Chapter 5

Multipath Wave Propagation and Fading

5.1 Multipath Propagation

In wireless telecommunications, multipath is the propagation phenomenon that results in radio signals reaching the receiving antenna by two or more paths. Causes of multipath include atmospheric ducting, ionospheric reflection and refraction, and reflection from water bodies and terrestrial objects such as mountains and buildings. The effects of multipath include constructive and destructive interference, and phase shifting of the signal. In digital radio communications (such as GSM) multipath can cause errors and affect the quality of communications. We discuss all the related issues in this chapter.

5.2 Multipath & Small-Scale Fading

Multipath signals are received in a terrestrial environment, i.e., where different forms of propagation are present and the signals arrive at the receiver from transmitter via a variety of paths. Therefore there would be multipath interference, causing multipath fading. Adding the effect of movement of either Tx or Rx or the surrounding clutter to it, the received overall signal amplitude or phase changes over a small amount of time. Mainly this causes the fading.

5.2.1 Fading

The term **fading**, or, small-scale fading, means rapid fluctuations of the amplitudes, phases, or multipath delays of a radio signal over a short period or short travel distance. This might be so severe that large scale radio propagation loss effects might be ignored.

5.2.2 Multipath Fading Effects

In principle, the following are the main multipath effects:

1. Rapid changes in signal strength over a small travel distance or time interval.
2. Random frequency modulation due to varying Doppler shifts on different multipath signals.
3. Time dispersion or echoes caused by multipath propagation delays.

5.2.3 Factors Influencing Fading

The following physical factors influence small-scale fading in the radio propagation channel:

(1) Multipath propagation – Multipath is the propagation phenomenon that results in radio signals reaching the receiving antenna by two or more paths. The effects of multipath include constructive and destructive interference, and phase shifting of the signal.

(2) Speed of the mobile – The relative motion between the base station and the mobile results in random frequency modulation due to different doppler shifts on each of the multipath components.

(3) Speed of surrounding objects – If objects in the radio channel are in motion, they induce a time varying Doppler shift on multipath components. If the surrounding objects move at a greater rate than the mobile, then this effect dominates fading.

(4) Transmission Bandwidth of the signal – If the transmitted radio signal bandwidth is greater than the "bandwidth" of the multipath channel (quantified by *coherence bandwidth*), the received signal will be distorted.

5.3 Types of Small-Scale Fading

The type of fading experienced by the signal through a mobile channel depends on the relation between the signal parameters (bandwidth, symbol period) and the channel parameters (rms delay spread and Doppler spread). Hence we have four different types of fading. There are two types of fading due to the time dispersive nature of the channel.

5.3.1 Fading Effects due to Multipath Time Delay Spread

Flat Fading

Such types of fading occurs when the bandwidth of the transmitted signal is less than the coherence bandwidth of the channel. Equivalently if the symbol period of the signal is more than the rms delay spread of the channel, then the fading is flat fading.

So we can say that flat fading occurs when

$$B_S \ll B_C \tag{5.1}$$

where B_S is the signal bandwidth and B_C is the coherence bandwidth. Also

$$T_S \gg \sigma_\tau \tag{5.2}$$

where T_S is the symbol period and σ_τ is the rms delay spread. And in such a case, mobile channel has a constant gain and linear phase response over its bandwidth.

Frequency Selective Fading

Frequency selective fading occurs when the signal bandwidth is more than the coherence bandwidth of the mobile radio channel or equivalently the symbols duration of the signal is less than the rms delay spread.

$$B_S \gg B_C \tag{5.3}$$

and

$$T_S \ll \sigma_\tau \tag{5.4}$$

At the receiver, we obtain multiple copies of the transmitted signal, all attenuated and delayed in time. The channel introduces inter symbol interference. A rule of thumb for a channel to have flat fading is if

$$\frac{\sigma_\tau}{T_S} \leq 0.1 \tag{5.5}$$

5.3.2 Fading Effects due to Doppler Spread

Fast Fading

In a fast fading channel, the channel impulse response changes rapidly within the symbol duration of the signal. Due to Doppler spreading, signal undergoes frequency dispersion leading to distortion. Therefore a signal undergoes fast fading if

$$T_S \gg T_C \tag{5.6}$$

where T_C is the coherence time and

$$B_S \gg B_D \tag{5.7}$$

where B_D is the Doppler spread. Transmission involving very low data rates suffer from fast fading.

Slow Fading

In such a channel, the rate of the change of the channel impulse response is much less than the transmitted signal. We can consider a slow faded channel a channel in which channel is almost constant over atleast one symbol duration. Hence

$$T_S \ll T_C \tag{5.8}$$

and

$$B_S \gg B_D \tag{5.9}$$

We observe that the velocity of the user plays an important role in deciding whether the signal experiences fast or slow fading.

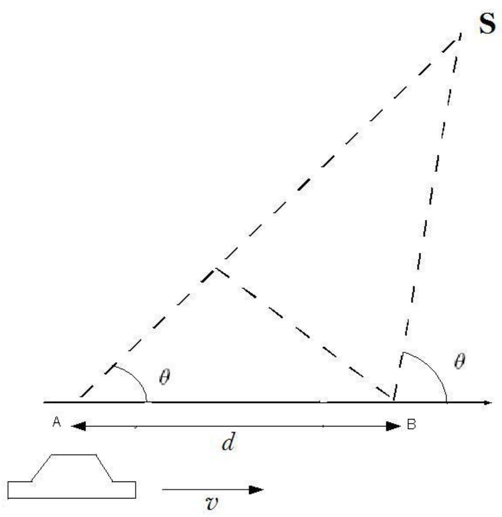


Figure 5.1: Illustration of Doppler effect.

5.3.3 Doppler Shift

The Doppler effect (or Doppler shift) is the change in frequency of a wave for an observer moving relative to the source of the wave. In classical physics (waves in a medium), the relationship between the observed frequency f and the emitted frequency f_o is given by:

$$f = \left(\frac{v \pm v_r}{v \pm v_s} \right) f_0 \tag{5.10}$$

where v is the velocity of waves in the medium, v_s is the velocity of the source relative to the medium and v_r is the velocity of the receiver relative to the medium.

In mobile communication, the above equation can be slightly changed according to our convenience since the source (BS) is fixed and located at a remote elevated level from ground. The expected Doppler shift of the EM wave then comes out to be $\pm \frac{v_r}{c} f_o$ or, $\pm \frac{v_r}{\lambda}$. As the BS is located at an elevated place, a $\cos\phi$ factor would also be multiplied with this. The exact scenario, as given in Figure 5.1, is illustrated below.

Consider a mobile moving at a constant velocity v, along a path segment length d between points A and B, while it receives signals from a remote BS source S. The difference in path lengths traveled by the wave from source S to the mobile at points A and B is $\Delta l = d \cos\theta = v \Delta t \cos\theta$, where Δt is the time required for the mobile to travel from A to B, and θ is assumed to be the same at points A and B since the source is assumed to be very far away. The phase change in the received signal due to the difference in path lengths is therefore

$$\Delta\varphi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta \tag{5.11}$$

and hence the apparent change in frequency, or Doppler shift (f_d) is

$$f_d = \frac{1}{2\pi}.\frac{\Delta\varphi}{\Delta t} = \frac{v}{\lambda}.\cos\theta. \tag{5.12}$$

Example 1

An aircraft is heading towards a control tower with 500 kmph, at an elevation of 20°. Communication between aircraft and control tower occurs at 900 MHz. Find out the expected Doppler shift.

Solution As given here,

$$v = 500\, kmph$$

the horizontal component of the velocity is

$$v' = v\cos\theta = 500 \times \cos 20^\circ \;= 130\, m/s$$

Hence, it can be written that

$$\lambda = \frac{900 \times 10^6}{3 \times 10^8} = \frac{1}{3}m$$

$$f_d = \frac{130}{1/3} = 390\, Hz$$

If the plane banks suddenly and heads for other direction, the Doppler shift change will be $390\, Hz$ to $-390\, Hz$.

5.3.4 Impulse Response Model of a Multipath Channel

Mobile radio channel may be modeled as a linear filter with time varying impulse response in continuous time. To show this, consider time variation due to receiver motion and time varying impulse response $h(d, t)$ and $x(t)$, the transmitted signal. The received signal $y(d, t)$ at any position d would be

$$y(d,t) = x(t) * h(d,t) = \int_{-\infty}^{\infty} x(\tau)\, h(d, t-\tau)\, d\tau \tag{5.13}$$

For a causal system: $h(d, t) = 0$, for $t < 0$ and for a stable system $\int_{-\infty}^{\infty} |h(d,t)|\, dt < \infty$

Applying causality condition in the above equation, $h(d, t-\tau) = 0$ for $t - \tau < 0$ $\Rightarrow \tau > t$, i.e., the integral limits are changed to

$$y(d,t) = \int_{-\infty}^{t} x(\tau)\, h(d, t-\tau)\, d\tau.$$

Since the receiver moves along the ground at a constant velocity v, the position of the receiver is $d = vt$, i.e.,

$$y(vt,t) = \int_{-\infty}^{t} x(\tau)\, h(vt, t-\tau)\, d\tau.$$

Since v is a constant, $y(vt,t)$ is just a function of t. Therefore the above equation can be expressed as

$$y(t) = \int_{-\infty}^{t} x(\tau)\, h(vt, t-\tau)\, d\tau \; = x(t) * h(vt,t) = x(t) * h(d,t) \tag{5.14}$$

It is useful to discretize the multipath delay axis τ of the impulse response into equal time delay segments called *excess delay bins*, each bin having a time delay width equal to $(\tau_{i+1} - \tau_i) = \Delta\tau$ and $\tau_i = i\Delta\tau$ for $i \in \{0, 1, 2, ..N-1\}$, where N represents the total number of possible equally-spaced multipath components, including the first arriving component. The useful frequency span of the model is $^2/_{\Delta\tau}$. The model may be used to analyze transmitted RF signals having bandwidth less than $^2/_{\Delta\tau}$.

If there are N multipaths, maximum excess delay is given by $N\Delta\tau$.

$$\{y(t) = x(t) * h(t, \tau_i) | i = 0, 1, ...N-1\} \tag{5.15}$$

Bandpass channel impulse response model is

$$x(t) \rightarrow h(t,\tau) = Re\{h_b(t,\tau)e^{j\omega_c t} \rightarrow y(t) = Re\{r(t)e^{j\omega_c t}\} \tag{5.16}$$

Baseband equivalent channel impulse response model is given by

$$c(t) \rightarrow \frac{1}{2}h_b(t,\tau) \rightarrow r(t) = c(t) * \frac{1}{2}h_b(t,\tau) \tag{5.17}$$

Average power is

$$\overline{x^2(t)} = \frac{1}{2}|c(t)|^2 \tag{5.18}$$

The baseband impulse response of a multipath channel can be expressed as

$$h_b(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau) \exp[j(2\pi f_c \tau_i(t) + \varphi_i(t,\tau))]\delta(\tau - \tau_i(t)) \tag{5.19}$$

where $a_i(t,\tau)$ and $\tau_i(t)$ are the real amplitudes and excess delays, respectively, of the ith multipath component at time t. The phase term $2\pi f_c \tau_i(t) + \varphi_i(t,\tau)$ in the above equation represents the phase shift due to free space propagation of the ith multipath component, plus any additional phase shifts which are encountered in the channel.

If the channel impulse response is wide sense stationary over a small-scale time or distance interval, then

$$h_b(\tau) = \sum_{i=0}^{N-1} a_i \exp[j\theta_i]\delta(\tau - \tau_i) \tag{5.20}$$

For measuring $h_b(\tau)$, we use a probing pulse to approximate $\delta(t)$ i.e.,

$$p(t) \approx \delta(t - \tau) \tag{5.21}$$

Power delay profile is taken by spatial average of $|h_b(t,\tau)|^2$ over a local area. The received power delay profile in a local area is given by

$$p(\tau) \approx \overline{k|h_b(t;\tau)|^2}. \tag{5.22}$$

5.3.5 Relation Between Bandwidth and Received Power

In actual wireless communications, impulse response of a multipath channel is measured using channel sounding techniques. Let us consider two extreme channel sounding cases.

Consider a pulsed, transmitted RF signal

$$x(t) = Re\{p(t)e^{j2\pi f_c t}\} \tag{5.23}$$

where $p(t) = \sqrt{\frac{4\tau_{\max}}{T_{bb}}}$ for $0 \le t \le T_{bb}$ and 0 elsewhere. The low pass channel output is

$$\begin{aligned} r(t) &= \frac{1}{2}\sum_{i=0}^{N-1} a_i \exp[j\theta_i] p(t - \tau_i) \\ &= \sum_{i=0}^{N-1} a_i \exp[j\theta_i].\sqrt{\frac{\tau_{\max}}{T_{bb}}} rect(t - \frac{T_b}{2} - \tau_i). \end{aligned}$$

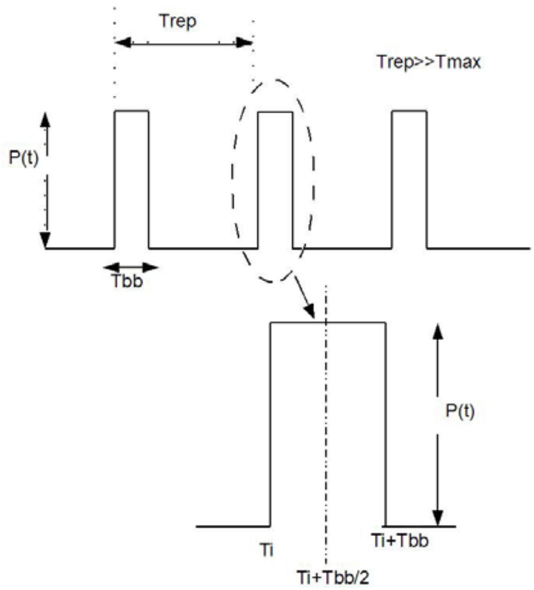


Figure 5.2: A generic transmitted pulsed RF signal.

The received power at any time t_0 is

$$\begin{aligned}
|r(t_0)|^2 &= \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} r(t) r^*(t) dt \\
&= \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} \frac{1}{4} \left(\sum_{k=0}^{N-1} a_k^2(t_0) p^2(t - \tau_k) \right) dt \\
&= \frac{1}{\tau_{\max}} \sum_{k=0}^{N-1} a_k^2(t_0) \int_0^{\tau_{\max}} \left(\sqrt{\frac{\tau_{\max}}{T_{bb}}} rect(t - \frac{T_b}{2} - \tau_i) \right)^2 dt \\
&= \sum_{k=0}^{N-1} a_k^2(t_0).
\end{aligned}$$

Interpretation: If the transmitted signal is able to resolve the multipaths, then average small-scale receiver power is simply sum of average powers received from each multipath components.

$$E_{a,\theta}[P_{WB}] = E_{a,\theta}[\sum_{i=0}^{N-1} |a_i \exp(j\theta_i)|^2] \approx \sum_{i=0}^{N-1} \overline{a_i^2} \tag{5.24}$$

Now instead of a pulse, consider a CW signal, transmitted into the same channel and for simplicity, let the envelope be $c(t) = 2$. Then

$$r(t) = \sum_{i=0}^{N-1} a_i \exp[j\theta_i(t, \tau)] \tag{5.25}$$

and the instantaneous power is

$$|r(t)|^2 = |\sum_{i=0}^{N-1} a_i \exp[j\theta_i(t,\tau)]|^2 \tag{5.26}$$

Over local areas, a_i varies little but θ_i varies greatly resulting in large fluctuations.

$$\begin{aligned} E_{a,\theta}[P_{CW}] &= E_{a,\theta}[\sum_{i=0}^{N-1} |a_i \exp(j\theta_i)|^2] \\ &\approx \sum_{i=0}^{N-1} \overline{a_i^2} + 2\sum_{i=0}^{N-1}\sum_{i,j\neq i}^{N} r_{ij}\overline{\cos(\theta_i - \theta_j)} \end{aligned}$$

where $r_{ij} = E_a[a_i a_j]$.

If, $r_{ij} = \overline{\cos(\theta_i - \theta_j)} = 0$, then $E_{a,\theta}[P_{CW}] = E_{a,\theta}[P_{WB}]$. This occurs if multipath components are uncorrelated or if multipath phases are i.i.d over $[0, 2\pi]$.

Bottomline:

1. If the signal bandwidth is greater than multipath channel bandwidth then fading effects are negligible

2. If the signal bandwidth is less than the multipath channel bandwidth, large fading occurs due to phase shift of unresolved paths.

5.3.6 Linear Time Varying Channels (LTV)

The time variant transfer function(TF) of an LTV channel is FT of $h(t,\tau)$ w.r.t. τ.

$$H(f,t) = FT[h(\tau,t)] = \int_{-\infty}^{\infty} h(\tau,t)e^{-j2\pi f\tau}\,d\tau \tag{5.27}$$

$$h(\tau,t) = FT^{-1}[H(f,t)] = \int_{-\infty}^{\infty} H(f,t)e^{j2\pi f\tau}\,df \tag{5.28}$$

The received signal

$$r(t) = \int_{-\infty}^{\infty} R(f,t)e^{j2\pi ft}\,df \tag{5.29}$$

where $R(f,t) = H(f,t)X(f)$.

For flat fading channel, $h(\tau,t) = Z(t)\delta(\tau - \tau_i)$ where $Z(t) = \sum \alpha_n(t)e^{-j2\pi f_c\tau_n(t)}$. In this case, the received signal is

$$r(t) = \int_{-\infty}^{\infty} h(\tau,t)x(t-\tau)\,d\tau = Z(t)x(t-\tau_i) \tag{5.30}$$

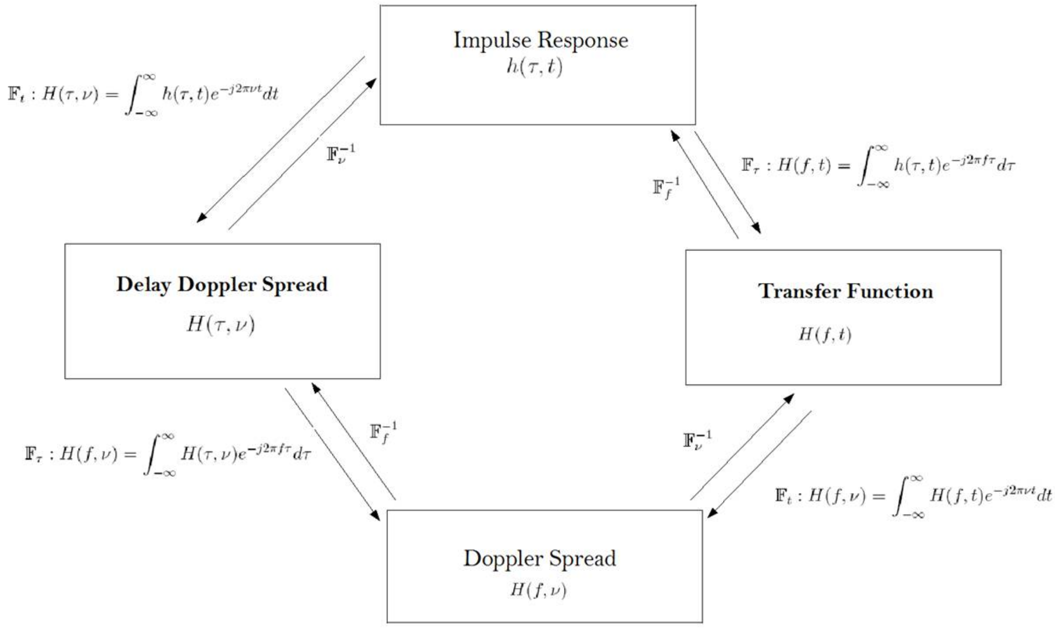


Figure 5.3: Relationship among different channel functions.

where the channel becomes multiplicative.

Doppler spread functions:

$$H(f,\nu) = FT[H(f,t)] = \int_{-\infty}^{\infty} H(f,t)e^{-j2\pi\nu t}dt \tag{5.31}$$

and

$$H(f,t) = FT^{-1}[H(f,\nu)] = \int_{-\infty}^{\infty} H(f,\nu)e^{j2\pi\nu t}d\nu \tag{5.32}$$

Delay Doppler spread:

$$H(\tau,\nu) = FT[h(\tau,t)] = \int_{-\infty}^{\infty} h(\tau,t)e^{-j2\pi\nu t}dt \tag{5.33}$$

5.3.7 Small-Scale Multipath Measurements

Direct RF Pulse System

A wideband pulsed bistatic radar usually transmits a repetitive pulse of width T_{bb} s, and uses a receiver with a wide bandpass filter ($BW = \frac{2}{T_{bb}}$ Hz). The signal is then amplified, envelope detected, and displayed and stored on a high speed oscilloscope. Immediate measurements of the square of the channel impulse response convolved with the probing pulse can be taken. If the oscilloscope is set on averaging mode, then this system provides a local average power delay profile.

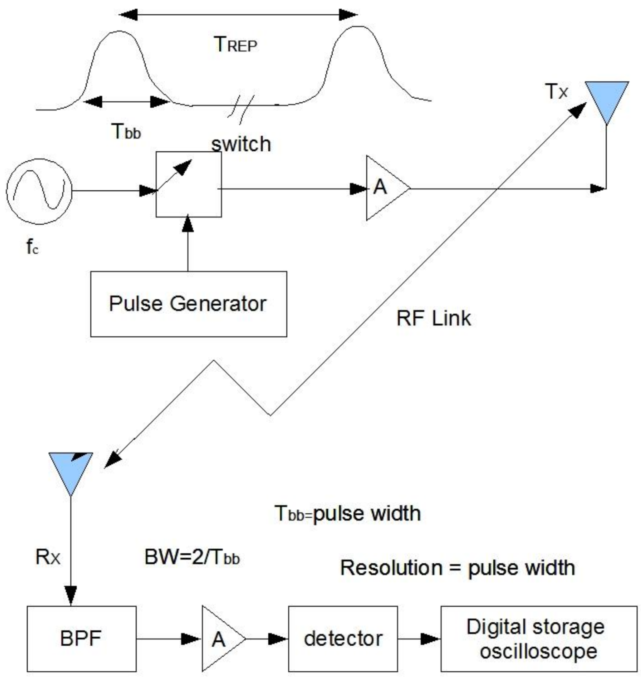


Figure 5.4: Direct RF pulsed channel IR measurement.

This system is subject to interference noise. If the first arriving signal is blocked or fades, severe fading occurs, and it is possible the system may not trigger properly.

Frequency Domain Channel Sounding

In this case we measure the channel in the frequency domain and then convert it into time domain impulse response by taking its inverse discrete Fourier transform (IDFT). A vector network analyzer controls a swept frequency synthesizer. An S-parameter test set is used to monitor the frequency response of the channel. The sweeper scans a particular frequency band, centered on the carrier, by stepping through discrete frequencies. The number and spacing of the frequency step impacts the time resolution of the impulse response measurement. For each frequency step, the S-parameter test set transmits a known signal level at port 1 and monitors the received signal at port 2. These signals allow the analyzer to measure the complex response, $S_{21}(\omega)$, of the channel over the measured frequency range. The $S_{21}(\omega)$ measure is the measure of the signal flow from transmitter antenna to receiver

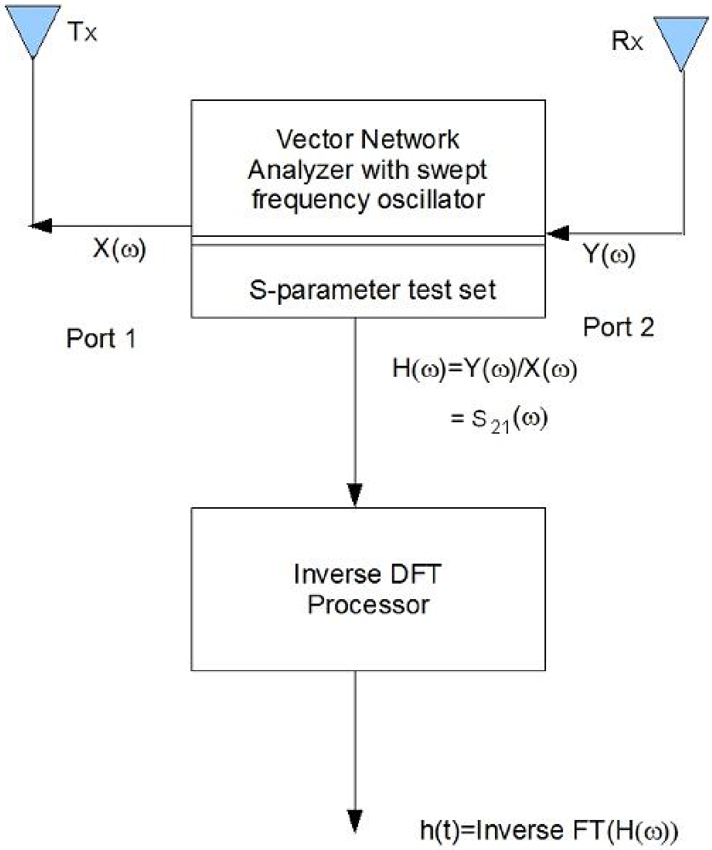


Figure 5.5: Frequency domain channel IR measurement.

antenna (i.e., the channel).

This system is suitable only for indoor channel measurements. This system is also non real-time. Hence, it is not suitable for time-varying channels unless the sweep times are fast enough.

5.4 Multipath Channel Parameters

To compare the different multipath channels and to quantify them, we define some parameters. They all can be determined from the power delay profile. These parameters can be broadly divided in to two types.

5.4.1 Time Dispersion Parameters

These parameters include the mean excess delay,rms delay spread and excess delay spread. The mean excess delay is the first moment of the power delay profile and is defined as

$$\bar{\tau} = \frac{\sum a_k^2 \tau_k}{\sum a_k^2} = \frac{\sum P(\tau_k)\tau_k}{\sum P(\tau_k)} \tag{5.34}$$

where a_k is the amplitude, τ_k is the excess delay and $P(\tau_k)$ is the power of the individual multipath signals.

The mean square excess delay spread is defined as

$$\bar{\tau^2} = \frac{\sum P(\tau_k)\tau_k^2}{\sum P(\tau_k)} \tag{5.35}$$

Since the rms delay spread is the square root of the second central moment of the power delay profile, it can be written as

$$\sigma_\tau = \sqrt{\bar{\tau^2} - (\bar{\tau})^2} \tag{5.36}$$

As a rule of thumb, for a channel to be flat fading the following condition must be satisfied

$$\frac{\sigma_\tau}{T_S} \leq 0.1 \tag{5.37}$$

where T_S is the symbol duration. For this case, no equalizer is required at the receiver.

Example 2

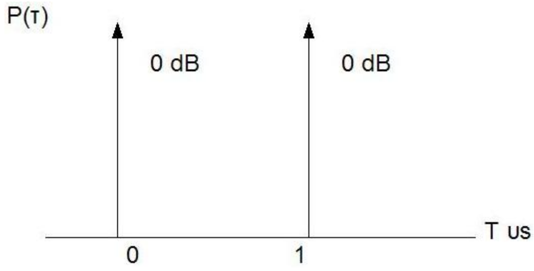
1. Sketch the power delay profile and compute RMS delay spread for the following:
   $P(\tau) = \sum_{n=0}^{1} \delta(\tau - n \times 10^{-6})$ (in watts)
2. If BPSK modulation is used, what is the maximum bit rate that can be sent through the channel without needing an equalizer?

Solution

1. $P(0) = 1$ watt, $P(1) = 1$ watt

$$\overline{\tau} = \frac{(1)(0) + (1)(1)}{1+1} = 0.5\mu s$$

$$\overline{\tau^2} = 0.5\mu s^2 \quad \sigma_\tau = \ 0.5\mu s$$

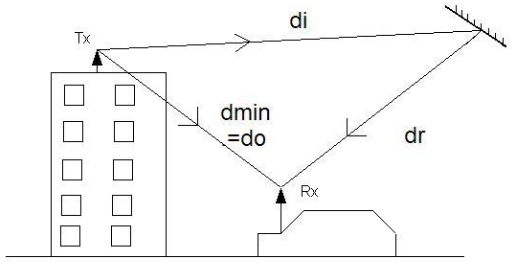


2. For flat fading channel, we need $\frac{\sigma_\tau}{T_s} 0.1 \quad \Rightarrow R_s = \frac{1}{T_s} = 0.2 \times 10^4 \ = 200 \, kbps$

   For BPSK we need $R_b = R_s = 200 \, kbps$

Example 3 A simple delay spread bound: **Feher's upper bound**

Consider a simple worst-case delay spread scenario as shown in figure below.



Here $d_{min} = d_0$ and $d_{max} = d_i + d_r$

Transmitted power = P_T, Minimum received power = $P_{R_{\min}} = P_{Threshold}$

$$\frac{P_{R_{\min}}}{P_T} = G_T G_R \left(\frac{\lambda}{4\pi d_{\max}}\right)^2$$

Put $G_T = G_R = 1$ i.e., considering omni-directional unity gain antennas

$$d_{\max} = \left(\frac{\lambda}{4\pi}\right)\left(\frac{P_T}{P_{R_{\min}}}\right)^{\frac{1}{2}}$$

$$\tau_{\max} = \frac{d_{\max}}{c} = \left(\frac{\lambda}{4\pi c}\right)\left(\frac{P_T}{P_{R_{\min}}}\right)^{\frac{1}{2}}$$

$$\tau_{\max} = \left(\frac{1}{4\pi f}\right)\left(\frac{P_T}{P_{R_{\min}}}\right)^{\frac{1}{2}}$$

5.4.2 Frequency Dispersion Parameters

To characterize the channel in the frequency domain, we have the following parameters.

(1) Coherence bandwidth: it is a statistical measure of the range of frequencies over which the channel can be considered to pass all the frequency components with almost equal gain and linear phase. When this condition is satisfied then we say the channel to be flat.

Practically, coherence bandwidth is the minimum separation over which the two frequency components are affected differently. If the coherence bandwidth is considered to be the bandwidth over which the frequency correlation function is above 0.9, then it is approximated as

$$B_C \approx \frac{1}{50\sigma_\tau}. \tag{5.38}$$

However, if the coherence bandwidth is considered to be the bandwidth over which the frequency correlation function is above 0.5, then it is defined as

$$B_C \approx \frac{1}{5\sigma_\tau}. \tag{5.39}$$

The coherence bandwidth describes the time dispersive nature of the channel in the local area. A more convenient parameter to study the time variation of the channel is the coherence time. This variation may be due to the relative motion between the mobile and the base station or the motion of the objects in the channel.

(2) Coherence time: this is a statistical measure of the time duration over which the channel impulse response is almost invariant. When channel behaves like this, it is said to be slow faded. Essentially it is the minimum time duration over which two received signals are affected differently. For an example, if the coherence time is considered to be the bandwidth over which the time correlation is above 0.5, then it can be approximated as

$$T_C \approx \frac{9}{16\pi f_m} \tag{5.40}$$

where f_m is the maximum doppler spread given be $f_m = \frac{\nu}{\lambda}$.

Another parameter is the Doppler spread (B_D) which is the range of frequencies over which the received Doppler spectrum is non zero.

5.5 Statistical models for multipath propagation

Many multipath models have been proposed to explain the observed statistical nature of a practical mobile channel. Both the first order and second order statistics

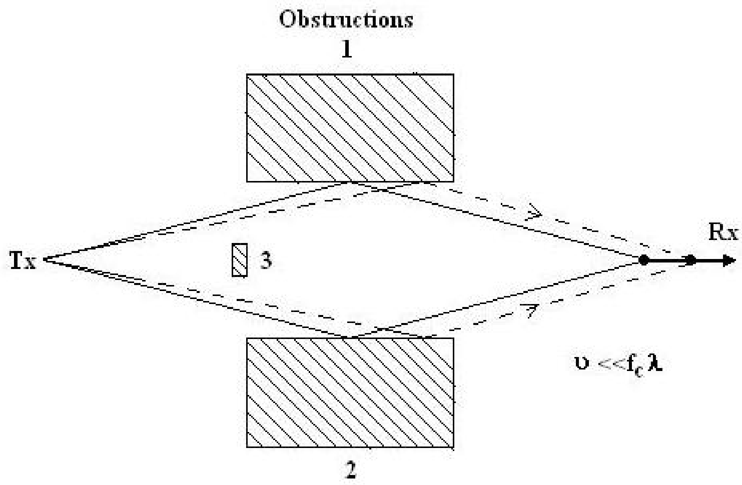


Figure 5.6: Two ray NLoS multipath, resulting in Rayleigh fading.

have been examined in order to find out the effective way to model and combat the channel effects. The most popular of these models are Rayleigh model, which describes the NLoS propagation. The Rayleigh model is used to model the statistical time varying nature of the received envelope of a flat fading envelope. Below, we discuss about the main first order and second order statistical models.

5.5.1 NLoS Propagation: Rayleigh Fading Model

Let there be two multipath signals S_1 and S_2 received at two different time instants due to the presence of obstacles as shown in Figure 5.6. Now there can either be constructive or destructive interference between the two signals.

Let E_n be the electric field and Θ_n be the relative phase of the various multipath signals.So we have

$$\tilde{E} = \sum_{n=1}^{N} E_n e^{j\theta_n} \tag{5.41}$$

Now if N$\rightarrow \infty$(i.e. are sufficiently large number of multipaths) and all the E_n are IID distributed, then by Central Limit Theorem we have,

$$\lim_{N\rightarrow\infty} \tilde{E} = \lim_{N\rightarrow\infty} \sum_{n=1}^{N} E_n e^{j\theta_n} \tag{5.42}$$

$$= Z_r + jZ_i = Re^{j\phi} \tag{5.43}$$

where Z_r and Z_i are Gaussian Random variables. For the above case

$$R = \sqrt{Z_r^2 + Z_i^2} \tag{5.44}$$

and

$$\phi = \tan^{-1} \frac{Z_i}{Z_r} \tag{5.45}$$

For all practical purposes we assume that the relative phase Θ_n is uniformaly distributed.

$$E[e^{j\theta_n}] = \frac{1}{2\pi} \int_0^{2\pi} e^{j\theta} d\theta = 0 \tag{5.46}$$

It can be seen that E_n and Θ_n are independent. So,

$$E[\tilde{E}] = E[\sum E_n e^{j\theta_n}] = 0 \tag{5.47}$$

$$E[\left|\tilde{E}\right|^2] = E[\sum E_n e^{j\theta_n} \sum E_n^* e^{-j\theta_n}] = E[\sum_m \sum_n E_n E_m e^{j(\theta_n - \theta_m)}] = \sum_{n=1}^{N} E_n^2 = P_0 \tag{5.48}$$

where P_0 is the total power obtained. To find the Cumulative Distribution Function(CDF) of R, we proceed as follows.

$$F_R(r) = P_r(R \leq r) = \int\int_A f_{Z_i,Z_r}(z_i, z_r) dz_i dz_r \tag{5.49}$$

where A is determined by the values taken by the dummy variable r. Let Z_i and Z_r be zero mean Gaussian RVs. Hence the CDF can be written as

$$F_R(r) = \int\int_A \frac{1}{\sqrt{2\pi}\sigma^2} e^{\frac{-(Z_r^2+Z_i^2)}{2\sigma^2}} dZ_i dZ_r \tag{5.50}$$

Let $Z_r = p\cos(\Theta)$ and $Z_i = p\sin(\Theta)$ So we have

$$F_R(r) = \int_0^{2\pi}\int_0^{2\pi} \frac{1}{\sqrt{2\pi}\sigma^2} e^{\frac{-p^2}{2\sigma^2}} p dp d\theta \tag{5.51}$$

$$= 1 - e^{\frac{-r^2}{2\sigma^2}} \tag{5.52}$$

Above equation is valid for all $r \geq 0$. The pdf can be written as

$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \tag{5.53}$$

and is shown in Figure 5.7 with different σ values. This equation too is valid for all $r \geq 0$. Above distribution is known as Rayleigh distribution and it has been derived

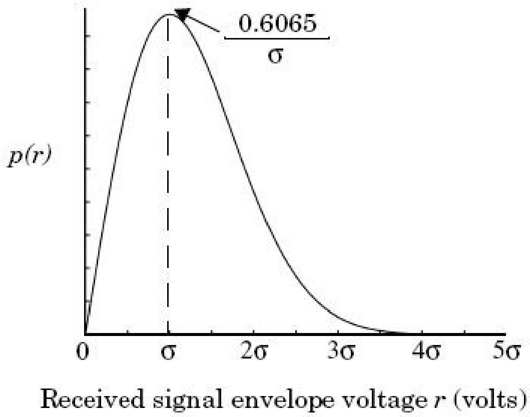


Figure 5.7: Rayleigh probability density function.

for slow fading. However, if $f_D \ll 1$ Hz, we call it as Quasi-stationary Rayleigh fading. We observe the following:

$$E[R] = \sqrt{\frac{\pi}{2}}\sigma \tag{5.54}$$

$$E[R^2] = 2\sigma^2 \tag{5.55}$$

$$\mathrm{var}[R] = (2 - \frac{\pi}{2})\sigma^2 \tag{5.56}$$

$$median[R] = 1.77\sigma. \tag{5.57}$$

5.5.2 LoS Propagation: Rician Fading Model

Rician Fading is the addition to all the normal multipaths a direct LOS path.

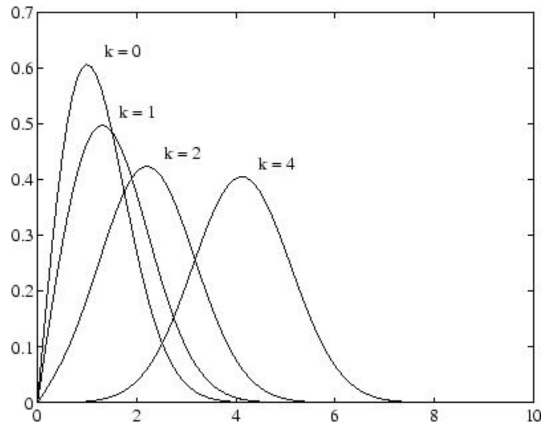


Figure 5.8: Ricean probability density function.

$$f_R(r) = \frac{r}{\sigma^2} e^{\frac{-(r^2+A^2)}{2\sigma^2}} I_0(\frac{A_r}{\sigma^2}) \tag{5.58}$$

for all $A \geq 0$ and $r \geq 0$. Here A is the peak amplitude of the dominant signal and $I_0(.)$ is the modified Bessel function of the first kind and zeroth order.

A factor K is defined as

$$K_{dB} = 10 \log \frac{A^2}{2\sigma^2} \tag{5.59}$$

As A $\rightarrow 0$ then $K_{dB} \rightarrow \infty$.

5.5.3 Generalized Model: Nakagami Distribution

A generalization of the Rayleigh and Rician fading is the Nakagami distribution.

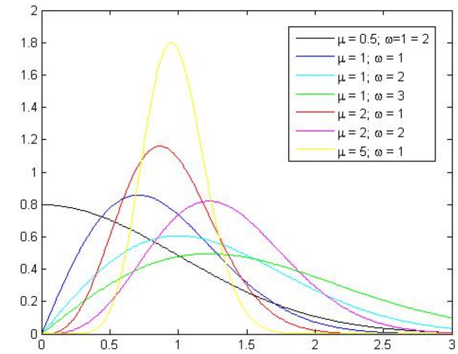


Figure 5.9: Nakagami probability density function.

Its pdf is given as,

$$f_R(r) = \frac{2r^{m-1}}{\Gamma(m)} (\frac{m^m}{\Omega^m}) e^{\frac{-mr^2}{\Omega}} \tag{5.60}$$

where,

$\Gamma(m)$ is the gamma function

Ω is the average signal power and

m is the fading factor.It is always greater than or equal to 0.5.

When m=1, Nakagami model is the Rayleigh model.

When

$$m = \frac{(M+1)^2}{2M+1}$$

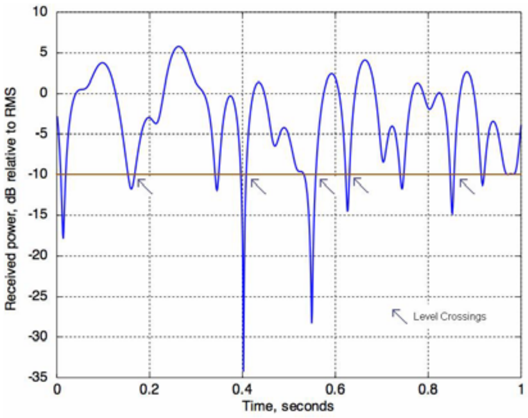


Figure 5.10: Schematic representation of level crossing with a Rayleigh fading envelope at 10 Hz Doppler spread.

where

$$M = \frac{A}{2\sigma}$$

Nakagami fading is the Rician fading.

As m $\rightarrow \infty$ Nakagami fading is the impulse channel and no fading occurs.

5.5.4 Second Order Statistics

To design better error control codes, we have two important second order parameters of fading model, namely the **level crossing rate** (LCR) and **average fade duration** (AFD). These parameters can be utilized to assess the speed of the user by measuring them through the reverse channel. The LCR is the expected rate at which the Rayleigh fading envelope normalized to the local rms amplitude crosses a specific level 'R' in a positive going direction.

$$N_R = \int_0^\infty \dot{r} p(R, \dot{r}) d\dot{r} = \sqrt{2\pi} f_D \rho e^{-\rho^2} \tag{5.61}$$

where $\dot{r}$ is the time derivative of r(t), f_D is the maximum Doppler shift and ρ is the value of the specified level R, normalized to the local rms amplitude of the fading envelope.

The other important parameter, AFD, is the average period time for which the receiver power is below a specified level R.

$$\bar{\tau} = \frac{1}{N_r} P_r(r \leq R) \tag{5.62}$$

As

$$P_r(r \leq R) = \int_0^R p(r)dr = 1 - e^{-\rho^2}, \tag{5.63}$$

therefore,

$$\bar{\tau} = \frac{1 - e^{-\rho^2}}{\sqrt{2\pi} f_D \rho e^{-\rho^2}} \tag{5.64}$$

$$= \frac{e^{-\rho^2} - 1}{\sqrt{2\pi} f_D \rho}. \tag{5.65}$$

Apart from LCR, another parameter is fading rate, which is defined as the number of times the signal envelope crosses the middle value (r_m) in a positive going direction per unit time. The average rate is expressed as

$$N(r_m) = \frac{2v}{\lambda}. \tag{5.66}$$

Another statistical parameter, sometimes used in the mobile communication, is called as depth of fading. It is defined as the ratio between the minimum value and the mean square value of the faded signal. Usually, an average value of 10% as depth of fading gives a marginal fading scenario.

5.6 Simulation of Rayleigh Fading Models

5.6.1 Clarke's Model: without Doppler Effect

In it, two independent Gaussian low pass noise sources are used to produce in-phase and quadrature fading branches. This is the basic model and is useful for slow fading channel. Also the Doppler effect is not accounted for.

5.6.2 Clarke and Gans' Model: with Doppler Effect

In this model, the output of the Clarke's model is passed through Doppler filter in the RF or through two initial baseband Doppler filters for baseband processing as shown in Figure 5.11. Here, the obtained Rayleigh output is flat faded signal but not frequency selective.

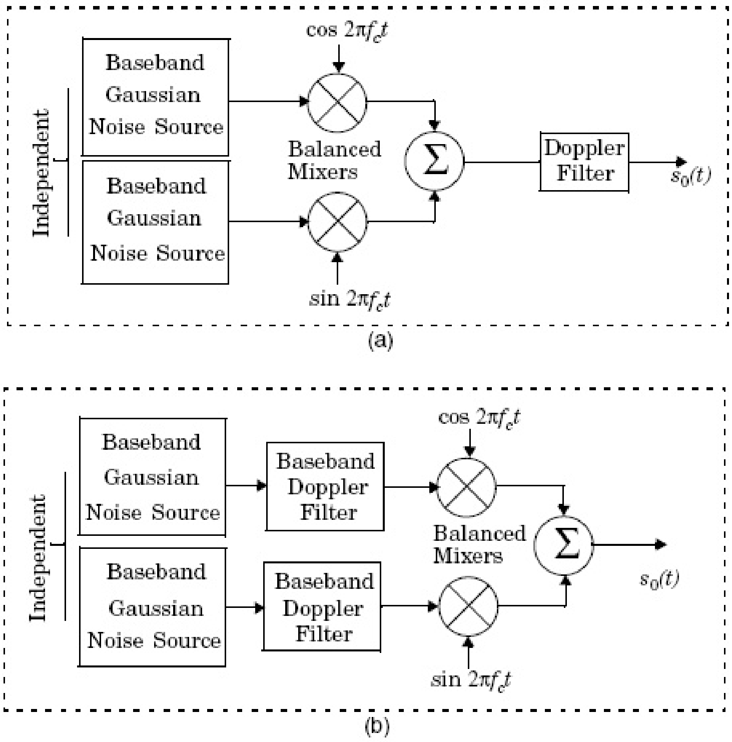


Figure 5.11: Clarke and Gan's model for Rayleigh fading generation using quadrature amplitude modulation with (a) RF Doppler filter, and, (b) baseband Doppler filter.

5.6.3 Rayleigh Simulator with Wide Range of Channel Conditions

To get a frequency selective output we have the following simulator through which both the frequency selective and flat faded Rayleigh signal may be obtained. This is achieved through varying the parameters a_i and τ_i, as given in Figure 5.12.

5.6.4 Two-Ray Rayleigh Faded Model

The above model is, however, very complex and difficult to implement. So, we have the two ray Rayleigh fading model which can be easily implemented in software as shown in Figure 5.13.

$$h_b(t) = \alpha_1 e^{j\phi_1}\delta(t) + \alpha_2 e^{j\phi_2}\delta(t-\tau) \tag{5.67}$$

where α_1 and α_2 are independent Rayleigh distributed and ϕ_1 and ϕ_2 are independent and uniformaly distributed over 0 to 2π. By varying τ it is possible to create a wide range of frequency selective fading effects.

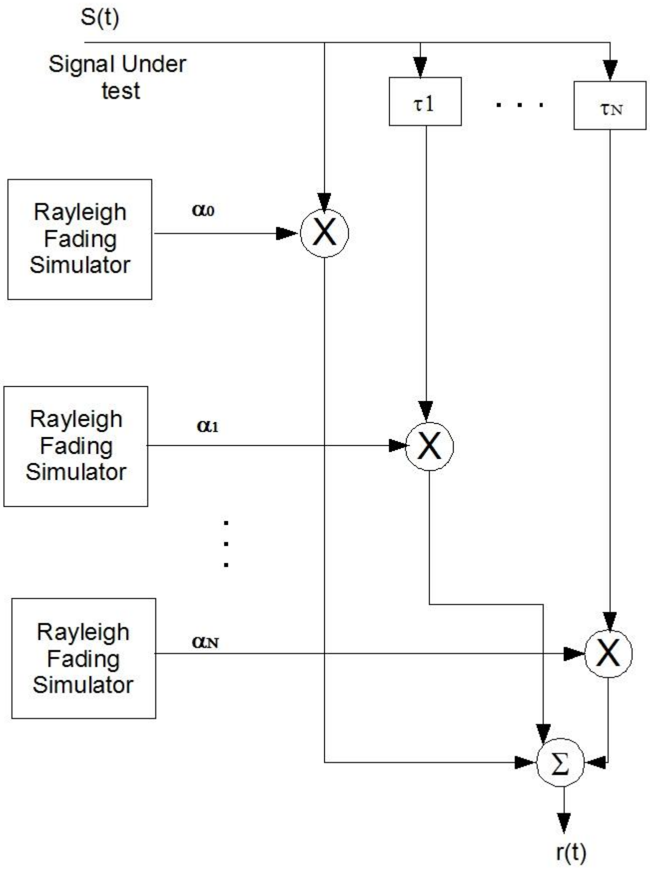


Figure 5.12: Rayleigh fading model to get both the flat and frequency selective channel conditions.

5.6.5 Saleh and Valenzuela Indoor Statistical Model

This method involved averaging the square law detected pulse response while sweeping the frequency of the transmitted pulse. The model assumes that the multipath components arrive in clusters. The amplitudes of the received components are independent Rayleigh random variables with variances that decay exponentially with cluster delay as well as excess delay within a cluster. The clusters and multipath components within a cluster form Poisson arrival processes with different rates.

5.6.6 SIRCIM/SMRCIM Indoor/Outdoor Statistical Models

SIRCIM (Simulation of Indoor Radio Channel Impulse-response Model) generates realistic samples of small-scale indoor channel impulse response measurements. Sub-

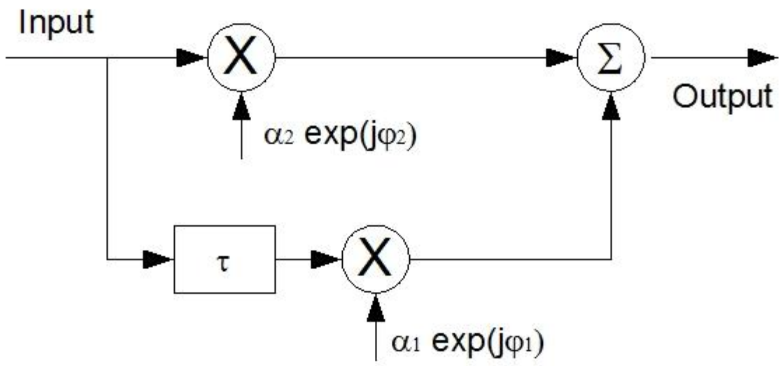


Figure 5.13: Two-ray Rayleigh fading model.

sequent work by Huang produced SMRCIM (Simulation of Mobile Radio Channel Impulse-response Model), a similar program that generates small-scale urban cellular and micro-cellular channel impulse responses.

5.7 Conclusion

In this chapter, the main channel impairment, i.e., fading, has been introduced which becomes so severe sometimes that even the large scale path loss becomes insignificant in comparison to it. Some statistical propagation models have been presented based on the fading characteristics. Mainly the frequency selective fading, fast fading and deep fading can be considered the major obstruction from the channel severity view point. Certain efficient signal processing techniques to mitigate these effects will be discussed in Chapter 7.

5.8 References

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