MA 201 Complex Analysis
Lecture 2: Open and Closed set
Some Basic Definitions

- **Open disc**: Let $z_0 \in \mathbb{C}$ and $r > 0$ then, $B(z_0, r) = \{z \in \mathbb{C} : |z - z_0| < r\}$ is an open disc centered at $z_0$ with radius $r$.

- **Deleted Neighborhood of $z_0$**: Let $z_0 \in \mathbb{C}$ and $r > 0$ then, $B(z_0, r) - \{z_0\} = \{z \in \mathbb{C} : 0 < |z - z_0| < r\}$ is called the **deleted neighborhood** of $z_0$.

- **Interior point**: A point $z_0$ is called an **interior point** of a set $S \subset \mathbb{C}$ if we can find an $r > 0$ such that $B(z_0, r) \subset S$.

- **Boundary points**: If $B(z_0, r)$ contains points of $S$ and points of $S^c$ every $r > 0$, then $z_0$ is called a **boundary point** of a set $S$.

- **Exterior points**: If a point is not an interior point or boundary point of $S$, it is an exterior point of $S$. 

Some Basic Definitions

- **Open Set:** A set $S \subset \mathbb{C}$ is **open** if every $z_0 \in S$ there exists $r > 0$ such that $B(z_0, r) \subset S$.

- **Exercise:** Show that a set $S$ is an open set if and only if every point of $S$ is an interior point.

- **Connected Set:** An open set $S \subset \mathbb{C}$ is said to be **connected** if each pair of points $z_1$ and $z_2$ in $S$ can be joined by a polygonal line consisting of a finite number of line segments joined end to end that lies entirely in $S$.

- **Domain/Region:** An open, connected set is called a **domain**. A domain together with some, none or all of its boundary points is called a **region**.
Some Basic Definitions

- **Bounded Set:** A set $S \subset \mathbb{C}$ is **bounded** if there exists a $K > 0$ such that $|z| < K \ \forall \ z \in S$. We say $S$ is **unbounded** if $S$ is not bounded.

- **Limit point/Accumulation point:** Let $\zeta$ is called an limit point of a set $S \subset \mathbb{C}$ if every deleted neighborhood of $\zeta$ contains at least one point of $S$.

- **Closed Set:** A set $S \subset \mathbb{C}$ is **closed** if $S$ contains all its limit points.

- **Exercise:** Show that a set $S$ is closed if and only if $S^c$ is open.

- **Closure of a Set:** The closure of a set $S \subset \mathbb{C}$, denoted by $\bar{S}$, defined by the set $S$ together with all its limit points.

- **Exercise:** Show that a set $S$ is closed if and only if $\bar{S} = S$. 