-consecutive turn including the NNCT-1 and NNCT-2 is tabulated below:

Table 3: Algorithm Steps for Non-Consecutive Turning

<table>
<tr><td>Estimation (or calculation) for</td><td colspan="2">Initialize with first flight strip ($i=1$) and length of travel ($L'=0$)</td></tr>
<tr><td>Calculation of constant values</td><td colspan="2">Calculate T_m, ψ_T, r by equations (5a), (5b), (6)

Calculate h_t', l_f, l_r, j, L_f, L_r by equations (17a), (18a), (18b), (19), (20a) and (20b)</td></tr>
<tr><td rowspan="2">Difference between ends of two flight lines</td><td colspan="2">Change the direction after every short or long-turn

Do not change the direction after an extra turn</td></tr>
<tr><td>If $(direction = forward)$

If $(rotation = clockwise)$</td><td>If $(direction = reverse)$

If $(rotation = clockwise)$</td></tr>
</table>

	$L_i' = \left\| X_i^R - (X_{i+l_f}^R + V\, t_C) \right\|$ If $(rotation = counterclockwise)$ $L_i' = \left\| X_i^L - (X_{i+l_f}^L - V\, t_C) \right\|$ For an extra turn: $L_i' = \left\| X_i^R - (X_{i+1}^L - V\, t_C) \right\|$	$L_i' = \left\| X_i^L - (X_{i-l_r}^L - V\, t_C) \right\|$ or $(rotation = counterclockwise)$ $L_i' = \left\| X_i^R - (X_{i-l_r}^R + V\, t_C) \right\|$ For an extra turn: $L_i' = \left\| X_i^L - (X_{i+1}^R + V\, t_C) \right\|$
Sum the difference between ends of two flight lines	$L' = L' + L_i'$	
Number of turns in total length of travel	Detect the possibility of NNCT-1 and consider it Count number of short-turns (n_r), long-turns (n_f), extra turns (n_e)	
Total turn time	$T_T = \frac{1}{V}(n_f\, L_f + n_r\, L_r + n_e\, L_e + L')$	

6. Conclusion

This report has demonstrated the conceptualization and modelling of consecutive turning and non-consecutive turning mechanisms. Variants of non-consecutive turning mechanisms, i.e. natural non-consecutive turning mechanism and hybrid non-consecutive turning mechanism, are also presented. A flight planning algorithms that determine the optimal turning mechanism that corresponds to minimum turning time is illustrated. For implementation of the flight planning algorithm with all type of turning mechanism, pseudo codes are also presented.

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