DEPARTMENT OF MATHEMATICS
Indian Institute of Technology Guwahati

MA101S: Mathematics-I	MidSem
Instructor: Rajesh Srivastava	June 10, 2018
Time duration: 02 hours	Maximum Marks: 40

N.B. Answer without proper justification will attract zero mark.

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1. (a) What is the infimum of the set $A = \left\{ e^{-n} + \frac{(-1)^n}{n} \,:\, n \in \mathbb{N} \right\}$? **[1]**

   (b) Does there exist a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f(e^{-n^2}) = f(\cos n)$ for all $n \in \mathbb{N}$? **[1]**

   (c) Let $f : (0,1) \to \mathbb{R}$ be differentiable. For $c \in (0,1)$ to be a point of inflection, is it necessary that $f''(c) = 0$? **[1]**

   (d) Does there exist a power series $\sum a_n x^n$ that converges only at two points in $\mathbb{R}$? **[1]**

   (e) Let $f : [a,b] \to \mathbb{R}$ be continuous and satisfying $\int_a^x f(t)dt = \int_x^b f(t)dt$. Does it imply that f is constant? **[1]**

2. Whether the series $\sum_{n=1}^{\infty} \frac{3^n + 2^{n+1}}{5^n}$ is convergent? If yes, find the sum of the series. **[2]**

3. Find all $\alpha \in \mathbb{R}$ such that the sequence $x_n = \sqrt{(n+1)^\alpha - n^\alpha}$ is convergent. **[3]**

4. Determine all values of $x \in \mathbb{R}$ such that the power series $\sum\limits_{n=2}^{\infty} \frac{(x-4)^n}{n(\log n)^2}$ is convergent. **[4]**

5. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable at $x = 0$ and $f'(0) > 0$. If $f(0) = 0$, then show that there exists $\delta > 0$ such that $f(x) \neq 0$ for all $x \in (-\delta, \delta) \setminus \{0\}$. **[4]**

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $|f(x)| < 1$ for all $x \in \mathbb{R}$. Prove that there exists $c \in \mathbb{R}$ such that $f^2(c) + f^4(c) = 2c$. **[4]**

7. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) = x^3 + 1$ for for all $x \in \mathbb{Q}$. Find the value of $f(\sqrt{2}) + f(\sqrt{3})$. **[3]**

8. Let f be a continuous function on $[0,1]$ and differentiable on $(0,1)$. If $f'(x) > f(x)$ for all $x \in (0,1)$ and $f(0) = 0$, then show that $f(x) > 0$ for all $x \in (0,1]$. **[4]**

9. Find the Taylor series of $\cos x$ around $x = 0$ that converges to $\cos x$ on $(-1,1)$. **[4]**

10. Let $f, g : \mathbb{R} \to \mathbb{R}$ be two functions satisfying $|f(x)| \leq |g(x)|$ for all $x \in (-\delta, \delta)$ and for some $\delta > 0$. If g is differentiable at 0 and $g'(0) = 0 = g(0)$, then show that f is differentiable at 0. **[3]**

11. Examine whether the improper integral $\int\limits_0^{\infty} \frac{dx}{2x^2 + \sqrt{x}}$ is convergent? **[4]**

END