DEPARTMENT OF MATHEMATICS
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

Course: MA224: Real Analysis — **Quiz II**
Instructor: Rajesh Srivastava — **Date:** April 17, 2026
Duration: 1.5 hours — **Maximum Marks:** 10

Note: Answers lacking proper justification will not be awarded marks.

---

1. (a) Let O be an open subset of $\mathbb{R}^2$. Let $f : O \to \mathbb{R}$ be such that $f_x(X) = f_y(X) = 0$ for all $X \in O$. Does it imply that f is constant on O? **[1]**

   (b) Does there exist a linear transformation $A : \mathbb{R}^2 \to \mathbb{R}^2$ such that the strict inequality $\|A^2\| < \|A\|^2$ holds? **[1]**

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be map $f(x,y) = \begin{cases} \sqrt{x^2+y^2}\ \sin\left(\dfrac{y^2}{x-y}\right) & \text{if } x \neq y, \\ 0 & \text{if } x = y. \end{cases}$

   Show that f is continuous at $(0,0)$. Does f differentiable at $(0,0)$? **[2]**

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be map $f(x,y) = \begin{cases} (x+y)\log(x^2+y^2) & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

   Show that f is continuous at $(0,0)$. Find all possible directions $\mathbf{v}$ in which the directional derivative $D_{\mathbf{v}} f(0,0)$ exist. **[1]**

4. Let $\mathbb{D} = \{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 < 1\}$. Define a function $f : \mathbb{D} \to \mathbb{R}$ by $f(x_1, x_2) = \sin(x_1^2 + x_2)$. Show, by using appropriate MVT, that

$$|f(X) - f(Y)| \leq \sqrt{5}\|X - Y\|_2$$

   for every $X, Y \in \mathbb{D}$. **[2]**

5. Show that equation $x^2 + yz - \cos(xz) = 0$ can be solved for x in some neighborhood of $(1,1,0)$. Whether it can be solved for y in a neighborhood of $(1,1,0)$? **[3]**

END