DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA4128: Measure Theory	Quiz - I
Instructor: Rajesh Srivastava	September 5, 2026
Time duration: 1.5 hours	Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Let $E \in \mathcal{M}(\mathbb{R})$ and $m(E) < \infty$. Is it possible that $m(E \cap (n, n+1)) \geq 1$ for all $n \in \mathbb{Z}$? $\boxed{\mathbf{1}}$

   (b) If $E \subset \mathbb{R}$ is Lebesgue measurable, does it imply that $\bigcup_{x \in E} [x-1, x+1]$ is Lebesgue measurable? $\boxed{\mathbf{1}}$

2. Let S be a σ-algebra on a non-empty set X. For $x_1, x_2 \in X$, define $\delta_{x_i} : S \to [0, \infty]$ by $\delta_{x_i}(A) = \begin{cases} 0 & \text{if } x_i \notin A, \\ 1 & \text{if } x_i \in A. \end{cases}$
   Show that $\mu = \delta_{x_1} + \delta_{x_2}$ is a measure on S. Is $\nu = \delta_{x_1} - \delta_{x_2}$ a measure on S? $\boxed{\mathbf{2}}$

3. For $E \subset (0, 1)$ write $E^2 = \{x^2 : x \in E\}$. Show that $m^*(E^2) \leq 2m^*(E)$. $\boxed{\mathbf{2}}$

4. Let C be the Cantor ternary set. Show that for each pair of points $x, y \in C$, there exists $z \notin C$ such that $x < z < y$. $\boxed{\mathbf{2}}$

5. Let (X, S, μ) be a measure space. For a sequence of sets $A_n \in S$, if we define $\underline{\lim} A_n = \cup_{k \geq 1} (\cap_{n \geq k} A_n)$, then show that $\mu(\underline{\lim} A_n) \leq \underline{\lim}\, \mu(A_n)$. $\boxed{\mathbf{2}}$

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