**DEPARTMENT OF MATHEMATICS**
**Indian Institute of Technology Guwahati**

MA101S: Mathematics-I — EndSem
Instructor: Rajesh Srivastava — July 8, 2018
Time duration: 03 hours — Maximum Marks: 55

**N.B.** Answer without proper justification will attract zero mark.

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1. (a) Does the image of a circle under any $2 \times 2$ invertible matrix is a circle? **[1]**

   (b) Let $A$ be a $m \times n$ matrix such that $Ax = b$ has two solutions for every $b \in \mathbb{R}^m$. Does it imply that $Ay = 0$ has infinitely many solutions? **[1]**

   (c) Does there exist a non-zero diagonalizable matrix having a zero eigenvalue? **[1]**

   (d) Let $f : [-1, 1] :\to \mathbb{R}$ be defined by $f(x) = 0$ if $-1 \leq x < 0$ and $f(x) = 1$ if $0 \leq x \leq 1$. Let $F(x) = \int_{-1}^{x} f(t)dt$. Whether $F$ is differentiable at $x = 0$? **[1]**

2. . Let $A = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix}$. Find a matrix $E = (e_{ij})_{3\times 3}$ with $e_{ij} \in \{0, 1\}$ such that $EA$ is an upper triangular matrix. **[3]**

3. Show that a square matrix $A$ of order $n$ is invertible if and only if $\det(A) \neq 0$. **[4]**

4. Let $\{x_0, x_1, \ldots, x_n\}$ be a set in $\mathbb{R}$. Show that there exists a polynomial $p$ of degree $n$ that satisfies $p(x_i) = i;\ i = 0, 1, \ldots, n$. **[3]**

5. Let $W_1 = \{(x, y, z) : x + y + z = 0\}$ and $W_2 = \{(x, y, z) : x + 2y + 3z = 0\}$. Show that $W_1 + W_2 = \mathbb{R}^3$. What is the dimension of $W_1 \cap W_2$? **[4]**

6. Let $\mathbb{P}_2(\mathbb{R})$ be the space of all polynomials of degree at most 2. Find the co-ordinates of $1 + 2x + x^2$ with respect to $\{1 + x, 1 - x, 1 - x + x^2\}$. **[3]**

7. Let $A$ be an $n \times n$ invertible matrix and let $\{v_1, \ldots, v_n\}$ a be basis for $\mathbb{R}^n$. Then show that $\mathbb{R}^n = \text{span}\{Av_1, \ldots, Av_n\}$. **[2]**

8. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T(1, 0, 0) = (1, 1, 0)$, $T(1, 1, 0) = (0, 1, 0)$ and $T(1, 1, 1) = (1, 2, 0)$. Find $R(T)$ and $N(T)$. **[4]**

9. Find a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $R(T) = \text{span}\{(1, 2, 3), (3, 2, 1)\}$ and $N(T) = \text{span}\{(1, 1, 0)\}$. **[3]**

10. Let $\mathbb{R}^3$ be equipped with the usual inner product $\langle (x, y, z), (x', y', z') \rangle = xx' + yy' + zz'$. Find a basis for the orthogonal complement of the set $\{(1, 1, 1), (2, 1, 0)\}$. **[4]**

11. Let $\langle ., . \rangle$ be the usual inner product on $\mathbb{R}^2$. If $A = (a_{ij})_{2\times 2}$ matrix satisfies $\langle Ax, x \rangle = 0$ for all $x \in \mathbb{R}^2$, then show that $a_{11} = a_{22} = 0$ and $a_{12} + a_{21} = 0$. **5**

12. Let $A, B$ and $C$ be real symmetric square matrices of order $n$ such that $A^2+B^2+C^2 = 0$. Show that $A = B = C = 0$. **4**

13. Show that the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$ is diagonalizanle. Whether $A$ is a nilpotent matrix? **5**

14. Find the $\lim\limits_{n\to\infty} \frac{n}{n^2+1} \left\{ \sin\frac{1}{n} + \sin\frac{2}{n} + \cdots + \sin\frac{n}{n} \right\}$. **3**

15. Let $f$ and $g$ be Riemann integrable functions on $[0, 1]$ such that $\int_0^1 f(t)dt = \int_0^1 g(t)dt$. If $h$ is a function on $[0, 1]$ satisfying $f(t) \leq h(t) \leq g(t)$ for all $t \in [0, 1]$, then show that $h$ is Riemann integrable on $[0, 1]$. **4**

**END**