DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA4128: Measure Theory — Quiz - II
Instructor: Rajesh Srivastava — November 1, 2026
Time duration: 1.5 hours — Maximum Marks: 10

N.B. Answer without proper justification will attract zero mark.

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1. (a) Does there exist $f \in L^1(\mathbb{R}, M, m)$ such that f is unbounded over a set of positive measure? **1**

   (b) Let $f_n(t) = t^n$. Does $\int\limits_B f_n \, dm \to 0$ for every Borel set B in $[-1, 1]$? **1**

   (c) If $L^+(X, S, \mu)$ consists of only simple functions, then what is the cardinality of σ-algebra S? **1**

2. Let $f, g, fg \in L^1(X, S, \mu)$. Show that the set function $\lambda : S \to [0, \infty]$ defined by $\lambda(E) = \int\limits_E f^+ d\mu$ is a measure on (X, S) and $\int\limits_X g \, d\lambda = \int\limits_X f^+ g d\mu$. **3**

3. Let $f \in L^1([0, \infty), M, m)$ be continuous. Prove that the function F defined by $F(x) = \int\limits_{[0,x)} f \, dm$ is differentiable and $F' = f$. **2**

4. Let $f_n(t) = t\, e^{-n|t|} \chi_{[n,n+1)}(t)$. Show that $\lim\limits_{n\to\infty} \int\limits_{\mathbb{R}} f_n \, dm$ exists and is equal to zero. **2**

END