DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA550: Measure Theory — Quiz - II
Instructor: Rajesh Srivastava — April 16, 2024
Time duration: 1.5 hours — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Let $f : (X, S, \mu) \to [0, \infty]$ be such that $\mu\{x \in X : f(x) > n\} < \sin \frac{1}{n}$ for all $n \geq 1$. Does it imply that $\{x \in X : f(x) = \infty\}$ is a measurable set? **[1]**

   (b) Does there exist a sequence of Lebesgue measurable functions f_n on $[0, \pi]$ to $\mathbb{R}$ such that $\int_{[0,\pi]} f_n dm \to 0$, but $\lim \int_{[0,\pi]} |f_n| dm > 0$? **[1]**

2. Let $f : \mathbb{R} \to [0, \infty]$ be such that for each $\epsilon > 0$, there exists a closed set F such that $m(F) < \epsilon$ and f is continuous on $\mathbb{R} \setminus F$. Show that f is Lebesgue measurable. **[2]**

3. Let μ_n be sequence of measures on (X, S), and write $\mu(E) = \sum_{n=1}^{\infty} \mu_n(E)$ for each $E \in S$. If $f : X \to [0, \infty]$ is such that $\int_X f d\mu_n \leq \frac{1}{n^2}$ for each n$\in \mathbb{N}$, then show that $f \in L^1(\mu)$. **[2]**

4. Let $f, f_n : \mathbb{R} \to \bar{\mathbb{R}}$ be such that $f_n \to f$ point wise and $\inf_{n \geq 1} \int_{\mathbb{R}} |f_n| dm \leq L < \infty$. Show that $f \in L^1(m)$. **[2]**

5. Evaluate the Lebesgue integral $\lim_{n\to\infty} \int_{[0,\infty)} \frac{ne^{-x}}{n+x} dm$. **[2]**

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