DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA549: Topology — EndSem
Instructor: Rajesh Srivastava — November 21, 2023
Time duration: 03 hours — Maximum Marks: 50

N.B. Answer without proper justification will attract zero mark.

---

1. (a) Wether every continuous function from a topological space X onto topological space Y is a quotient map? **[1]**
   (b) Is it possible to extend every continuous function on a topological space X into its one-point compactification? **[1]**
   (c) Whether $\{\frac{1}{n} : n \in \mathbb{N}\}$ is compact with respect to K-topology on $\mathbb{R}$? **[1]**
   (d) Whether a non-constant continuous image of compact and connected topological space X into $\mathbb{R}$ is a closed interval? **[1]**
   (e) Let A be a countable dense set in an uncountable topological space X. Does it necessarily implies that X is connected? **[1]**
   (f) Let $f(z) = \sin z$. Wether $\mathbb{C} \setminus f^{-1}\{0\}$ is path connected in $\mathbb{C}$? **[1]**
   (g) Does every compact topological space is normal? **[1]**

2. Find continuous maps which transfer the torus onto a cylinder, and the cylinder into rectangle $[0,1] \times [0,1]$. **[3]**

3. Let X be a topological space. Let $\mathcal{F} = \{A \subsetneq X : A \text{ is compact in } X\}$. If $\mathcal{F} = P(X) \setminus \{X\}$, then show that X is compact. **[4]**

4. Let X and Y be two topological spaces. If $f : X \to Y$ be such that $G_f = \{(x, f(x)) : x \in X\}$ is closed. Prove/disprove that f is continuous. **[4]**

5. Construct an example of non-compact metric spaces in which every open cover has Lebesgue number. **[3]**

6. Show that Cantor ternary set has no isolated point and totality disconnected. **[3]**

7. Let X be a connected topological space. If there exists a non-constant continuous function $f : X \to [0,1]$, then show that X is uncountable. **[4]**

8. Let A be a subspace of a topological space X. Let $: A \to X$ be continuous. Show that if graph of f is connected, then A is connected. Does converse hold true? **[5]**

9. Let $Gl_n(\mathbb{R})$ denote the topological space of all non-singular $n \times n$ matrices. Find all connected components of $Gl_n(\mathbb{R})$ with respect to subspace topology inherited from the usual topology on $\mathbb{R}^{n^2}$. **[3]**

10. Let $\mathbb{R}_l$ denote the lower limit topology on $\mathbb{R}$. Show that $\mathbb{R}_l^2 = \mathbb{R}_l \times \mathbb{R}_l$ is not a Lindelöf space. **4**

11. Let X be topological space. If $x_n \in A \subset X$ is such that $x_n \to x$, then $x \in \bar{A}$. Does converse hold true? **3**

12. Let X be a Hausdorff space. If $f : X \to X$ is continuous and satisfying $f \circ f = f$, then show that $f(X)$ is closed. **4**

13. Let f be a continuous function on $[a, b]$. Use Urysohn's lemma to deduce that for each $\epsilon > 0$, there exists a continuous function g on $\mathbb{R}$ such that $g = f$ on $[a, b]$ and $g = 0$ on complement of an open set O containing $[a, b]$. **3**

END