DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Course: MA6001: Real Analysis - I
Instructor: Rajesh Srivastava
Duration: 1.5 hours

Quiz II
Date: November 4, 2026
Maximum Marks: 10

Note: Answers lacking rigorous justification will not be awarded marks.

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1. (a) Does there exist a continuous function $f : \mathbb{R} \to \mathbb{R}^2$ such that $f(\sin n) = (n, \frac{1}{n})$ for all $n \in \mathbb{N}$? **1**

   (b) Let $f : [1, \infty) \to [1, \infty)$ be defined by $f(x) = x + \frac{1}{x}$. Whether f is a contraction map? **1**

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = \begin{cases} \frac{x\sqrt{x^2+y^2}}{|y|} & \text{if } y \neq 0, \\ 0 & \text{if } y = 0. \end{cases}$
   Show that f has all directional derivative at $(0, 0)$ but not continuous at $(0, 0)$. Further, show that the map $\varphi : S^1 \to \mathbb{R}$ defined by $\varphi(v) = D_v f(0, 0)$ is onto. **2**

3. Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a differetiable map whose each of the partial derivative is bounded by 1. Show that $\|f(x) - f(y)\|_2 \leq n\|x - y\|_2$. **2**

4. Let $f : \mathbb{R}^n \to \mathbb{R}$ be continuous such that $f(x) > \|x\|$ for all $x \in \mathbb{R}^n$. Show that $f^{-1}(K)$ is compact, whenever K is compact in $\mathbb{R}$. **2**

5. Show that the system of equations $xy - 1 = 0$ and $y^2 + z^2 - 1 = 0$ can be solved for x and y in terms of z near $(2, \frac{1}{2}, \frac{\sqrt{3}}{2})$ as $x = \varphi(z)$ and $y = \psi(z)$. Further, find the values of $\varphi'(\frac{\sqrt{3}}{2})$ and $\psi'(\frac{\sqrt{3}}{2})$. **2**

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