DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA747: Measure Theory — Quiz I
Instructor: Rajesh Srivastava — February 23, 2016
Time duration: 1.5 hours — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Does there exist an unbounded set $A \subset \mathbb{R}$ such that $m^*(A) = 0$ but $m(\overline{A}) = 1$, where $\overline{A}$ denotes the usual closure of the set A? **1**

   (b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be linear map. Does it imply that the image of an F_σ-set in $\mathbb{R}^2$ is an F_σ-set ? **1**

2. Show that $E \subset \mathbb{R}$ is Lebesgue measurable if and only if for each $\epsilon > 0$ there exist close set F and open set O such that $F \subseteq E \subseteq O$ and $m(O \setminus F) < \epsilon$. **2**

3. Let $E \in M(\mathbb{R})$ with $m(E) < \infty$. For O to be a finite union of open intervals, show that $f(x) = m(E \cap (O + x))$ is a uniformly continuous function on $\mathbb{R}$. **2**

4. Let $E \in M(\mathbb{R})$ and $m(E) > 0$. Show that for each $n \in \mathbb{N}$, there exists an open interval I such that $(n+1)m(E \cap I) \geq m(I)$. **2**

5. Let $E = \mathbb{N} \times \{0\}$ and let $O_n = \left\{x \in \mathbb{R}^2 : d(x, E) < \frac{1}{n}\right\}$. Show that $\lim_{n\to\infty} m(O_n) = \infty$ and $m\left(\bigcap_{n=1}^{\infty} O_n\right) = 0$. **2**

END