MA550: Measure Theory   Quiz - I
Instructor: Rajesh Srivastava   February 17, 2024
Time duration: 1.5 hours   Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

---

1. (a) Whether the complement of the Cantor set C in $[0,1]$ is dense in $[0,1]$? $\boxed{1}$

   (b) Let $A_n \subseteq \mathbb{R}$ be sequence of sets such that $n^2 > m^*(A_n) > n$. It is possible that $m^*\left(\bigcup\limits_{n=1}^{\infty} \frac{1}{2^n} A_n\right) < \infty$? $\boxed{1}$

2. Show that $\sigma\{(a,b) : a,b \in \mathbb{Q}\} = \sigma\{K \subset \mathbb{R} : K \text{ is compact}\}$. $\boxed{1}$

3. Let $\mathcal{A} = \{A \subseteq \mathbb{R} : A \text{ is finite or } A^c \text{ is finite}\}$. Define a set function μ_o on $\mathcal{A}$ such that

$$\mu_o(A) = \begin{cases} 0 & A \text{ is finite;} \\ 1 & A^c \text{ is finite.} \end{cases}$$

   Prove/disprove that μ_o is a pre-measure on $\mathcal{A}$. $\boxed{2}$

4. If E is subset of $[1,2]$ such that $m^*(E) = 0$, then Show that $E' = \{x + x^2 : x \in E\}$ is measurable. $\boxed{3}$

5. For $A \subseteq \mathbb{R}$, show that Lebesgue outer measure of A can be given by $m^*(A) = \inf\left\{\sum\limits_{n=1}^{\infty} l(I_n) : A \subseteq \cup I_n, I_n \text{ are open intervals with } l(I_n) < 1\right\}$. $\boxed{2}$

END