DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA747: Measure Theory — Quiz II
Instructor: Rajesh Srivastava — April 20, 2016
Time duration: 2.0 hours — Maximum Marks: 14

N.B. Answer without proper justification will attract zero mark.

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1. (a) Let $f_n(t) = t^n$. Does $\int_B f_n \, dm \to 0$ for every Borel set B in $[0, 1]$? **[1]**

   (b) Does there exist $f \in L^1(\mathbb{R}, M, m)$ such that f is unbounded over a set of positive measure? **[1]**

   (c) If $L^+(X, S, \mu)$ consists of only simple functions, then what is the cardinality of σ-algebra S? **[1]**

2. Let $f, g, fg \in L^1(X, S, \mu)$. Show that the set function $\lambda : S \to [0, \infty]$ defined by $\lambda(E) = \int_E f^+ d\mu$ is a measure on (X, S) and $\int_X g \; d\lambda = \int_X f^+ g d\mu$. **[1+3]**

3. Let $f \in L^1(X, S, \mu)$ be arbitrary and let $E_n = \{x \in X : |f(x)| \geq n\}$. If $0 < p \leq 1$, then show that $\lim_{n\to\infty} n^p \, \mu(E_n) = 0$. Does the conclusion hold if $p > 1$? **[2+2]**

4. Let $f_n(t) = t \, e^{-n|t|} \chi_{[n,n+1)}(t)$. Show that $\lim_{n\to\infty} \int_{\mathbb{R}} f_n \; dm$ exists and is equal to zero. **[3]**

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